



Arizona's Draft Standards Mathematics

Sixth Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
Draft Standards for Public Comment

Sixth Grade Overview

Ratios and Proportional Relationships (RP)

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System (NS)

- Apply and extend previous understandings of multiplication and division to divide fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations (EE)

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry (G)

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability (SP)

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Sixth Grade: Critical Areas

In sixth grade, instructional time should focus on five critical areas:

- 1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.**
- 2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers.**
- 3. Writing, interpreting, and using expressions and equations.**
- 4. Developing understanding of statistical thinking.**
- 5. Reasoning about area, surface area and volume.**

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and, by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus, students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets

of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

(5) Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Sixth Grade: Mathematics Standards

Ratios and Proportional Relationships (RP)

6.RP.A	Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)
6.RP.A.3	Use ratio and rate reasoning to solve mathematical problems and problems in a real-world context. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System (NS)

6.NS.A	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.A.1	Interpret and compute quotients of fractions to solve mathematical problems and problems in a real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. (In general, $(a/b) \div (c/d) = ad/bc$.)

6.NS.B	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.B.2	Fluently divide multi-digit numbers using a standard algorithm.
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.
6.NS.B.4	<p>Understand the greatest common factor, understand the least common multiple, and use the distributive property.</p> <p>a. Find the greatest common factor of two whole numbers less than or equal to 100.</p> <p>b. Find the least common multiple of two whole numbers less than or equal to 12.</p> <p>c. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.</p>
6.NS.C	Apply and extend previous understandings of numbers to the system of rational numbers.
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.C.6	<p>Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p>

6.NS.C.7	<p>Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line.</p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.</p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world contexts.</p> <p>d. Distinguish comparisons of absolute value from statements about order, especially when considering values in context.</p>
6.NS.C.8	<p>Solve mathematical problems and problems in a real-world context by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>
<u>Expressions and Equations (EE)</u>	
6.EE.A	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.
6.EE.A.2	<p>Write, read, and evaluate algebraic expressions.</p> <p>a. Write expressions that record operations with numbers and variables.</p> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity.</p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in a real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</p>
6.EE.A.3	Apply the properties of operations to generate equivalent expressions.
6.EE.A.4	Identify when two expressions are equivalent.

6.EE.B	Reason about and solve one-variable equations and inequalities.
6.EE.B.5	Understand solving an equation or inequality as a process of reasoning to find the value(s) which make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.B.6	Use variables to represent numbers and write expressions to solve mathematical problems and problems in a real-world context; understand that a variable can represent an unknown number or any number in a specified set.
6.EE.B.7	Solve mathematical problems and problems in a real-world context by writing and solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and $p/x = q$ for cases in which p , q and x are all non-negative rational numbers.
6.EE.B.8	Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in a real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number line.
6.EE.C	Represent and analyze quantitative relationships between dependent and independent variables.
6.EE.C.9	Use variables to represent two quantities to solve mathematical problems and problems in a real-world context that change in relationship to one another; write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

<u>Geometry (G)</u>	
6.G.A	Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.A.1	Find the area of polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in a real-world context.
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \cdot w$) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in a real-world context.
6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context.
6.G.A.4.	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in a real-world context.
<u>Statistics and Probability (SP)</u>	
6.SP.A	Develop understanding of statistical variability.
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers.
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a variation measurement uses a single number to describe the spread of the data set.

6.SP.B	Summarize and describe distributions.
6.SP.B.4	Display and interpret numerical data in plots on a number line including dot plots, histograms, and box plots.
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: <ul style="list-style-type: none"> a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
6.MP	Standards for Mathematical Practice
6.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.

6.MP.2	<p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>
6.MP.3	<p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p>
6.MP.4	<p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>

6.MP.5	<p>Use appropriate tools strategically.</p> <p>Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.</p>
6.MP.6	<p>Attend to precision.</p> <p>Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.</p>
6.MP.7	<p>Look for and make use of structure.</p> <p>Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.</p>
6.MP.8	<p>Look for and express regularity in repeated reasoning.</p> <p>Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.</p>