



# Arizona's Draft Standards Mathematics

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## High School Plus Standards

ARIZONA DEPARTMENT OF EDUCATION  
HIGH ACADEMIC STANDARDS FOR STUDENTS  
Draft Standards for Public Comment

## Plus Standards Overview

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### NUMBER AND QUANTITY -N

#### The Complex Number System (N-CN)

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.
- Use complex numbers in polynomial identities and equations.

#### Vector and Matrix Quantities (N-VM)

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

### ALGEBRA (A)

#### Arithmetic with Polynomials and Rational Expressions (A-APR)

- Use polynomial identities to solve problems.
- Rewrite rational expressions.

#### Reasoning with Equations and Inequalities (A-REI)

- Solve systems of equations.

### FUNCTIONS (F)

#### Interpreting Functions (F-IF)

- Analyze functions using different representations.

#### Building Functions (F-BF)

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

#### Trigonometric Functions (F-TF)

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.

### GEOMETRY (G)

#### Similarity, Right Triangles, and Trigonometry (G-SRT)

- Apply trigonometry to general triangles.

#### Circles (G-C)

- Understand and apply theorems about circles.

#### Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation for a conic section.

#### Geometric Measurement and Dimension (G-GMD)

- Explain volume formulas and use them to solve problems.

### STATISTICS AND PROBABILITY (S)

#### Conditional Probability and the Rules of Probability (S-CP)

- Use the rules of probability to compute probabilities of compound events in a uniform probability model.
- Calculate expected values and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

### CONTEMPORARY MATHEMATICS (CM)

#### Discrete Mathematics (CM-DM)

- Understand and apply vertex-edge graph topics.

### Standards for Mathematical Practices (MP)

## Plus Standards: High School

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The plus (**P**) standards are standards that are found outside the limits of a high school Algebra 1, Algebra 2, or Geometry minimum course of study as defined by the standards. The plus standards are represented through coding that begins with **P** instead of a course designation. The plus standards are intended to be included in honors, accelerated, advanced courses, fourth credit courses, as well as extensions of the regular courses (Algebra 1, Algebra 2, and Geometry).

*The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Mathematical modeling is integrated throughout Algebra 2 course by utilizing real world context.*

DRAFT

<b>Plus Standards</b>	
<b>Number and Quantity -N</b>	
<b><u>The Complex Number System (N–CN)</u></b>	
<b>P.N-CN.A</b>	<b>Perform arithmetic operations with complex numbers.</b>
P.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
<b>P.N-CN.B</b>	<b>Represent complex numbers and their operations on the complex plane.</b>
P.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
P.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument $120^\circ$ .
P.N-CN.B.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
<b>P.N-CN.C</b>	<b>Use complex numbers in polynomial identities and equations.</b>
P.N-CN.C.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .
P.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

<b>Vector and Matrix Quantities (N–VM)</b>	
<b>P.N-VM.A</b>	<b>Represent and model with vector quantities.</b>
P.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$ , $ \mathbf{v} $ , $\ \mathbf{v}\ $ , $v$ ).
P.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
P.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
<b>P.N-VM.B</b>	<b>Perform operations on vectors.</b>
P.N-VM.B.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$ , where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$ , with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
P.N-VM.B.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ . b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\  =  c \mathbf{v} $ . Compute the direction of $c\mathbf{v}$ knowing that when $ c \mathbf{v} \neq 0$ , the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$ ) or against $\mathbf{v}$ (for $c < 0$ ).
<b>P.N-VM.C</b>	<b>Perform operations on matrices and use matrices in applications.</b>
P.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

<b>P.N-VM.C.7</b>	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
<b>P.N-VM.C.8</b>	Add, subtract, and multiply matrices of appropriate dimensions.
<b>P.N-VM.C.9</b>	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
<b>P.N-VM.C.10</b>	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
<b>P.N-VM.C.11</b>	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
<b>P.N-VM.C.12</b>	Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
<b>Algebra - A</b>	
<b>Arithmetic with Polynomials and Rational Expressions (A-APR)</b>	
<b>P.A-APR.A</b>	<b>Use polynomial identities to solve problems.</b>
P.A-APR.C.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)
<b>P.A-APR.D</b>	<b>Rewrite rational expressions.</b>
P.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

<b>Reasoning with Equations and Inequalities (A-REI)</b>	
<b>P.A-REI.C</b>	<b>Solve systems of equations.</b>
P.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.
P.A-REI.C.9	Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).
<b>Functions - F</b>	
<b>Interpreting Functions (F-IF)</b>	
<b>P.F-IF.C</b>	<b>Analyze functions using different representations.</b>
P.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
<b>Building Functions (F-BF)</b>	
<b>P.F-BF.A</b>	<b>Build a function that models a relationship between two quantities.</b>
P.F-BF.A.1	Write a function that describes a relationship between two quantities. c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
<b>P.F-BF.B</b>	<b>Build new functions from existing functions.</b>
P.F-BF.B.4	Find inverse functions. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.

P.F-BF.B.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
<b><u>Trigonometric Functions (F-TF)</u></b>	
<b>P.F-TF.A</b>	<b>Extend the domain of trigonometric functions using the unit circle.</b>
P.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$ , $\pi+x$ , and $2\pi-x$ in terms of their values for $x$ , where $x$ is any real number.
P.F-TF.A.4	Use the units circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
<b>P.F-TF.B</b>	<b>Model periodic phenomena with trigonometric functions.</b>
P.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
P.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
P.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
<b>Geometry - G</b>	
<b><u>Similarity, Right Triangles, and Trigonometry (G-SRT)</u></b>	
<b>P.G-SRT.D</b>	<b>Apply trigonometry to general triangles.</b>
P.G-SRT.D.9	Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
P.G-SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.

P.G-SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
<b><u>Circles G-C</u></b>	
<b>P.G-C.A</b>	<b>Understand and apply theorems about circles.</b>
P.G-C.A.4	Construct a tangent line from a point outside a given circle to the circle.
<b><u>Expressing Geometric Properties with Equations (G-GPE)</u></b>	
<b>P.G-GPE.A</b>	<b>Translate between the geometric description and the equation for a conic section.</b>
P.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
P.G-GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
<b><u>Geometric Measurement and Dimension (G-GMD)</u></b>	
<b>P.G-GMD.A</b>	<b>Explain volume formulas and use them to solve problems.</b>
P.G-GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
<b>Statistics and Probability - S</b>	
<b><u>Making Inferences and Justifying Conclusions (S-IC)</u></b>	
<b>P.S-IC.B</b>	<b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b>
P.S-IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
P.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
P.S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
P.S-IC.B.6	Evaluate reports based on data.

<b>Conditional Probability and the Rules of Probability (S-CP)</b>	
<b>P.S-CP.B</b>	<b>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</b>
P.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$ , and interpret the answer in terms of the model.
P.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
<b>Using Probability to Make Decisions (S-MD)</b>	
<b>P.S-MD.A</b>	<b>Calculate expected values and use them to solve problems.</b>
P.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
P.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
P.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
P.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

<b>P.S-MD.B</b>	<b>Use probability to evaluate outcomes of decisions.</b>
P.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
P.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
P.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
<b>Contemporary Mathematics - CM</b>	
<b>Discrete Mathematics - (CM-DM)</b>	
<b>P.CM-DM.A</b>	<b>Understand and apply vertex-edge graph topics</b>
P.CM-DM.A.1	Study the following topics related to vertex-edge graph: Euler circuits, Hamilton circuits, the Travelling Salesperson Problem (TSP), minimum weight spanning trees, shortest path, vertex coloring, and adjacency matrices.
P.CM-DM.A.2	Understand, analyze, and apply vertex-edge graph to model and solve problems related to path, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.
P.CM-DM.A.3	Devise, analyze, and apply algorithms for solving vertex-edge graph problems.
P.CM-DM.A.4	Extend work with adjacency matrices for graph, such as interpreting row sums and using the $n$ th power of the adjacency matrix to count path of length $n$ in a graph.

P.MP	Standards for Mathematical Practice
P.MP.1	<p>Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.</p>
P.MP.2	<p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>

P.MP.3	<p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p>
P.MP.4	<p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>
P.MP.5	<p>Use appropriate tools strategically.</p> <p>Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.</p>

P.MP.6	<p>Attend to precision.</p> <p>Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.</p>
P.MP.7	<p>Look for and make use of structure.</p> <p>Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.</p>
P.MP.8	<p>Look for and express regularity in repeated reasoning.</p> <p>Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.</p>