Review of Arizona Draft Standards in Mathematics

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Arizona state standards in mathematics, which are currently effectively the Common Core state standards, are undergoing a revision, and the state has produced a draft rewrite. This discussion addresses this (fall of 2016) Arizona draft rewrite.

General

Academic **Content** Standards should, as their name implies, first and foremost address academic content. In other words, they should deal largely with the "what" rather than with the "how." How demanding the standards are depends on the aspirations of those who define them, yet like in many other endeavors we like to compare ourselves with the best educational achievers in the world. In mathematics, this typically means East Asian countries such as Singapore, South Korea, or Japan.

Yet there are another two essential characteristics of good academic standards.

Clarity: without clarity, the document is bound to remain a dead letter on the page, as their readers – teachers, administrators, and parents – will not be able to grasp the underlying content or understand its implications.

Measurability: standards must allow us to measure – relatively accurately, on large-scale assessments – whether students mastered the content that is specified in them.

The above is generally not controversial and – indeed – the **Arizona Standards Revision Process of 2016** mentions them explicitly and prominently.

Yet that document adds a few more requirements on the standards that are more controversial. For example, it makes a baffling statement that they can be used to "***build student understanding***." How can they do that if they strive to be just the content – the "what" – rather than the process, the sequence, the "how"? After all, there is no order among the standards within a grade, and they refer to the point of mastery of a skill – when it can be measured – rather than to its development that can occur in previous months or years. We certainly hope that students will build understanding, but that role should be delegated to the curriculum and the pedagogy that develop students to master the standards.

But there is a much deeper issue with another dimension that the Revision Process document expects from the standards: their cognitive demand. It writes:

- The standard has complexity of reasoning
- The cluster contains a range of cognitive demand/complexity of reasoning

As already mentioned, standards are about content and not about students. Yet complexity of reasoning is a student characteristic, not a characteristic of the content. The standard can be challenging in that it

demands significant prerequisite knowledge, yet this does not make it have "complexity of understanding" – proficient students may demonstrate its effortless mastery.

In fact, this requirement of cognitive demand and complexity of reasoning also flies in the face of the more fundamental need of measurability from the standards. We simply do not have the ability to easily or reliably measure cognitive demand on students, as any competent psychometrician will readily acknowledge. What may seem like an exercise in complex reasoning to one student may be a trivial matter of recall to another that happened to see such problems in the past. What we can measure reliably – and quickly – is what students can DO, but not what goes in their minds while they are doing it. In fact, much of the problems with PARCC and SBAC are directly attributable to this point.

Which brings me to a key problem: the rewrite did not attempt to modify the existing variant of Common Core standards beyond in a trivial way, reflected in adding 3 minor standards in K-3, adding two minor standards in high school, and removing two Arizona-specific standards, where the current standards diverged from the original Common Core. All other efforts of the rewrite went essentially into an effort to "clarify" the existing standards and make them more "measurable." More on that in a moment.

Standards' Expectations and Philosophy

Both the current Arizona standards as well as the draft rewrite reflect essentially the Common Core expectations. Yet the essential flaws of Common Core standards that have been recognized over the passing years both in terms of experience gained, and in view of new findings of cognitive science, have not been addressed in the rewrite at all.

Specifically, I refer to the three following aspects.

- Focus on understanding in early grades and delaying facility with fact memorization.

  Common Core – and Arizona – standards attempt to develop arithmetic understanding and skills with whole numbers and fractions through spending large amount of time – typically a year or even two – in expecting students to apply variety of strategies and modeling to learn to "understand" the four arithmetic operations before expecting students to routinely use calculations with them. In contrast, memorization of the addition facts comes late in grade 2 and multiplication table memorization is expected in grade 3, and using standard algorithms for them is expected in grades 4 and 5 respectively.

  In recent years cognitive science firmly established that because our Working Memory (WM), where we perform our "thinking," can hold only a very small number – on the order of 3-5 – of "facts," it is critical that memorization of facts and storing them into our Long Term Memory (LTM), which is effectively unlimited and readily usable, will come early in the process of learning. In this way students can readily retrieve the math facts necessary to allow them to solve more complex problems, rather than incessantly – and unnecessarily – struggle with them. Here is what the five leading cognitive scientists in the county have said on this:

> *At all ages, there are several ways to improve the functional capacity of working memory. The most central of these is the achievement of automaticity, that is, the fast, implicit, and automatic retrieval of a fact or a procedure from long-term memory … to obtain the maximal benefits of automaticity in support of complex problem solving, arithmetic facts and fundamental algorithms should be thoroughly mastered, and indeed, over-learned, rather than merely learned to a moderate degree of proficiency. [National Advisory Mathematics Panel: Report of the Task Group on Learning Processes, 2008]*

The wisdom of this delay of memorizing facts also came into question in recent years from parents and professional outside education that observed young students struggling and getting frustrated with trivial arithmetic.

- Delaying the standards arithmetic algorithms in the quest for "understanding."

  In a sense, this is the continuation of the effort to develop understanding ahead of facility. Yet it manifests itself in a strange way, where the ability of student to perform a rather conceptually simple operation such as addition in multiple ways is taken as a representation of a deeper cognitive knowledge than the ability to fluently and accurately add. This is akin to expecting a violin student to be able to sound a few simple notes with the bow in his right hand, in his left hand, when the violin leans on the ground, under his chin, or when he lies down, instead of expecting the student to play well music of increasing complexity in a single most effective position. And we see, indeed, classroom students today who worry more about which "strategy" they are expected to use for a given problem, rather than correctly solve the mathematics of the problem in the most obvious and convenient manner.

- Unambitious expectations when compared to educationally high-achieving nations.

  It is broadly acknowledged even by Common Core supporters that high-achieving countries teach algebra (or algebra and geometry) to their eight grade students. Yet, inexplicably, by default Common Core defers the first algebra course to the high school.

  Similarly, the default high school mathematics that Common Core offers (i.e., without the Plus standards) is much below what high achieving countries teach their students. A large number of basic topics, from sequences and series through combinatorics, most of trigonometry, conic sections, and to geometry of circles are eliminated from Common Core regular high school content.

Consequently, tinkering with Common Core – and Arizona – standards language, but without addressing at least some of these major deficiencies, cannot significantly improve the quality of what Arizona will expect from its students, or bring them closer to international expectations. A meaningful rewrite will require shifting some content to lower grades to prepare all seven grade students for an algebra class like our international competitors do, and restoring some of the eliminated high school content to the regular mathematics courses.

Draft Rewrite Characteristics

As already mentioned, rather than address the major problem with the current standards, the draft focused almost exclusively on language editing with the declared goal of clarifying the existing standards and making them more measurable.

Unfortunately, the rewrite falls short even on these unambitious goals. The attached review annotates (red text in the "Notes" column) many, even if not all, of the problems with the various rewriting suggestions. I will touch here on the major issues only, while the interested readers are encouraged to find the details there.

- The Issue of Examples as "Limiting"

  Many of the original standards included examples to illustrate and clarify the meaning of the – rather abstract – language of the standards. With a very few exceptions, these standards were, indeed, illustrative and helpful.

  The authors of the rewrite seem to have decided that an example or two embedded in the standards are in some way "limiting" the standard, and hence eliminated essentially all of them. Perhaps they did not distinguish between the "e.g." meaning (as used by the standards) and the "i.e." meaning. Perhaps something else confused them. Whatever the reason, the result is that the rewrite is much less clear than the original and few practitioners will be able to read the new standards and understand what they actually mean.

  To add insult to injury, in a few cases (e.g., 7.RP.A.3, A2.F-IF.C.8) a list of examples was turned into an exclusive list of what the standard expects, doing precisely what it claimed it was not doing – limiting the standard to just those examples.

  Almost all examples should be restored. That will greatly improve the clarity of the standards.

- The Issue of Examples as "Instructional Practice"

  In a similar way, some examples were eliminated under the claim that they "illustrate instruction" (or are an "instructional practice"). This seems wrong-headed as these are examples, not standards, and illustrating with an example how to do something does not impose it on anyone.

  Restore such illustrative examples.

- Replacement of "the standard algorithms" language by "a standard algorithm."

  The rewrite eliminated the definite article and replaced it by the indefinite one. This seems to promote the vision the there is a multiplicity of "standard" algorithms (for arithmetic). This is what the American Mathematical Society had to say about that:

"It is highly desirable that… nearly everyone…. learn a standard way of doing the four basic arithmetic operations. (The standard algorithms need not be absolutely unique, just as there are variant spellings between, say, the U.S. and England, but too much variation leads to difficulties.) We do not think it is wise for students to be left with untested private algorithms for arithmetic operations…."[Notices of the AMS, Feb. 1998]

The standard algorithms are "standard" because they are unique to each arithmetic operation, and their variations across the world are minimal and cosmetic. Pretending this isn't so is disingenuous, even if it comes from people like Karen Fuson or Sybilla Beckmann.

- Cases of Mathematical Confusion and Inadvertently Changing Standards

  In quite a few instances the rewrite seems to have misunderstood the original standard, or changed its focus through seemingly-innocuous language changes. Details are spread throughout the attached document, but their presence calls into question the competence of the rewrite.

- Missing Content (some examples)

  Solving and manipulating equations that include absolute expressions is missing from Algebra 1, yet facility with, and understanding of, absolute functions is expected in multiple standards.

  Logarithms are essentially completely neglected and students aren't expected to know what they are, how to work with them, or their characteristics. The sum total of knowledge about logarithm is in standard A2.F-LE.A.4 which essentially defines logarithms as "the solution to $ab^{ct}=d$". Yet logarithmic functions make frequent appearance where the standard litany of applicable functions is mentioned. How are the students expected to handle them if they are never taught what logarithms are?

  Arithmetic and geometric sequences are essentially ignored beyond their basic definition. No expressions for general terms or sums are developed.

  Conic sections have been included in any decent Algebra 2 course for years. This rewrite decided, instead, to remove even the last vestige of it and re-labeled it as a "plus" standard. This seems like Algebra 2 for poets.

The Issue of Standards for Mathematical Practice

I have already mentioned that the rewrite authors professed an interest in avoiding embedding instructional methods in the standards. Despite that, they religiously copied the eight Standards for Mathematical Practice into each and every grade, as if they were given from Mount Sinai. Yet what are those "standards" for mathematical practices if not instructional guidance par excellence? They are all about how student act, communicate, and discuss rather than about the content they are supposed to master. And much of what they discuss is difficult or impossible to measure on large-scale assessments.

Those SMPs should be deprecated and ideally eliminated from **content** standards – they have nothing to do with content. Their place is in auxiliary documents such as curriculum frameworks or model curricula.

Conclusion

The proposed rewrite is flawed in a very many ways, as detailed in the attached review. Its flaws, however, are not limited to making all these errors, although that made the draft significantly worse than the original standards. Rather, the rewrite flaws are more fundamental in that the rewrite did not even attempt to correct the many fundamental flaws in the original standards, both in terms of their low grade 2-8 content expectations and their embedded pedagogy and cognitive approach.