Arizona Mathematics Standards Revision – Expert Panel Review

Reviewer Name Elizabeth Pope

As you conduct your review of the **introduction**, please consider the following questions.

A. Does the introduction provide sufficient information and guidance on how to read the standards?
B. Does the introduction provide sufficient information on how the standards are structured?
C. Is there anything missing that should be included in the introduction?

1. Please provide feedback on the introduction section. Include strengths as well as suggestions for refinements.

A. The introduction does a good job clearly defining some key details and differences that should aid in reading, understanding and implementing the mathematics standards. Detailed information about the intended purpose of the standards, how they were created, and important research documents that were consulted in creating the standards is given. The introduction provides clear and detailed information about the "two types" of standards that compose the mathematics standards including definitions for each of the Mathematical Practice Standards that are consistent expectations across all grade levels. The inclusion of the Addition/Subtraction and Multiplication/Division Problem Types table seems misplaced. The text preceding the table explaining that students should have opportunities to "experience" each of these problem types and situations is a logical inclusion in the introduction but the actual tables themselves could be moved to the "Glossary" to make them more easily accessible when referencing them throughout the school year. The same comment applies to the "Fluency Progression" table. If the authors feel these tables are necessary in the introduction then perhaps they can also include them in the "Glossary" as an Appendix at the end of the standards document for quick reference/easy access.

B. The detailed information about the structure of the standards on pages 8, 9, and 10 is very clear and helpful. The tables and diagrams explaining how to read the standards as well as the various components that comprise each standard are clear and easy to understand. The information in this section of the introduction for the standards is much more comprehensive and clear than the information in the same section of the introduction of the ELA standards. It would make sense to have consistent information and consistent names for the components of the standards in both sets of standards as they are structured the same. These types of inconsistencies make using the standards unnecessarily complicated.

C. Aside from editing the information about reading the standards consistent between the Math and ELA standards it may also be helpful to somehow link this to the language used in the older/other standards that still use the Concept, Strand, PO language. This may be outside the scope of the Math and ELA Standards revision project but I know that it is something some pre-service teachers and new teachers find confusing and difficult and again makes the implementation of all of the content standards unnecessarily confusing.

As you conduct your review of the **glossary**, please consider the following questions.

A. Does the glossary identify key terms and resources?
B. Do the definitions provide sufficient guidance for practitioners?
C. Is there anything missing that should be included in the glossary?

2. Please provide feedback on the glossary section. Include strengths as well as suggestions for refinements.

A. In terms of terminology the glossary seems quite comprehensive and provides definitions of some of the mathematical terms referenced in the standards as well as qualitative terms used to describe student performance such as "fluency". At the beginning of the glossary of terms is a note that states that a term that is in bold in the glossary will be the acceptable definition for assessment but the only term included in the glossary that is in bold is "trapezoid". Is this correct? Is this the only term which this note applies to? If so it would make more sense to include this as a note or a footnote on that individual term to ensure that it is seen/read. The glossary is also titled "Mathematics Terms, Tables & Illustrations" but there are no tables or illustrations (beyond examples of common equations included as part of the definition for various terms) included. As mentioned in the review of the introduction it would make sense to include the Problem Type/Situation tables (Tables 1 and 2) and the table that outlines the progression of fluency across grade levels (Table 3) in the glossary for quick reference. In addition, if no illustrations beyond those that are part of a definition are going to be included in the glossary that should be omitted from the title to avoid confusion.

B. Definitions provided in the glossary are succinct and provide practitioners with basic information needed to understand and implement important aspects of the Mathematics standards. The definitions for "contextualize" and "decontextualize" may need to include examples. If a practitioner needs to refer to the glossary for a definition of what either "contextualize" or "decontextualize" means, they are likely wondering what constitutes a "context". The definitions "to place (as a word or activity) in a context" and "to remove from a context" are likely not going to be very helpful. Providing an example of what "a context" may be would likely be a more helpful and complete these definitions. The rest of the definitions of mathematical terms are clear and provide the basic information a practitioner may need to understand some of the terminology used in the standards. Including the definitions of more subjective terms referenced in the standards, such as "fluency" and "mathematical reasoning" will also likely be very helpful practitioners. These are terms that are frequently used in education but often have different meanings. Including the definition for what these terms mean in the Arizona Mathematics Standards will help ensure consistent expectations and implementation across the state.

C. As mentioned above including the Tables 1-3 from the introduction in the glossary would be helpful and a logical place for practitioners to be able to find them for quick/easy reference.

As you conduct your review of the **standards**, please consider the following questions.

A. Does each standard clearly state what students should know and be able to do?
B. Can the standards be measured?
C. Is there clarity in the standards? Are there any ambiguous or unclear words/phrases (some, a few, follow, understand…)?
D. Do the standards in each domain have sufficient **breadth of content or skill**?
E. Do the standards within a domain represent a range of **cognitive demand and rigor**?
F. Is there meaningful alignment and development of skills/knowledge allowing students to build understanding from one grade level to the next?
G. Are the standards written with clear student expectations that would be interpreted and implemented consistently across the state?

3. Please provide feedback on the Counting and Cardinality (CC) Domain (Kindergarten only). Include strengths as well as suggestions for refinements.

A. The majority of the standards in the kindergarten domain of Counting and Cardinality state what students should know and be able to do. Standard K.CC.B. "Understand the relationship between numbers and quantities" is a bit vague. The word "understand" is used both in the cluster name as well as in parts B and C of standard K.CC.B.4. In neither place is "understand" expanded upon or explained (how are practitioners expected to know if students "understand"? What types of things are students expected to do or demonstrate that show their "understanding"?).

B. Almost all of the standards in this domain can be easily measured. Once the term "understand" is defined or operationalized in standards K.CC.B all of the standards should be able to be measured and assessed easily as they will all clearly state the expected student behaviors.

C. Standard K.CC.B (and parts B and C of K.CC.B.4) are the only standards in this domain that contain vague or ambiguous language. As mentioned above it is important to define what students are expected to do in order to show their "understanding" of the relationship between numbers and quantities? For example, is the student expected to state that the last number named when counting objects is how many there are? Are students expected to explain how they know that each successive number name refers to a larger quantity?

D./E. The breadth and depth of skill students are required to master for the Counting and Cardinality standards seems developmentally appropriate given the age and skill level of most students in kindergarten. The standards address basic knowledge/recall skills such as being able to count to 100 and write numbers from 0-20 as well as more complex skills such as comparing quantities between two groups. The skills addressed in this strand represent some of the basic concepts key to learning mathematics which students will need to learn and master in order to develop mathematical competence in any area.

F. Since kindergarten is the only grade level that contains the Counting and Cardinality domain there isn't a direct/literal progression from this standard in kindergarten to the next grade level. However, as stated above, the skills in this strand are foundational skills that students will continue to reference and build upon year after year. The skills in this standard are the basic concepts on which the standards in all other grade levels are built. In that sense the progression of skills in this strand allow students to build understanding from one grade level to the next.

G. The only standard that may be difficult for practitioners to interpret and implement consistently across the state is K.CC.B (specifically K.CC.B.4, parts B and C). What one practitioner considers acceptable demonstration of "understanding" may be completely different from another practitioner and both may not align with the skill level intended by the state. Clarifying the language in this standard to include observable and measurable student behaviors will increase the ease of implementation and consistency of student expectations.

* It is noted in the introduction that "all domains are underlined" (page 8) but none of the domains are underlined in the Kindergarten standards.

4. Please provide feedback on the Operations and Algebraic (OA) Thinking Domain (Grades K-5). Include strengths as well as suggestions for refinements.

In particular reference to the standards for grades K-3 in the Operations and Algebraic Thinking domain:

A. Almost all of the actual standards in this domain clearly state what students are to know and be able to do. Most of the standards clearly state the behaviors that students are to demonstrate even if the Cluster is somewhat ambiguous. For example K.OA.A states that students will "understand addition as putting together and adding to, and understand subtraction as taking apart…" but then the standards that follow are all clearly stated, observable and measureable tasks/behaviors that students would perform indicating their understanding. 1.OA.B.4, 1.OA.D.6, and 3.OA.B.6 all use the term "understand" to describe the student behavior and do not include any further, more specific and clear actions that would demonstrate student understanding.

B. Aside from standards 1.OA.B.4, 1.OA.D.6, and 3.OA.B.6 the standards in this domain can all be measured. Expanding upon the term "understand" or replacing it with an observable and measureable verb would allow practitioners to assess student performance in relation to these standards.

C. 1.OA.B.4, 1.OA.D.6, and 3.OA.B.6 are the only standards in this domain that use ambiguous terminology without providing any further information about the kinds of skills students are expected to perform in order to show their understanding of the topics/concepts addressed by each of those standards.

D./E. The breadth of the standards in this domain is narrower at the lower grade levels and increasingly more broad, including more skills (such as those related to multiplication and division) with each grade level. The narrower focus in the earlier grade levels makes sense as the focus is on mastering some of the foundational skills needed to be able to perform more complex tasks. The complexity of skills included in this domain increases as well with each successive grade level. In the lower grades students are expected to expand upon basic skills (add and subtract fluently through 10 when in first grade as opposed to through 5 in kindergarten) and are gradually introduced to new, more cognitively challenging skills as well. Presumably as students become more proficient with the basic skills more challenging tasks are introduced. While all of the tasks included in the standards seem to follow typical developmental patterns it should be noted that students may struggle in forming the desired deeper conceptual understanding related to some of the skills (such as the inverse relationship between addition and subtraction) even though they are able to reiterate rules that have been taught or follow a sequence of steps.

F. On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, it is slightly confusing as someone reading the standards that the clusters aren't necessarily related from one grade level to the next. For example, 1.OA.C is "Add and subtract fluently through 10" and 2.OA.C is "Work with groups of objects to gain foundations for multiplication" and 3.OA.C is "Multiply and divide through 100". While all of these standards relate to arithmetic skills there is no consistent or common thread among skills addressed at each grade level in this cluster (OA.C). This is especially confusing given the way the ELA standards are structured with Anchor Standards. It's possible that some practitioners would assume or expect the math standards to follow a similar structure.

G. Once the term "understand" is defined in a clear and student centered way the standards in the Operations and Algebraic Thinking domain should be easy for practitioners to interpret and implement with consistent expectations across the state.

5. Please provide feedback on the Number and Operations in Base Ten (NBT) Domain (Grades K-5). Include strengths as well as suggestions for refinements.

In particular reference to the standards for grades K-3 in the Number and Operations in Base Ten domain:

A. Almost all of the standards in this domain clearly state what students are to know and be able to do. Standard K.NBT.B.2. does not provide any actual behavior or skill that students are to do. The standard reads that students will "demonstrate their conceptual understanding of addition and subtraction through 10 using a variety of strategies". There is no clear directive in terms of what types of strategies would accurately show a students' conceptual understanding. Can students use any strategy to model addition and subtraction and would that count as a demonstration of conceptual understanding for a kindergarten student? There needs to be more information given so that practitioners know what kind of evidence to look for (how can they tell if a student has developed an appropriate conceptual understanding? What does that look like?)

B. Several of the standards in this domain use the word "understand" without providing any further explanation as to how students are to demonstrate their understanding. In addition to K.NBT.B.2, 1.NBT.B.2, and 2.NBT.A.1. The portion of these last two standards that uses the word "understand" almost seems unnecessary. In reading these standards it appears as though the concepts and skills that these standards address are stated in parts A, B, and C following the statements about "understanding". If verbs were added to parts A, B, and C (show, tell, explain 10 can be represented by a group of ten ones called a "ten") these skills could then easily be measured.

C. K.NBT.B.2, 1.NBT.B.2, and 2.NBT.A.1. are the only standards in this domain that use ambiguous terminology . K.NBT.B.2 is completely ambiguous with no real tasks or skills included. 1.NBT.B.2 and 2.NBT.A.1 use the word "understand" where more precise and observable verbs could be used to define exactly what students should be expected to do to demonstrate their understanding.

D./E./F. The breadth and depth of the standards in this domain seems reasonably appropriate at each grade level in grades K-3. The concepts related to the base ten number system are so crucial to mathematical fluency and to the type of conceptual understanding discussed in the introduction of the standards. It makes sense to begin by introducing students to ideas such as place value in very concrete ways (as with base ten blocks) to illustrate that ten ones also make "a ten" then teach them how to apply these skills in various mathematical contexts (such as rounding and estimating). The progression of the breadth of application of skills related to base ten as well as the complexity of the tasks students are asked to perform based on principles of the base ten number system follow a logical sequence from one grade level to the next.

G. Once the term "understand" is defined in a clear and student centered way throughout this domain, the standards in the Number Operations in Base Ten domain should be easy for practitioners to interpret and implement with consistent expectations across the state.

6. Please provide feedback on the Measurement and Data (MD) Domain (Grades K-5). Include strengths as well as suggestions for refinements.

In particular reference to the standards for grades K-3 in the Measurement and data domain:

A. Almost all of the standards in this domain clearly state what students are to know and be able to do. Standards 2.MD.A.2., 3.MD.C.6, and 3.MD.C.8 all begin with statements which begin with the word "understand". Standard 2.MD.A.2 contains further information explaining what students are expected to do in order to demonstrate their understanding but standards 3.MD.C.6 and 3.MD.C.8 do not include specific descriptions of what students are expected to do to show their understanding.

B. These two standards in this domain (3.MD.C.6 and 3.MD.C.8) use the word "understand" without providing any further explanation as to how students are to demonstrate their understanding. Many of the standards throughout the document use the phrase "demonstrate understanding…" and then provide measureable behaviors that students can do and practitioners can observe as a means to assess progress. It may be worthwhile to rephrase all of the standards that begin only with "Understand" in a similar manner to make them measureable.

C. 2.MD.A.2., 3.MD.C.6, and 3.MD.C.8 are the only standards in this domain that use ambiguous terminology . All three standards use the word "understand" where more precise and observable verbs could be used to define exactly what students should be expected to do to demonstrate their understanding.

D./E. The breadth and depth of the standards in this domain seems reasonably appropriate at each grade level in grades K-3. The exclusion of standards related to time and money in the previous version of the mathematics standards was alarming. The addition of standards addressing these skills is an important change that adds to the breadth of the standards in this domain in grades 1-4 in a positive way. The progression of the cognitive demand associated with the skills in this domain of standards seems developmentally appropriate when considering cognitive development of students in grade K-3. An example of this is standard 2.MD.A.2, which addresses the idea that the length of an object remains the same regardless of units used. This is loosely related to the ideas of conservation and that there can be multiple representations for the same idea that students typically begin to master around grade 2.

F. On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, the content within each of the Clusters is again sort of random when looking at the standards in this domain from one grade level to the next. As an entire concept the progression of the skills related to Measurement and Data is logical but there isn't any clear connection of the standards in a Cluster between grade levels. As a whole the skills in the domain build upon one another but the skills addressed by individual standards or clusters do not necessarily relate and build upon one another from one grade to the next.

G. Once the term "understand" is defined in a clear and student centered way throughout this domain, the standards in the Measurement and Data domain should be easy for practitioners to interpret and implement with consistent expectations across the state.

7. Please provide feedback on the Number and Operations-Fractions (NF) Domain (Grades 3-5). Include strengths as well as suggestions for refinements.

In particular reference to the standards for grades 3 in the Number and Operations-Fractions domain:

A. Some of the standards in the Number and Operations-Fractions domain provide clear descriptions of what students are expected to know and do. "Understand" is used repeatedly throughout the standards in this domain. Some of the standards also contain further information explaining how students are expected to demonstrate their understanding (3.NF.A.2 and parts C and D of 3.NF.A.3) and others do not (3.NF.A.1 and parts A and B of 3.NF.A.3).

B. Information needs to be added to 3.NF.A.1 and 3.NF.A.3 to clarify how students are expected to demonstrate their understanding of the corresponding concepts so that they can be measured. Fractions have been identified as a critical building block for more abstract algebraic thinking skills so it is imperative that practitioners be provided with clear standards that can be measured so that any misunderstandings and misconceptions can be addressed early on in the learning process. Without clear, measureable standards the skills deemed as acceptable evidence of "understanding" will be too widely varied and may not be evidence of conceptual understanding at all.

C. Standards 3.NF.A.1 and 3.NF.A.3 use the ambiguous terminology of "understand" as discussed above. More precise and observable verbs should be used to define exactly what students should be expected to do to demonstrate their understanding.

D./E. The breadth and depth of the standards in this domain at grade 3 seems appropriate as this is the first time that students are being explicitly taught/introduced to fractions. Limiting the content expectations to fractions with denominators of 2, 3, 4, 6, and 8 helps keep the breadth of the content manageable for students who are learning something new that is more abstract than much of the content they have been exposed to so far in school. This also allows students to do a more in depth exploration of how fractions with these denominators are related and compare to one another (as outlined in the standards), which are more complex cognitive tasks.

F. Grade 3 is an appropriate time (developmentally) to begin introducing fractions to students, as it is a more abstract concept than those previously introduced. Even though there aren't specific skills related to fractions in grades K-2, many of the skills and concepts that students have been working on align with concepts that will support their learning and understanding of fractions.

G. Once the term "understand" is defined in a clear and student centered way throughout this domain, the standards in the Number and Operations-Fractions domain should be easy for practitioners to interpret and implement with consistent expectations across the state.

8. Please provide feedback on the Geometry (G) Domain (Grades K-8). Include strengths as well as suggestions for refinements.

In particular reference to the standards for grades 3 in the Geometry domain:

A. Almost all of the standards in the Geometry domain are written in a manner that clearly states what students are expected to know/do. The only standard that is not quite clear is 3.GA.1. Including examples of the categories and subcategories that students are expected to be able to list would be very helpful and make the expected student behavior much clearer. For example are students expected to know that isosceles and right triangles are both types of triangles?

B. The standards in the Geometry domain are written with clearly observable and measureable verbs. As mentioned above, including some examples of the categories and subcategories mentioned in 3.GA.1 may make measurement of that particular standard even easier.

C. None of the Geometry standards use ambiguous language but the way standard 3.GA.1 is worded makes it more confusing than necessary. Clarifying by simplifying the statement or including examples would help practitioners identify which categories and subcategories students are expected to know.

D./E .The breadth and depth of the standards in this domain are appropriate when taking cognitive, language, and motor skill development into consideration. The skills highlighted in the standards for kindergarten introduce students to basic geometric concepts, teach them new vocabulary associated with shapes (naming, describing, comparing) and support the development of their motor skills through drawing and representing these shapes. Beginning in grade 1 the focus of the Geometry standards becomes slightly narrower as students learn more nuanced information such as attributes of shapes, the difference between two and three-dimensional shapes, and partitioning shapes into equal sized pieces. The concepts remain the same from grades 1-3 but students are asked to perform increasingly more complex tasks with these skills and concepts.

F. The Geometry standards seem to be the only ones with a very clear, explicit progression of skills associated with each cluster from grades 1-3. The kindergarten standards in Geometry are slightly different, but from a developmental standpoint that makes logical sense. The skills addressed by the kindergarten standards are of greater variety but are basic, foundational skills that students then begin to apply with increasingly more complex tasks through grades 1-3. The inclusion of standards related to dividing shapes into equal parts is a great way to introduce fractional concepts to students in a very concrete way before they actually "learn" about fractions in grade 3.

G. The Geometry standards should be relatively easy for practitioners to interpret and implement consistently across the state. Providing specific examples or listing out the categories and subcategories of shapes that students are expected to know by the end of grade 3 is the only suggested revision that may help increase the consistency of expectations for students in relation to the Geometry standards.

9. Please provide feedback on the Ratio and Proportion (RP) Domain (Grades 6-7). Include strengths as well as suggestions for refinements.

N/A

10. Please provide feedback on the Number Systems (NS) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

N/A

11. Please provide feedback on the Expressions and Equations (EE) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

N/A

12. Please provide feedback on the Statistics and Probability (SP) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

N/A

13. Please provide feedback on the Functions (F) Domain (Grades 8). Include strengths as well as suggestions for refinements.

N/A

14. Please provide feedback on the Algebra 1 (A1) standards. Include strengths as well as suggestions for refinements.

N/A

15. Please provide feedback on the Geometry (G) standards. Include strengths as well as suggestions for refinements.

N/A

16. Please provide feedback on the Algebra 2 (A2) standards. Include strengths as well as suggestions for refinements.

N/A

17. Please provide any additional comments about this draft that you want the revision committee to consider.

In general it is important to keep in mind the nature of cognitive development among students in grades K-3. There is heavy emphasis in the Mathematics standards on helping students gain a deep conceptual understanding of mathematical concepts even before learning more basic skills such as algorithms and equations. Conceptual understanding by its very definition requires the ability to think and reason in an abstract manner. Students in grades K-3 are not yet thinking and organizing knowledge in this manner. Students at these age/grade levels are still concrete and logical thinkers. They are developing skills that will later lead them to more formal and abstract reasoning (such as coming to understand reversibility and complex relationships among objects, numbers, etc.) but it is unreasonable to expect students in this age range to be able to think and reason about mathematics in the same way that older students do. Younger students don't simply know less about mathematics and mathematical concepts than older students, they actually think about them in a different way. Students in grades K-3 can be introduced to more abstract, conceptual ideas but it is not necessarily appropriate developmentally to expect them to have a complete conceptual understanding of the principles at work behind those concepts. For example, one can certainly teach a kindergarten student to count to 100 by 10s but a kindergartener is not likely to be able to understand and connect this idea to the greater ideas of a base 10 number system. Similarly students of this age/grade can be taught that addition and subtraction are the inverse of one another, but cognitively they may not truly be ready to master the concepts behind reversibility until they are in first or second grade. Furthermore, measuring true conceptual understanding can be difficult as children can easily learn key words or phrases or follow a series of steps that may make it appear as though they understand underlying concepts but the student may not actually have a true understanding that allows them to apply and adapt skills flexibly as described in the standards.

It is important that the standards allow for varying levels of "conceptual understanding" that align with the ways children think and reason at various stages of development and that these levels be clearly outlined or described for teachers and students. The National Research Council's report, *Adding it Up: Helping Children Learn Mathematics* is referenced in the introduction of the mathematics standards as one of the documents referenced when creating the standards. The approach to developing fluency discussed in this report is a good example of how the continuous mastery of skills that are appropriate for a child's developmental level can be defined as fluency for a given grade level and the mastery of these skills is what leads to greater abilities and more or differing levels of fluency at the next grade level. Fluency is described as a skill that is developed over the long term but markers of skill development along the way that coincide with the cognitive development and abilities of students at various ages/grades. I believe that the intent of these Mathematics Standards was to take a similar approach to the idea of developing "conceptual understandings" of mathematical concepts. There are pieces of this approach throughout the standards but there is still a lack of specificity that may make teaching and assessing for conceptual understanding in a developmentally appropriate manner difficult for practitioners. Using vague words such as "understand" in the standards leave teachers guessing at what students are truly expected to know how to do in order to demonstrate that they are on the right track to developing the desired conceptual understanding. Providing clear standards with observable and measureable tasks for students to demonstrate and teachers to assess can only benefit students in the development and refinement of mathematical skills and knowledge.