Arizona Mathematics Standards Revision – Expert Panel Review

Reviewer Name Fabio Augusto Milner

As you conduct your review of the **introduction**, please consider the following questions.

A. Does the introduction provide sufficient information and guidance on how to read the standards?
B. Does the introduction provide sufficient information on how the standards are structured?
C. Is there anything missing that should be included in the introduction?

1. Please provide feedback on the introduction section. Include strengths as well as suggestions for refinements.

This section is very detailed, informative and clear. It conveys all information needed to read the standards and understand how they are structured.

There are a few words that are hyphenated but are not (e.g. credit-bearing, problem-solving).

On Table 3, the last row specifies what may very well be the single most important standard in all K-12 mathematics REQUIRING fluency, and yet is so badly neglected that even a majority of college students have trouble with it. They do not see the order of operations in complex expressions, and they do not know how to correctly rewrite them using properties of operations. It is very important to point this out somewhere to raise teacher and student sensitivity towards this fact.

The same table contains twice the erroneous word “multiple” where “multiply” should be used.

As you conduct your review of the **glossary**, please consider the following questions.

A. Does the glossary identify key terms and resources?
B. Do the definitions provide sufficient guidance for practitioners?
C. Is there anything missing that should be included in the glossary?

2. Please provide feedback on the glossary section. Include strengths as well as suggestions for refinements.

The glossary is not good. It almost always violates the criteria allegedly used to decide what terms to include in it. It is missing essential terms (e.g. imaginary number), contains mathematical and English language errors, equivocal and circular "definitions". It really needs a lot of additional work. Many essential terms are missing, such as imaginary number, abscissa, ordinate,

As you conduct your review of the **standards**, please consider the following questions.

A. Does each standard clearly state what students should know and be able to do?
B. Can the standards be measured?
C. Is there clarity in the standards? Are there any ambiguous or unclear words/phrases (some, a few, follow, understand…)?
D. Do the standards in each domain have sufficient **breadth of content or skill**?
E. Do the standards within a domain represent a range of **cognitive demand and rigor**?
F. Is there meaningful alignment and development of skills/knowledge allowing students to build understanding from one grade level to the next?
G. Are the standards written with clear student expectations that would be interpreted and implemented consistently across the state?

3. Please provide feedback on the Counting and Cardinality (CC) Domain (Kindergarten only). Include strengths as well as suggestions for refinements.

K.CC.B.5 contains the common usage, "a number from 1-20", that is much better expressed in formal English as "a number from 1 to 20".

This domain is well covered though I have a concern that teachers may never assess K.CC.B.4c.

4. Please provide feedback on the Operations and Algebraic (OA) Thinking Domain (Grades K-5). Include strengths as well as suggestions for refinements.

This domain would be strengthened by the introduction of the concept of a “unit” or “neutral element” in a binary operation. That allows defining “inverses” and thus understanding subtraction as addition of the additive inverse (“opposite”) and division as multiplication by the multiplicative inverse (“reciprocal”).

In 3.OA.D.9 “…perform operations in the conventional order when there are no parentheses to specify a particular order-Order of Operations” needs to be rewritten as “…perform operations in the conventional order–Order of Operations–when there are no parentheses to specify a particular order”

3.OA.D.10 needs the example that was deleted

5.OA.A.1 With the removal of brackets and braces, is the intention that those never be used? When (if so) will they be introduced?

5. Please provide feedback on the Number and Operations in Base Ten (NBT) Domain (Grades K-5). Include strengths as well as suggestions for refinements.

The new K.NBT.B.2 does not belong in NBT since it does not involve place value at all. In fact, when talking about the number 10, the conceptualization at this level is only as ten “ones” and not as one “ten”.

1.NBT.A.1 and 2.NBT.A.1 should have consistent language: the former uses “groups” of tens, while the latter uses “amounts”.

In 2.NBT.B.5 fluency should be expected but has been removed.

For 2.NBT.B.8 the draft does not show any difference with the 2010 standard.

3.NBT.A.3 should require fluency to 1000 not to 100.

In 5.NBT.B.7 the change introduced is ill-conceived: even when multiplying or dividing two decimals, the relationship needed in a standard algorithm is between addition and subtraction.

6. Please provide feedback on the Measurement and Data (MD) Domain (Grades K-5). Include strengths as well as suggestions for refinements.

2.MD.C.8 needs to specify the coins because other countries have coins of different denominations than the US. Moreover, the "dollars" in the draft allow for dollar bills of any denomination, which is not appropriate for this grade.

2.MD.C.8 and 3.MD.A.2 have inconsistent notation for cents. The latter needs better explanation of the decimal point. When we write $12.00 we are using the decimal point but there are no cents. What is probably meant is "1 ¢ = $0.01".

In 3.MD. B.5, "quarter inch" needs to be hyphenated.

In 3.MD.C.7b the last part, "and represent whole-number products as rectangular areas in mathematical reasoning", was removed but it should really stay. It is an important example of "creating" as opposed to "applying". Moreover, it naturally leads to the commutativity of multiplication.

In 3.MD.C.9 "mathematical problems" should not be removed. Moreover, the end of the old standard, "exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters", is very important and should not be removed (Table 1 does not include such problems that are at a higher cognitive level). In the Notes, the word *extraneous* is misspelled.

In 4.MD.A.1 the Notes have the word "involved" misspelled.

In 4.MD.A.3 the end of the proposed standard, "including problems with unknown side lengths", should rather specify "see Tables 1 and 2" for consistency with other standards.

Why is "real world" kept in 5.MD.C.5 but changed to "in a real world context" in other standards? In part a. the removal of "Represent threefold whole-number products as volumes" detracts from the standard (same comment made above for 3.MD.C.7b). In part c. "applying this technique" is meaningless because no technique is mentioned.

7. Please provide feedback on the Number and Operations-Fractions (NF) Domain (Grades 3-5). Include strengths as well as suggestions for refinements.

The deletion in 3.NF.A.2 is ill-guided. As proposed it never defines the location of 1/b on the number line.

In 3.NF.A.3c it is important to "recognize" fractions that are whole numbers as well as understanding them.

In 4.NF.C.7 the concept of decimal fractions is used but has not been introduced. Also, "decimals with tenths and hundredths" is not what is intended since they may be lacking one or the other. The 2010 wording "decimals to hundredths" is better, albeit not best.

In 5.NF.B.7 it is desirable to keep "create a story context for a whole number divided by a fraction". It cognitively more difficult to create the problem than to solve it.

8. **Please provide feedback on the Geometry (G) Domain (Grades K-8). Include strengths as well as suggestions for refinements.**

The proposed K.G.B.4 is a duplicate of the proposed (and current) K.G.B.1. I recommend to keep the existing K.G.B.4 removing the examples therein.

The proposed K.G.B.6 is a duplicate of the proposed K.G.B.5. I recommend to keep the existing K.G.B.6 removing the example therein.

1.G.A.1 has awkward wording, “draw shapes to possess defining attributes.” I would reword as “draw shapes that possess prescribed (or given) attributes.”

1.G.A.3 should include “quarters” as a synonym of “fourths”.

The proposed 2.G.A.1 should end with “Draw two-dimensional shapes having specified attributes.”

3.G.A.2 should read “Partition shapes into *b* parts with equal areas. Express the area of each part as a unit fraction 1/b of the whole. (Grade 3 expectations are limited to fractions with denominators *b* = 2,3,4,6,8).”

5.G.A.1 should include the names *abscissa* and *ordinate*. “Understand that the first number (x, called *abscissa*) indicates the distance traveled on the horizontal axis, and the second number (y, called *ordinate*) indicates the distance traveled on the vertical axis.”

7.G.A.3 contains examples that should be removed for consistency with many other standards in which examples were removed. Examples are not included within a standard unless an example would provide limits to the standard or clarification to the standard, which here they do not.

7.G.B.5 should have “a multi-step problem” in plural.

A good refinement to add here would be the non-commutativity of transformations. For example: “Understand that two plane transformations of a figure may produce different resuts when applied in different order.”

9. Please provide feedback on the Ratio and Proportion (RP) Domain (Grades 6-7). Include strengths as well as suggestions for refinements.

6.RP.A.1 needs to be placed in a real-world context. Otherwise ANY two numbers *a* and *b* are "in a ratio relationship", namely *a:b*.

In 6.RP.A.2 the concept of unit rate is not introduced properly. The essential component, *two co-varying quantities*, is missing. The examples suggest this but were removed and, moreover, this needs to be explicitly included rather than suggested. In the first deleted example, the numbers of cups of sugar and cups of flour are related variables/quantities (linearly, in fact), and those numbers are related by the latter being equal to the former multiplied by the unit rate of flour-to-sugar. ***Rate cannot be defined for just two numbers***; it **requires** *two co-varying quantities* (in a proportional relationship at this stage).

The fundamental concepts are introduced in the wrong order. Rates need to be defined AFTER proportional relationships. This fact becomes crystal clear in 7.RP.A.2b.

In 7.RP.A.3 the examples were meant to indicate desirable applications, not comprehensive limits. Why preclude the ratio of legs to people, for example?

10. Please provide feedback on the Number Systems (NS) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

6.NS.B.4 is not one standard because there is no unifying thread across its three parts. It is the ill-advised merging of three distinct standards into one. There should be one separate standard for each part, including a justification for why the lcm and gcd exist for any finite set of positive integers.

6.NS.C.5 should not contain the word “directions” because the standards do not ever mention or define the *direction of a number*.

6.NS.C.6c should not mention integers separately because at this grade level students already know that integers are rational numbers.

6.NS.C.9 should not be removed!!! Students have no clue about the meaning of the symbol “%” (i.e. “ % = 1/100 ”). It could be reworded “Understand that fractions, decimals, and percents are three different ways of representing numbers, and fluently convert from one way to another.” 7.NS.A.2 contains conversion of fraction to decimal.

7.NS.A.1 should begin “Apply and extend previous understandings of addition and subtraction to add and subtract any rational numbers.”

7.NS.A.2 should begin “Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide any rational numbers, except division by zero.”

8.NS.A.1 should rather end with “Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational.”

8.NS.A.2 should end with “and estimate the value of expressions involving irrational numbers.”

A good refinement of this domain would be the addition of a standard about the density of rational and of irrational numbers among the reals. For example, “8.NS.A.3: Understand that given any two distinct rational numbers, $a < b$ say, there exist a rational number c and and irrational number d such that $a < c < b$ and $a < d < b$. Similarly, given any two distinct irrational numbers, $a < b$ say, there exist a rational number c and and irrational number d such that $a < c < b$ and $a < d < b$.”

11. Please provide feedback on the Expressions and Equations (EE) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

In the proposed 6.EE.B.7, *p/x = q* should rather be *x/p = q* since this standard is about *linear* equations.

In the proposed 6.EE.B.8, the last words "on number line" should either be "on number lines" or "on a number line."

In the proposed 6.EE.C.9, the wording is in an incorrect semantic order, it should read "Use variables to represent two quantities that change in relationship to one another to solve mathematical problems and problems in a real-world context."

In the proposed 7.EE.A.2, the wording should be changed to "Rewrite an expression in a problem context in different forms and understand the connection between the structures of the different forms and its meaning in the particular context."

In the proposed 7.EE.B.4b, non-strict inequalities should be included for consistency with 6.EE.B.8.

In 8.EE.A.1, the adjective *numerical* is an essential qualifier of *expressions* because this standard is not intended for variables.

In 8.EE.A.2a, the end should read "rewrite square roots of non-perfect squares in equivalent form." The same applied to part b for cube roots.

In 8.EE.C.8c, the wording should be "Solve mathematical problems and problems in a real-world context", for consistency.

A suggestion for an extremely important refinement concerns Order of Operations. I think it is imperative that students (and teachers alike… there are hundreds of them who believe that the agreed-upon order of operations follows from mathematical principles…) understand that ORDER OF OPERATIONS is a concept only in the sense that *whenever an expression contains more than one operation it becomes necessary to know the order in which we are to perform the given operations* in order to avoid ambiguity. **What that order is** not a concept, IT IS AN AGREEMENT/CHOICE and, as such, it cannot be learned conceptually; it needs to be memorized and applied carefully and accurately. College professors (e.g. Jennifer Bay-Williams from University of Louisville and Sherri Martinie from Kansas State University) publish outrageous garbage articles in top rated journals such as those of NCTM purporting to "prove" the order of operations. IT IS DIFFICULT TO IMAGINE CHANGING THE WAY OUR STUDENTS LEARN MATHEMATICS IF THE TEACHERS THEY HAVE DO NOT GET RID OF SUCH ELEMENTARY AND DEEP-SEATED MISCONCEPTIONS. I urge you to read the "article": Jennifer M. Bay-Williams and Sherri L. Martinie, Order of Operations: The Myth and the Math, Teaching Children Mathematics Vol. 22, Issue 1, August 2015.

12. Please provide feedback on the Statistics and Probability (SP) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

In 6.SP.A.2 “which” should rather be “that” (better English usage). Moreover, “can be described by its center, spread, and overall shape” is very problematic, indeed false. The standard should read “Understand that a set of data collected to answer a statistical question has a distribution whose general (or overall) characteristics can be described by its center, spread, and overall shape.”

In 6.SP.A.3 the word “variation” is used as a synomym for “spread” in 6.SP.A.2. In 6.SP.B.5 yet a third word, “variability” is introduced. Why are three different terms being used for one concept?

In 7.SP.B.4 the example is important as clarification for the standard; clearly the intention of the standard is not finding the means for each distribution and then mechanically saying that they are different and which is larger. The applications must be meaningful and avoid, for example comparing the mean height of students in a school with the mean annual rainfall in Seattle in the last 100 years.

In 7.SP.C.5 “Larger numbers indicate greater likelihood” is essential for the expected understanding. In the Notes the word “involved” is misspelled.

13. Please provide feedback on the Functions (F) Domain (Grades 8). Include strengths as well as suggestions for refinements.

8.F.A.2 needs clarification by example. Clearly the intention of the standard is not listing for the first function some of its properties and for the second function some of its properties and then mechanically saying which ones are properties of both functions and which are properties of one but not of the other.

14. Please provide feedback on the Algebra 1 (A1) standards. Include strengths as well as suggestions for refinements.

In A1.F-IF.B there is a contradiction of sense between “intervals where the function is increasing” and “exponential (functions) with integer exponents” since the former require the function to be defined over some interval while the latter precludes that.

Concerning average rate of change, A1.F-IF.B.6 and A2.F-IF.B.6, most teachers and students do not even understand the concept of “change” for a function. It would be a monumental step forward if the concept were specifically mentioned as a standard (“Change of a quantity is a difference between two values of the quantity”). Similarly, introduce the concept of rate of change of two variables that are related to each other and co-vary (vary together) from the words in the name: rate is a ratio (or quotient); if u and v are co-varying variables, the rate of change of u with respect to v as u varies from u_1 to u_2 and v varies from v_1 to v_2 is the ratio of their changes, that is $(u_2 - u_1) / (v_2 - v_1)$.

15. Please provide feedback on the Geometry (G) standards. Include strengths as well as suggestions for refinements.

In G.G-CO.D it is imperative that geometric constructions are presented also as theorems that need to be proved. For example, the method for bisecting an angle needs a proof before it can be accepted as actually bisecting it.

In G.G-SRT.B.4 the "conversely" should be explained rather than removed: it is meant to require that students learn the proof that if the other two sides are divided proportionally, then the lines are parallel.

In G.G-GPE.B.4 "proving or disproving geometric figures" is meaningless. Also, "proving or disproving if a specific point lies on a given circle" should read "proving or disproving that a specific point lies on a given circle".

In G.G-GMD.A.3 the word "context" should be plural.

16. Please provide feedback on the Algebra 2 (A2) standards. Include strengths as well as suggestions for refinements.

Algebra 2 (and the glossary) needs the concepts of *abscissa* and *ordinate* as first and second coordinates of an ordered pair of numbers. It is disgraceful that high school graduates can only refer to them (wrongly, of course) as "x-coordinates" and "y-coordinates" for points that are frequently plotted on coordinate axes that are not labeled x and y.

A2.N-RN.A.1 needs the example that was removed.

With the changes made, N-CN never defined complex numbers. In particular, the relation $i^2 = -1$ makes no sense for students who "know" that squares are never negative.

A good example for A2.A-SSE.B.3 may be $e^{2t} - 2e^t + 1 = (e^t - 1)^2$, to underscore that previously learned structures and concepts need now to be combined with newly learned ones.

In A2.A-APR.C.4 it will be very unclear to teachers and students what is meant by "and use them to describe numerical relationships".

A2.F-IF.C.9 needs clarification by example. Clearly the intention of the standard is not listing for the first function some of its properties and for the second function some of its properties and then mechanically say which ones are properties of both functions and which are properties of one but not of the other.

In A2.F-BF.A.1 the expression "standard function types" is used but never defined. It is imperative that the limits of this standard be explicit. Also, it is imperative that A2.F-BF.A.1a and A2.F-BF.A.1b specifically direct to be applied both to mathematical and real-life situations.

In A2.F-BF.B.4 the inverse function is confused with its graph. Moreover, the deletion of the composition of a function with its inverse completely obscures the essential defining condition of inverse.

In A2.F-LE.A.4 it should be specified that technology should be used to evaluate logarithms that are not readily found by hand or observation.

The A2 standards would improve if the concept of periodic function were introduced before trigonometric functions, since it is in fact unrelated to those. The way it is done students get the very wrong idea that all periodic functions are trigonometric.

A2.S-ID.A.4 needs to be rephrased from "…properties of a normal distribution to approximate a normal curve…" to "…properties of a normal distribution to approximate the given data by a normal curve…"

In A2.S-ID.B.6 "chooses" needs to be "choose".

The language chosen for A2.S-IC.A.1 is very poor: "making inferences to be made". CHANGE IT to good English usage.

17. Please provide any additional comments about this draft that you want the revision committee to consider.

The standards are very weak in the treatment of logarithms. They introduce logarithmic functions in A2 without ever introducing first the elementary-school level concept of logarithm. Just as the concepts of sum and difference, and those of product and quotient are introduced in elementary school by relating, respectively, three numbers *a, b,* and *c* via $a + b = c$ and learning that "*c* is the sum of *a* and *b*," "*b* is the difference of *c* and *a*," and "*a* is the difference of *c* and *b*" are equivalent statements (i.e. synonyms); or relating them via $a \times b = c$ and learning that "*b* is the quotient of *c* by *a*", "*a* is the quotient of *c* by *b*", and "*c* is the product of *a* and *b*" are equivalent statements (i.e. synonyms); it is entirely analogous and should be taught just as matter-of-factly that relating them via $a^b = c$ leads to the equivalent concepts that "*c* is the *b*-th power of *a*", "*a* is the *b*-th root of *c*" and "*b* is the logarithm to base *a* of *c*". Entirely analogous situations that our educational system separates into the "easy" ones that we teach our students and the "difficult" one that we don't… Thus **we perpetuate the myth of how hard logarithms and roots are just because we want them to look that way rather than making them look exactly like terms in a sum or difference and factors in a product or quotient, which is all they are**.

When separating the A1 standards from the corresponding A2 standards for exponential functions, in A1 the wording "exponential function with integer exponents" is used (and then reminded in A2). This restricts the functions to be sequences, which is not the intent of this standard. This needs to be fixed in many A1 and A2 standards (every time the issue appears).