Arizona Mathematics Standards Revision – Expert Panel Review

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Reviewer Name

As you conduct your review of the introduction, please consider the following questions.

- A. Does the introduction provide sufficient information and guidance on how to read the standards?
- B. Does the introduction provide sufficient information on how the standards are structured?
- C. Is there anything missing that should be included in the introduction?

1. Please provide feedback on the introduction section. Include strengths as well as suggestions for refinements.

I found the introduction very helpful. In particular, the description of the Standards for Mathematical Practice were explained very well, and after reading this I understood that these standards are identical across grade levels, that these standards express habits of mind that are fostered throughout mathematics education, and that these standards differ from the content standards which vary by grade. The narratives were extremely helpful. I also appreciate the clarity with which the Mathematical Content standards were presented, particularly with the figure on page 7 of the introduction. The explanation of the coding system was very clear. I had no problem understanding how to read the standards or interpreting the structure of the standards. I was able to anticipate the presentation of the content standards and mathematical practice standards from the introduction. I did not identify any information missing from the introduction, it seemed comprehensive to me. Table 1 and 2 were helpful, and when I read this section of the introduction, I interpreted these as containing common problem types/situations, rather than a comprehensive list of all of the problem types for the various operations. However, the notes on the standards seem to imply that these tables represent all of the problem types (e.g. 1.OA.A.1). Clarification on whether or not the tables contain a comprehensive list of problem types is warranted. In addition, while I found the numbering system for the standards very clear, the vertical alignment between the standards isn't a feature of the numbering system, which may confuse some readers. Perhaps a note indicating that the numbering at the end of each standard does not imply vertical alignment from one grade to the next would be helpful to the readers of the standards.

As you conduct your review of the **glossary**, please consider the following questions.

- A. Does the glossary identify key terms and resources?
- B. Do the definitions provide sufficient guidance for practitioners?
- C. Is there anything missing that should be included in the glossary?

2. Please provide feedback on the glossary section. Include strengths as well as suggestions for refinements.

The glossary is helpful and clear. Relevant definitions were included, and each definition was very clear. The examples, where provided, were illustrative. The only thing that I would change is to simply call this "Glossary" instead of "Glossary: Mathematical Terms, Tables & Illustrations", or to eliminate the reference to tables, since there aren't really any tables included in this section.

As you conduct your review of the **standards**, please consider the following questions.

- A. Does each standard clearly state what students should know and be able to do?
- B. Can the standards be measured?
- C. Is there clarity in the standards? Are there any ambiguous or unclear words/phrases (some, a few, follow, understand...)?
- D. Do the standards in each domain have sufficient breadth of content or skill?
- E. Do the standards within a domain represent a range of cognitive demand and rigor?
- F. Is there meaningful alignment and development of skills/knowledge allowing students to build understanding from one grade level to the next?
- G. Are the standards written with clear student expectations that would be interpreted and implemented consistently across the state?

3. Please provide feedback on the Counting and Cardinality (CC) Domain (Kindergarten only). Include strengths as well as suggestions for refinements.

Each standard in this domain is clearly stated and describes what students should know and be able to do. Each standard is measurable, has sufficient breadth and cognitive demand, and there are not ambiguous words or phrases included in any of the standards. The standards are written so that they will be unambiguously interpreted across the state. The refinements included in the current draft improve the clarity of the standards. The standards are developmental appropriate. I have no additional feedback on the standards in this domain.

4. Please provide feedback on the Operations and Algebraic (OA) Thinking Domain (Grades K-5). Include strengths as well as suggestions for refinements.

The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. However, the deletion of the mental strategies described in 1.OA.C.5 without reference to the full definition of fluency described in the introduction may alter or limit the cognitive processes engaged away from flexible mathematical thinking and toward rote memorization.

5. Please provide feedback on the Number and Operations in Base Ten (NBT) Domain (Grades K-5). Include strengths as well as suggestions for refinements.

The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, "Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate.

6. Please provide feedback on the Measurement and Data (MD) Domain (Grades K-5). Include strengths as well as suggestions for refinements.

The standards are written with clarity, are measurable, and have sufficient breadth and depth. The standard K.MD.A.2 is developmentally appropriate as long as the attribute being measured presents in a consistent way across cases. For example, the child would be able to compare a measurable attribute such as length for two objects with the same appearance (e.g. two straight lines) but not necessarily when the presentation of the attribute varies across objects (e.g. a straight line and a curved line) as the latter requires cognitive thinking skills that are not typically developed until around age 7. I suggest adding language to specify the equivalence of appearance of the attribute to this standard. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression.

7. Please provide feedback on the Number and Operations-Fractions (NF) Domain (Grades 3-5). Include strengths as well as suggestions for refinements.

The standards are measurable, clear, and contain breadth and depth of the content. The developmental progression is clear and apparent across grade levels. The clarification of the link between the standards and real world problem solving is an improvement.

8. Please provide feedback on the Geometry (G) Domain (Grades K-8). Include strengths as well as suggestions for refinements.

In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.

Removing the list of shapes from the Kindergarten standards is potentially problematic, since there are 2-D and 3-D shapes that are not included in this list (e.g. octagon, icosahedron), and yet the expectation at kindergarten is not for exhaustive knowledge of all 2-D and 3-D shapes. Therefore the scope of the expectations in these standards is left vague and potentially unreasonable for kindergarteners.

9. Please provide feedback on the Ratio and Proportion (RP) Domain (Grades 6-7). Include strengths as well as suggestions for refinements.

These standards are clear, measurable, and developmentally appropriate. The inclusion of the limits in standard 7.RP.A.3 are appropriate and useful. No suggestions for refinements were identified. The standards are written so that they will be unambiguously interpreted across the state.

10. Please provide feedback on the Number Systems (NS) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

These standards are clear, measurable, and contain sufficient breadth and depth. The vertical alignment of these standards is excellent, and the refinements made to the standards are useful. The standards are written so that they will be unambiguously interpreted across the state.

11. Please provide feedback on the Expressions and Equations (EE) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

In terms of developmental appropriateness, the 6th grade standards in this domain are likely to be quite challenging for 6th graders since students at this age are just beginning to be able to think representationally and abstractly, requirements for understanding algebraic expression and equation. Providing some limit to the complexity of the algebraic expressions would make these standards more developmentally appropriate, such as limiting the number of variables in an expression, or the types of operations included in the expressions. That said, the standards are written clearly and are measurable, and interpretation of these standards should be unambiguous across the state.

12. Please provide feedback on the Statistics and Probability (SP) Domain (Grades 6-8). Include strengths as well as suggestions for refinements.

The standards in this domain are very well written – they are clear, measurable, demonstrate a logical progression of knowledge in terms of breadth and depth, and are easily interpreted. Moving 8.SP.B.1 from 7th grade to 8th grade enhances the knowledge progression across grades. The standards are developmentally appropriate.

13. Please provide feedback on the Functions (F) Domain (Grades 8). Include strengths as well as suggestions for refinements.

The standards in this domain are measurable, interpretable, have good vertical and horizontal alignment, and are easily interpreted. I have no suggestions for refinement for the standards in this domain.

14. Please provide feedback on the Algebra 1 (A1) standards. Include strengths as well as suggestions for refinements.

The changes made to the standards in this domain help specify differences between expectations for Algebra 1 and 2. However, moving the conditional probability and rules of probability standards to A1 does not contribute to improvements in horizontal and veritical alignment; in my opinion these standards taught in conjunction with with the conditional probability standards located in A2 would lead to more conceptual coherence, in terms of breadth and depth of content knowledge.

15. Please provide feedback on the Geometry (G) standards. Include strengths as well as suggestions for refinements.

The instances of rewording in these standards (e.g. G.G.-SRT.B.5) place emphasis on conceptual mathematical thinking required, which is an improvement in the standards. Moving G.G-GPE.A.2&.3 to the plus standards is a sound decision, as this material seems beyond the depth and breadth of the rest of the Geometry standards. All standards are measurable, describe breadth and depth of content, demonstrate horizontal and vertical alignment, and are eaily interpretable.

16. Please provide feedback on the Algebra 2 (A2) standards. Include strengths as well as suggestions for refinements.

The changes made to these standards help define the differences between A1 and A2. The deletion of the Quantities (N-Q) standards makes sense, as these standards are indeed integrated throughout A1, A2 and Geometry. The standards are measurable, clear, describe breadth and depth of content, and are interpretable.

17. Please provide any additional comments about this draft that you want the revision committee to consider.

The verb "recognize" is typically considered measurable, since you can directly observe a student recognizing, for example, through objectively formatted test items where item options (the key and distractors) are presented to the learner. In addition, the use of recognize helps differentiate the level of cognitive complexity involved in a standard. It is simpler cognitively to recognize than to engage in other types of cognitive activities, such as to free recall or to demonstrate full understanding of a concept. Therefore removing this verb may change the level of cognitive engagement demanded by the standard.

I understand that the purpose of the deletion of examples from the standards is to remove instructional guidance as to how the standard should be taught, since the standards are not the curriculum. However, at times I felt that the deletion of some of the examples diminished the clarity of the standards.

Overall, I found the revisions to the standards sound. I commend the working group on their thoughtful and detailed analyses of these standards.