



Arizona's Draft Standards Mathematics

Geometry

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
Draft Standards for Public Comment

Geometry Overview

NUMBER AND QUANTITY - N

Quantity (N-Q)

- Reason quantitatively and use units to solve problems.

GEOMETRY - G

Congruence (G-CO)

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry (G-SRT)

- Understand similarity in terms of similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.

Circles (G-C)

- Understand and apply theorems about circles.
- Find arc lengths and area of sectors of circles.

Expressing Geometric Properties with Equations (G-CPE)

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension (G-GMD)

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry (G-MG)

- Apply geometric concepts in modeling situations.

Standards for Mathematical Practices (MP)

Geometry: Critical Areas

The fundamental purpose of the Geometry course is to formalize and extend students' geometric experiences from the middle grades. In this high school Geometry course, students explore more complex geometric situations and deepen their explanations of geometric relationships, presenting and hearing formal mathematical arguments. Important differences exist between this course and the historical approach taken in geometry classes. For example, transformations are emphasized in this course.

For the high school Geometry course, instructional time should focus on five critical areas:

- 1. Establish criteria for congruence of geometric figures based on rigid motions.**
- 2. Establish criteria for similarity of geometric figures based on dilations and proportional reasoning**
- 3. Develop informal explanations of circumference, area, and volume formulas**
- 4. Prove geometric theorems.**
- 5. Solve problems involving right triangles.**

(1) Students have prior experience with drawing triangles based on given measurements and performing rigid motions including translations, reflections, and rotations. They have used these to develop notions about what it means for two objects to be congruent. In this course, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. They apply reasoning to complete geometric constructions throughout the course and explain why these constructions work.

(2) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of geometric figures, use similarity to solve problems (including utilizing real-world contexts), and apply similarity in right triangles to understand right triangle trigonometry. When studying properties of circles, students develop relationships among segments on chords, secants, and tangents as an application of similarity.

(3) Students' experience with three-dimensional objects is extended to developing informal explanations of circumference, area, and volume formulas. Radians are introduced for the first time as a unit of measure – which prepares students for work done with the Unit Circle in the Algebra II course. Students have opportunities to apply their understanding of volume formulas to real-world modeling contexts. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

(4) Students prove theorems—using a variety of formats including deductive and inductive reasoning and proof by contradiction—and solve problems about triangles, quadrilaterals, circles, and other polygons. Relating back to work in previous courses, students apply the Pythagorean Theorem in the Cartesian coordinate system to prove geometric relationships and slopes of parallel and perpendicular lines. Continuing in the Cartesian coordinate system, students graph circles by manipulating their algebraic equations and apply techniques for solving quadratic equations – all of which relates back to work done in the Algebra I course.

(5) Students define the trigonometric ratios of sine, cosine, and tangent for acute angles using the foundation of right triangle similarity. Students use these trigonometric ratios with the Pythagorean Theorem to find missing measurements in right triangles and solve problems in real-world contexts – which prepares students for work done with trigonometric functions in the Algebra II course.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Mathematical modeling is integrated throughout Geometry by utilizing real world context.

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Geometry: Mathematics Standards	
Number and Quantity - N	
<u>Quantities (N-Q)</u>	
G.N-Q.A	Reason quantitatively and use units to solve problems.
G.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
G.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem solving opportunities utilizing real-world context.
G.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Geometry - G	
<u>Congruence (G-CO)</u>	
G.G-CO.A	Experiment with transformations in the plane.
G.G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.G-CO.A.2	Represent and describe transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
G.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.

G.G-CO.B	Understand congruence in terms of rigid motions.
G.G-CO.B.6	Use geometric definitions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G.G-CO.C	Prove geometric theorems.
G.G-CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
G.G-CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.
G.G-CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.
G.G-CO.D	Make geometric constructions.
G.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods. Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
<u>Similarity, Right Triangles, and Trigonometry (G-SRT)</u>	
G.G-SRT.A	Understand similarity in terms of similarity transformations.
G.G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.G-SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
G.G-SRT.B	Prove theorems involving similarity.
G.G-SRT.B.4	Prove theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally; the Pythagorean Theorem proved using triangle similarity.
G.G-SRT.B.5	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing a real-world context.
G.G-SRT.C	Define trigonometric ratios and solve problems involving right triangles.
G.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.

G.G-SRT.C.8	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles in applied problems.
<u>Circles (G-C)</u>	
G.G-C.A	Understand and apply theorems about circles.
G.G-C.A.1	Prove that all circles are similar.
G.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
G.G-C.B	Find arc lengths and areas of sectors of circles.
G.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.
<u>Expressing Geometric Properties with Equations (G-GPE)</u>	
G.G-GPE.A	Translate between the geometric description and the equation for a conic section.
G.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.G-CPE.B	Use coordinates to prove simple geometric theorems algebraically.
G.G-GPE.B.4	Use coordinates to prove or disprove simple geometric theorems algebraically. Theorems include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.
G.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.
G.G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
<u>Geometric Measurement and Dimension (G-GMD)</u>	
G.G-GMD.A	Explain volume formulas and use them to solve problems.
G.G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing real-world context.
G.G-GMD.B	Visualize relationships between two-dimensional and three dimensional objects.
G.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

<u>Modeling with Geometry (G-MG)</u>	
G.G-MG.A	Apply geometric concepts in modeling situations.
G.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects.
G.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations.
G.G-MG.A.3	Apply geometric methods to solve design problems.
G.MP	Standards for Mathematical Practice
G.MP.1	<p>Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.</p>
G.MP.2	<p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>

G.MP.3	<p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p>
G.MP.4	<p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>
G.MP.5	<p>Use appropriate tools strategically.</p> <p>Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.</p>

G.MP.6	<p>Attend to precision.</p> <p>Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.</p>
G.MP.7	<p>Look for and make use of structure.</p> <p>Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.</p>
G.MP.8	<p>Look for and express regularity in repeated reasoning.</p> <p>Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.</p>