Arizona's Draft Standards
Mathematics

Executive Summary

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
Draft Standards for Public Comment

Executive Summary

Process and Mathematics Workgroup Tasks

In November 2015, the Mathematics Workgroup commenced meetings to review public comments related to the 2010 mathematics standards. Participation in the Arizona Department of Education (ADE) Mathematics Workgroup was based on self-submitted applications. The workgroups functioned in an accordion model; members of the workgroup were fluid and participated at varying stages of the review process. Applications for workgroup involvement opened in May 2015 and remain open. To date over 100 Arizona educators have participated in the ADE Mathematics Workgroup meetings. Participation included Arizona K-12 Teachers and Administrators from public and charter schools and teachers and professors from Arizona community colleges and universities. Ten Arizona counties have been represented in the ADE Mathematics workgroups to date.

From November 2015 through January 2016 the Mathematics Workgroup reviewed and categorized all public comments that were received including comments that did not directly address the standards. After categorizing public comments, workgroups utilized public comments as an initial data source to begin the refinement or revision process of the 2010 Mathematics Standards.

Using the public comment data, academic research, and three criteria for the review process (clarity, cognitive demand, and measurability), workgroups reviewed all K-12 mathematics standards individually within domains and across domains and grade levels. The Mathematics Workgroups utilized the following definitions when reviewing all standards for clarity, cognitive demand and measurability:

CLARITY: Quality of being easily understood. *(Merriamwebster.com)*

- The standard is clear and understandable.
- The standard can be used by educators to clearly guide learning for students.
- The standard can be used by educators to build **student** understanding.
- Examples or parenthesis in/after the standard provide clarification or define the limit of the standard.

COGNITIVE DEMAND: Represents the type of thinking and level of complexity of thought we expect students to engage in when learning. Cognitive Demand is about high levels of reasoning and thinking. Standards are written at different levels/ranges of cognitive demand.

- The standard has complexity of reasoning.
- The cluster contains a range of cognitive demand/complexity of reasoning.

MEASUREABLE: Student progression towards mastery of the standards should be observable and verifiable.

- The standard can be measured through varied modes of assessments.

A final review by ***progression*** (purposeful sequencing of teaching and learning expectations across multiple developmental stages, ages, or grade levels[1]) of content within and across grade levels occurred in May and June 2016. After final rounds of revisions were completed, the workgroup conducted an additional review of all public comments to ensure that all data were utilized during the review period. The final meeting included final edits to the introduction, glossary, and standards with a final look at the progressions within and across domains and across grade levels.

Vote of support to release the draft for public comment

The final meeting also included an individual survey of all Mathematics Workgroup members present on July 11th to vote in support of the release of the draft Arizona Mathematics Standards for public review. All 40 members present (100%) responded with strongly agree/agree that they supported the release of the current draft of the Arizona Mathematics Standards for public review.

Key Issues Raised in Public Comments/ASDC and Workgroup Meetings and Addressed in the draft Mathematics Standards

1. **The standards are not the curriculum**.

   While the Arizona Mathematics Standards may be used as the basis for curriculum, the Arizona Academic Standards are not a curriculum. Therefore, identifying the sequence of instruction at each grade - what will be taught and for how long - requires concerted effort and attention at the district and school levels. The standards do not dictate any particular curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local governing board. The Arizona Department of Education defines standards, curriculum, and instruction as:

   Standards – **What** a student needs to know, understand, and be able to do by the end of each grade/course. Standards build across grade levels in a progression of increasing understanding and through a range of cognitive demand levels.

   Curriculum –The resources used for teaching and learning the standards. Curricula are adopted at the local level by districts and schools. Curriculum refers to the **how** in teaching and learning the standards.

   Instruction – The methods used by teachers to teach their students. Instructional techniques are employed by individual teachers in response to the needs of all the students in their classes to help them progress through the curriculum in order to master the standards. Instruction refers to the **how** in teaching and learning the standards.

   This information can be found on page 3 of the Mathematics Standards Introduction.

[1] http://edglossary.org/learning-progression/

2. **The standards are not instructional practices. This information can be found on page 1 of the introduction.**

   While the Arizona Mathematics Standards define the knowledge, understanding, and skills that need to be effectively taught and learned for *each and every* student to be college and workplace ready, the standards are not instructional practices. The educators and subject matter experts who worked on the Mathematics Standards Subcommittee and Workgroups ensured that the Arizona Mathematics Standards are free from embedded pedagogy and instructional practices. The Arizona Mathematics Standards do not define how teachers should teach and must be complimented by well-developed, aligned, and appropriate curriculum materials, as well as effective instructional practices.
   This information can be found on page 3 of the Mathematics Standards Introduction.

3. **The vertical alignments/mathematical progressions K through Algebra 2 were preserved.**

   The Mathematics Workgroup reviewed the mathematical progressions within and across grade levels. Modifications and refinements to standards were made if necessary to ensure a clear progression within and across domains and grade levels.
   This information can be found on page 7 of the Mathematics Standards Introduction.

4. **All examples were deleted from the K-Algebra 2 draft mathematics standards.**

   The 2010 Arizona Mathematics standards contained examples that did not define the limit of a specific standard or provide clarification to what students need to know by the end of a grade or course. Some of the examples provided instructional guidance on "how" the standard should be taught. The purpose of a standards document is to provide clear expectations of what students should know, understand and be able to do by the end of the grade or course in mathematics.

5. **Narratives were developed for each of the eight mathematical practices.**

   The eight mathematical practices had a single sentence descriptor in the 2010 Arizona Mathematics Standards. The Mathematics Workgroup developed individual narratives to provide clarification on each of the eight standards for mathematical practice of which mathematically proficient students should exhibit. These process skills should develop over the course of a student's K-12 career.
   Standards for Mathematical Practice narratives can be found on page 5 and 6 of the Mathematics Standards Introduction and at the end of each grade level/high school course standards document.

6. **Fluency was defined by the Mathematics Workgroup to encompass all grade levels K-Algebra 2.**

Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.[2]

- **Efficiency**—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
- **Flexibility**—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check.
- **Appropriately**—knows when to apply a particular procedure.

It is critical to note that fluency is not always defined in a standard by the word "fluently" being present. Sometimes fluency is implied. Fluent is something we strive for students to achieve whenever we want them to be efficient, accurate, flexible, and appropriate in their problem solving and thinking.
The fluency definition can be found on page 2 of the Mathematics Glossary and on page 15 of the Mathematics Standards Introduction.

7. **The fluency progressions were refined across each grade in K- 6th. Refinements include clarity of language and clear grade level expectations.**

It is important to note that some wording from the 2010 standards was preserved because the wording provided clear expectations and guidance in regards to grade level outcomes in regards to fluency.
The phrase ***"by the end of 2nd grade, students will know from memory"*** was preserved in the 2nd grade standard on addition facts.
The phrase ***"by the end of 3rd grade, students will know from memory"*** was preserved in the 3rd grade standard on multiplication and division facts.
The grade 2 fluency standard can be found on page 3 of the second grade math draft standards.
The grade 3 fluency standard can be found on page 4 of the third grade math draft standards.

[2] National Council of Teachers of Mathematics, Inc. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA. Diane Briars (2016) NCTM. djbmath@comcast.net. Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics, 7*(3), 154–158.

Table 3. Fluency Progression Across All Grade Levels.

Grade	Coding	Standard
K	**K.OA.A.5**	Fluently add and subtract through 5.
1	**1.OA.C.5**	Fluently add and subtract through 10.
2	**2.OA.B.2**	Fluently add and subtract through 20. By the end of 2nd grade, know from memory all sums of two one-digit numbers.
3	**3.NBT.A.3** **3.OA.C.7**	Fluently add and subtract through 100. Fluently multiply and divide through 100. By the end of 3rd grade, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.
4	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
5	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using a standard algorithm.
6	**6.NS.B.2** **6.NS.B.3** **6.EE.A.2**	Fluently divide multi-digit numbers using a standard algorithm. Fluently add, subtract, multiple, and divide multi-digit decimals using a standard algorithm for each operation. Write, read, and evaluate algebraic expressions.
7	**7.NS.A.1.d** **7.NS.A.2.c** **7.EE.B.4.a**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiple and divide rational numbers. Fluently solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$
8	**8.EE.C.7**	Solve linear equations and inequalities in one variable.
Algebra 1	**A1.A-APR.A** **A1.A-SSE.A.1b**	Perform arithmetic operations on polynomials. Interpret complicated expressions by viewing one or more of their parts as a single entity.
Geometry	**G.G-SRT.B.5** **G.G-GPE.B** **G.G-CO.D**	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing a real-world context. Use coordinates to prove simple geometric theorems algebraically. Make geometric constructions.
Algebra 2	**A2.A-SSE.A.2** **A2.F-BF.B**	Use the structure of an expression to identify ways to rewrite it. Build new functions from existing functions.

8. **Money progression was added and time progression was refined in elementary standards.**

<u>Money</u>

The majority of content specific public comment in elementary school surrounded the lack of clear progressions of standards related to time and money in the 2010 elementary Mathematics Standards. The Workgroup created a progression of standards related to money starting in first grade and culminating in fourth grade through application of understanding through problem solving.

Money Progression			
First Grade	**Second Grade**	**Third Grade**	**Fourth Grade**
1.MD.B.4 Identify coins by name and value (pennies, nickels, dimes and quarters).	**2.MD.C.8** Find the value of a collection of coins and dollars. Record the total using $ and ¢ appropriately.	**3. MD.A.2** Solve word problems involving money through $20.00, using symbols $, ₵, and "." as a distinction between dollars and cents.	**4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. **4.MD.A.2** Solve word problems in a real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations that feature a measurement scale.

Time

Public comment was also abundant around the need for standards related to understanding time. The Mathematics Workgroup refined the current time standards and clarified the progression from grades first through fourth.

Time Progression			
First Grade	**Second Grade**	**Third Grade**	**Fourth Grade**
1.MD.B.3 Tell and write time in hours and half-hours using analog and digital clocks.	**2.MD.C.7** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	**3.MD.A.1** Tell and write time to the nearest minute and solve word problems involving addition and subtraction of time intervals in minutes.	**4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. **4.MD.A.2** Solve word problems in a real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations that feature a measurement scale.

9. **Probability progression was refined between middle school and high school.**

The probability progression from seventh grade through high school was reviewed and some standards were moved to strengthen the progression across grade levels. Standard 7.SP.C.8 was moved to 8th grade to ensure that probability was taught consistently throughout the middle school grades (7th through 9th grade). The probability cluster – Conditional Probability was moved from Algebra 2 to Algebra 1 to also strengthen the probability progression from 7th to 8th to Algebra 1.

10. Created distinct Algebra 1 and Algebra 2 standards.

The 2010 Mathematics Standards contained "dual standards" in Algebra 1 and Algebra 2. These "dual standards" were standards that occurred in both courses and the limits between the two standards were defined in a table at the back of the high school standards documents. The Mathematics Workgroup and public comments felt it was essential that these "dual standards" were clarified and that unique standards were created for each course that clear limits between Algebra 1 and Algebra 2. Here is one example.

Algebra 1	Algebra 2
A1-A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. **Focus on numerical expressions**, such as recognizing $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the form (53+47)(53-47). **Focus on polynomial expressions in one variable**, such as seeing an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$.	A2-A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. **Extend polynomial expressions to multivariable expressions.** **Focus on rational or exponential expressions** seeing that $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.

In this example of a "dual standard" that has been refined to create two unique standards that are represented within two different courses. The ***"focus"*** of the standard in Algebra 1 shows the limit of the standard is on numerical expressions and on polynomial expressions in one variable. The ***"focus"*** of the standard in Algebra 2 shows the limit of the standard is on rational or exponential expressions and to ***"extend"*** polynomial expression to multivariable expressions. They key words help to define the limits between the two courses of study.

11. Plus Standards for accelerated, advanced, honors and fourth credit courses in high school mathematics.

The plus (**P**) standards are standards that are found outside the limits of a high school Algebra 1, Algebra 2, or Geometry minimum course of study as defined by the standards. The plus standards are represented through coding that begins with **P** instead of a course designation. The plus standards are intended to be included in honors, accelerated, advanced courses, fourth credit courses, as well as extensions of the regular courses (Algebra 1, Algebra 2, and Geometry).