Kindergarten Draft Mathematics Standards - Technical Review			
Code	**2010 Standards**	**Refinement/Draft**	**Notes**
Counting & Cardinality (CC)			
Counting & Cardinality	**Know number names and the count sequence.**		
K.CC.A.1	Count to 100 by ones and by tens.	No refinement needed to the existing standard	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of counting, cardinality, and number sense.
K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1)	Count forward beginning from a given number instead of having to begin at 1.	Removed "within the known sequence" because of possible misinterpretation. Removed parenthesis, and added "instead of having to begin at 1" to standard.
K.CC.A.3	Write numbers from 0–20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	No refinement needed to the existing standard	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
Counting & Cardinality	**Understand the relationship between numbers and quantities.**		
K.CC.B.4	Connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	No refinement needed to the existing standard	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of counting, cardinality, and number sense.
K.CC.B.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	Count to answer questions about “how many?” when 20 or fewer objects are arranged in a line, a rectangular array, or a circle or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	The standard was reworded to clarify the meaning and intent.
Counting & Cardinality	**Compare numbers and quantities.**		
K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects)	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to ten objects.)	The example was deleted from the standard because it did not promote clarity.
K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.	No refinement needed to the existing standard	No refinement needed to the existing standard as this is a neccesary standard in the progression of understanding of numeric representation of value. It meets criteria for clarity, cognitive demand, and measurability.
Operations and Algebraic Thinking (OA)			
Operations and Algebraic Thinking	**Understand addition as putting together and adding to, and understanding subtraction as taking apart and taking from.**		
K.0A.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problems. This applies wherever drawings are mentioned in the Standards.)	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.	The example was deleted from the standard because it did not promote clarity. The standard now meets criteria for clarity, cognitive demand, and measurability.
K.0A.A.2	Use addition and subtraction through 10 to solve word problems involving multiple problem types (See Table 1), using a variety of strategies.	Use addition and subtraction through 10 to solve word problems involving multiple problem types (See Table 1), using a variety of strategies.	The example was deleted from the standard because it did not promote clarity. "Using a variety of strategies" was added to maintain consistency. " (See Table 1)" was added to clarify problem types appropriate to Kindergarten.
K.0A.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings, and record each decomposition with a drawing or equation.	" e.g." was removed but "by using objects or drawings" remained for clarification of the standard. The word "by" was changed to "with a drawing or equation" to clarify the language of the standard. The example was deleted from the standard because it did not promote clarity.

K.0A.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings, and record the answer with a drawing or equation.	" e.g." was removed but "by using objects or drawings" remained for clarification of the standard.
K.0A.A.5	Fluently add and subtract within 5.	Fluently add and subtract through 5.	"Within", was changed to "through", to make sure the number 5 was understood to be included.
Number and Operations in Base Ten (NBT)			
Number & Operations in Base Ten	**Work with numbers 11-19 to gain foundations for place value.**		
K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Compose and decompose numbers from 11 to 19 into ten ones and additional ones by using objects or drawings and record each composition or decomposition with a drawing or equation.	The phrase, "understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones" was deleted because it is already stated within the existing standard. The word "further" was changed to "additional" to clarify the language of the standard. The word "by" was changed to "with" to clarify the language of the standard. The "e.g" was removed and "by using objects or drawings" remained. The example of the equation was deleted as it does not provide limits to the standard.
Number & Operations in Base Ten	**Use place value understanding and properties of operations to add and subtract.**		
K.NBT.B.2		Demonstrate conceptual understanding of addition and subtraction through 10 using a variety of strategies.	This is a new standard that was added to adhere to the progression of whole number fluency from Kindergarten-4th grade.
Measurement & Data (MD)			
Measurement & Data	**Describe and compare measurable attributes.**		
K.MD.A.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	Describe several measurable attributes of a single object such as length and weight.	Two sentences were combined due to redundancy in the standard and provide clarity.
K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.	Directly compare two objects with a measurable attribute in common, to see which object has “more of” or “less of” the attribute, and describe the difference.	The example was deleted because it does not provide clarity or limit of the standard.
Measurement & Data	**Classify objects and count the number of objects in categories.**	**Classify objects and count the number of objects in each category.**	Minor wording changes for clarity.
K.MD.B.3	Classify objects or people into given categoires; count the number in each cateogry and sort the cateogires by count. (Note: limit category counts to be less than or equal to 10.)	No refinement needed to the existing standard	Per public comment about sorting, it is included in this standard. Identification is inherent in classification and therefore is implied in the standard, so there was no refinement made to the standard. This is a necessary standard in the progression of measurement and data.
Geometry (G)			
Geometry	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**	**Identify and describe shapes.**	Removed the list of shapes since K.G.A.3 and K.G.B.4 state 2-D and 3-D shapes.
K.G.A.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	No refinement needed to the existing standard	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of geometry.
K.G.A.2	Correctly name shapes regardless of their orientation or overall size.	No refinement needed to the existing standard	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of geometry.
K.G.A.3	Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	No refinement needed to the existing standard	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of geometry.
Geometry	**Analyze, compare, create, and compose shapes.**		
K.G.B.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	The example was deleted from the standard because it did not promote clarity or limits to the standard.
K.G.B.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	Model shapes in the world by building and drawing shapes.	The examples were deleted as they limited the standard. The phrase "from components" was deleted as it did not provide clarity to the standard.

K.G.B.6	Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"	Model shapes in the world by building and drawing shapes.	The example was deleted from the standard because it did not promote clarity.
K.MP	**Standards for Mathematical Practice**		
K.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	
K.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	
K.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

K.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
K.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
K.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
K.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
K.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

1st Grade Draft Mathematics Standards - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Operations and Algebraic Thinking (OA)			
1.OA.A	**Represent and solve problems involving addition and subtraction.**		
1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Table 1.)	Use addition and subtraction through 20 to solve word problems involving multiple problem types (see Table 1) using a variety of strategies.	Individual problem types were removed to help with clarity with reference to Table 1 that includes all the problem types. The examples were deleted from the standard as they illustrate instruction. The word "within" was changed to "through" to encompass the number 20.
1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Table 1)	"e.g." and "by" were deleted but the descriptors were kept to maintain the scope of the standard. it meets criteria for clarity, cognitive demand, and measurability. It now meets criteria for clarity, cognitive demand, and measurability.
1.OA.B	**Understand and apply properties of operations and the relationship between addition and subtraction.**		
1.OA.B.3	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) (Students need not use formal terms for these properties.)	Apply properties of operations (commutative and associative properties of addition) as strategies to add and subtract through 20. (Students need not use formal terms for these properties.)	The examples were deleted from the standard as they did not clarify the limit of the standard. The standard was clarified to state "within 20" for consistency within the domain. The word "within" was changed to "through" to encompass the number 20.

Code	2010 Standards	Refinement/Draft	Notes
1.OA.B.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.	Understand subtraction through 20 as an unknown-addend problem. (See Table 1)	The example was deleted from the standard as it did not clarify the limit of the standard. "within 20" was added to the standard to provide consistency and clarity within the domain. The word "within" was changed to "through" to encompass the number 10.
1.OA.C	**Add and subtract within 20.**	**Add and subtract through 10.**	The number for the cluster was changed to 10 to reflect the fluency progression from kindergarten to second grade. The word "within" was changed to "through" to encompass the number 10.
1.OA.C.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	Remove this standard	This is a strategy used when adding and subtracting and does not meet the definition for a standard.
1.OA.C.5	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the	Fluently add and subtract through 10.	The examples were deleted from the standard as they did not clarify the limit of the standard. The examples illustrate instruction. This standard is only the fluency part of the original standard. See 1.NBT.C.7 for understanding of addition and subtraction through 20. The code changed due to removal of the previous standard. The word "within" was changed to "through" to encompass the number 10.
1.OA.D	**Work with addition and subtraction equations.**		
1.OA.D.6	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.	Understand the meaning of the equal sign, regardless of its placement within an equation, and determine if equations involving addition and subtraction are true or false.	"Regardless of its placement within an equation" was added to further clarify the scope of the standard. The examples were deleted as they do not clarify the limit of the standard. The code changed due to removal of a previous standard in this domain.

Code	2010 Standards	Refinement/Draft	Notes
1.OA.D.7	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations: 8 + ? = 11, 5 = □ – 3, 6 + 6 = □	Determine the unknown whole number in any position in an addition or subtraction equation relating three whole numbers.	The examples were deleted from the standard as they do not clarify the limit of the standard. The examples illustrated instruction. The phrasing "in any position" was added to further clarify the scope of the standard. The code changed due to removal of a previous standard in this domain.
Number and Operations in Base Ten (NBT)			
1.NBT.A	**Extend the counting sequence.**		
1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criterion for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of Number and Operations in Base Ten.
1.NBT.B	**Understand place value.**		
1.NBT.B.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	Understand that the two digits of a two-digit number represent groups of tens and some ones. Understand the following as special cases: a. 10 can be thought of as a group of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	"Groups of" and "some" were added to further emphasize that ten can represent a single entity (a group) and at the same time ten ones. "Group" replaced the word "bundle" for more precise mathematical language.
1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criterion for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of Number and Operations in Base Ten.

Code	2010 Standards	Refinement/Draft	Notes
1.NBT.C	**Use place value understanding and properties of operations to add and subtract.**		
1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	Add through 100 using models and/or strategies based on place value, properties of operations, and the relationship between addition and subtraction.	"Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten." was deleted as it indicates instruction. "relate the strategy to a written method and and explain the reasoning used " was deleted as it is included in the mathematical practice standards.
1.NBT.C.5.	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criterion for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of Number and Operations in Base Ten.
1.NBT.C.6	Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Add and subtract multiples of 10 through 100 using models and/or strategies based on place value, properties of operations, and the relationship between addition and subtraction.	"In the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences)," was deleted and 'within 100' was added to clarify the limit of the standard. "add" was added to the standard to clarify the cluster heading of adding and subtracting using place value. The word "within" was changed to "through" to encompass the number 100.
1.NBT.C.7		Demonstrate understanding of addition and subtraction through 20 using a variety of place value strategies, properties of operations, and the relationship between addition and subtraction.	Standard 1.OA.C.6 was separated into two standards to clearly show the difference between building understanding and fluency. This is the understanding portion of 1.OA.C.6 that leads to the fluency through 20 in grade 2.
Measurement and Data (MD)			

Code	2010 Standards	Refinement/Draft	Notes
1.MD.A	Measure lengths indirectly and by iterating length units.		
1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	Order three objects by length. Compare the lengths of two objects indirectly by using a third object.	The semicolon in this standard was replaced with a period to denote separate ideas. It meets criteria for clarity, cognitive demand, and measurability.
1.MD.A.2	Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.	Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	"Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps." was deleted as this is repetitive language already mentioned in the standard.
1.MD.B	**Tell and write time.**	**Work with time and money.**	The cluster title was revised to reflect the addition of money in first grade.
1.MD.B.3	Tell and write time in hours and half-hours using analog and digital clocks.	Tell and write time in hours and half-hours using analog and digital clocks.	No refinement needed on this standard since it meets criterion for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of Measurement and Data.
1.MD.B.4		Identify coins by name and value (pennies, nickels, dimes and quarters).	This new standard was added per public comment to include working with coins in 1st grade. It supports the progression of money from 1st to 4th grade.
Measurement & Data	**Represent and interpret data.**		

Code	2010 Standards	Refinement/Draft	Notes
1.MD.C.5	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	No refinement needed.	No refinement needed on this standard since it meets criterion for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of Measurement and Data. In previous standards, this was 1.MD.C.4, it is now 1.MD.C.5. Coding was changed to reflect the addition of 1.MD.C.4
Geometry (G)			
1.G.A	**Reason with shapes and their attributes.**		
1.G.A.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	Distinguish between defining attributes (open, closed, number of sides, vertices) versus non-defining attributes (color, orientation, size) for two-dimensional shapes; build and draw shapes to possess defining attributes.	"(e.g. triangles are closed and three-sided)" was deleted as it did not clarify the limit of the standard. "e.g." was deleted but the descriptors were kept to maintain the scope of the standard. "two-dimensional shapes" was added to clarify the expectation of the standard.
1.G.A.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as “right rectangular prism.”)	Compose two-dimensional shapes or three-dimensional shapes to create a composite shape and compose new shapes from the composite shape.	The examples (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) and (cubes, right rectangular prisms, right circular cones, and right circular cylinders) were deleted as they were limiting to the standard.
1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves and fourths. , and use the phrases half of, fourth of, and quarter of. Understand that decomposing into more equal shares creates smaller shares.	It now meets criteria for clarity, cognitive demand, and measurability.
1.MP	**Standards for Mathematical Practice**		

Code	2010 Standards	Refinement/Draft	Notes
1.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	
1.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

Code	2010 Standards	Refinement/Draft	Notes
1.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

Code	2010 Standards	Refinement/Draft	Notes
1.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
1.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	

Code	2010 Standards	Refinement/Draft	Notes
1.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
1.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
1.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

2nd Grade Draft Mathematrics Standards - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Operation and Algebraic Thinking (OA)			
Operations & Algebraic Thinking	**Represent and solve problems involving addition and subtraction.**		
2.OA.A.1.	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 1.)	Use addition and subtraction through 100 to solve one-step word problems. (See Table 1) Use addition and subtraction through 20 to solve two-step word problems. Represent a word problem as an equation with a symbol for the unknown.	The example was deleted as it is already included in Table 1. "Using single-digit addends" was added to clarify limits of two-step word problems. Changed within to through to include the number 100.
Operations & Algebraic Thinking	**Add and subtract within 20.**	Add and subtract through 20.	within' was changed to 'through' in the cluster heading for consistent language within the fluency progression.
2.OA.B.2.	Fluently add and subtract within 20 using mental strategies. (See standard 1.OA.6 for a list of mental strategies.)	Fluently add and subtract through 20. By the end of Grade 2, know from memory all sums of two one-digit numbers.	(See standard 1.OA.6 for a list of mental strategies.) and "using mental strategies" were removed as these examples do not clarify the limit of the standard. "Within" was changed to "through" for consistent language within the fluency progression.
Operations & Algebraic Thinking	**Work with equal groups of objects to gain foundations for multiplication.**		
2.OA.C.3.	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	Determine whether a group of objects (up to 20) has an odd or even number of members. Write an equation to express an even number as a sum of two equal addends.	The examples were deleted as they did not clarify the limit of the standard.

Code	2010 Standards	Refinement/Draft	Notes
2.OA.C.4.	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	Use addition to find the total number of objects arranged in rectangular arrays (with up to 5 rows and up to 5 columns). Write an equation to express the total as a sum of equal addends.	Parenthesis were added to define the limit of rectangular arrays used in 2nd grade
Number and Operations in Base Ten (NBT)			
Number & Operations in Base Ten	**Understand place value.**		
2.NBT.A.1.	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	The examples were deleted as they did not clarify the limit of the standard.
2.NBT.A.2.	Count within 1000; skip-count by 5s, 10s, and 100s.	Count to 1000 by 1s, 5s, 10s, and 100s from different starting points.	Parenthesis were added to clarify that students should skip count starting at different numbers. This addresses the public comment requesting that students "skip count from different starting points."
2.NBT.A.3.	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	No refinement needed to the existing standard.	No refinement needed to the existing standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard progression of Numbers and Operations in Base Ten.
2.NBT.A.4.	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	No refinement needed to the existing standard.	No refinement needed to the existing standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard progression of Numbers and Operations in Base Ten.
Number & Operations in Base Ten	**Use place value understanding and properties of operations to add and subtract.**		

Code	2010 Standards	Refinement/Draft	Notes
2.NBT.B.5.	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Demonstrate understanding of addition and subtraction through 100 using a variety of strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	"Demonstrate understanding" was added to follow the fluency progression in K-4. "Within" was changed to "through" for consistent language within the fluency progression.
2.NBT.B.6.	Add up to four two-digit numbers using strategies based on place value and properties of operations.	Add up to four two-digit numbers using strategies based on place value and properties of operations. **(Use of a standard algorithm is a 4th Grade standard see 4.NBT.B.4)**	"(Use of a standard algorithm is a 4th Grade standard see 4.NBT.B.4)" was added to clarify the progression of student strategies from K-4.
2.NBT.B.7.	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	**Demonstrate understanding** for addition and subtraction through 1000, **connecting** concrete models or drawings to strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written **form**.	"Demonstrate understanding" was added to follow the fluency progression in K-4. "using" was changed to "connecting" in order to clarify the connection between place value, properties of operations and the relationship between addition and subtraction. Changed the word within to through so the standard would include the number 100. "Written method" was changed to "written form" as it is more precise mathematical language. "Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds." was removed. This is an example of instructional support.
2.NBT.B.8.	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	Based on public comment, clarification of using different starting points was added to the standard.
2.NBT.B.9.	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)	No refinement needed to the existing standard.	No refinement needed on this standard, since it meets criteria for clarity, cognitive demand and measurability. This is a nescessary standard in the progression of Number and Operations in Base Ten.
Measurement and Data (MD)			
Measurement & Data	**Measure and estimate lengths in standard units.**		
2.MD.A.1.	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Measure the length of an object by selecting and using appropriate tools.	"Such as rulers, yardsticks, meter sticks, and measuring tapes" was deleted because it was limiting to the standard.
2.MD.A.2.	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	**Understand that the length of an object does not change regardless of the units used.** Measure the length of an object twice, using different **standard** length units for the two measurements; describe how the two measurements relate to the size of the unit chosen.	"Understand that the length of an object does not change regardless of the units used" was added to clarify the meaning of the standard. "Standard" was added to clarify the cluster heading.

Code	2010 Standards	Refinement/Draft	Notes
2.MD.A.3.	Estimate lengths using units of inches, feet, centimeters, and meters.	No refinement needed to the existing standard.	No refinement needed on this standard, since it meets criteria for clarity, cognitive demand and measurability. This is a nescessary standard in the progression of Measurement and Data.
2.MD.A.4.	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	No refinement needed to the existing standard.	No refinement needed on this standard, since it meets criteria for clarity, cognitive demand and measurability. This is a nescessary standard in the progression of Measurement and Data.
Measurement & Data	**Relate addition and subtraction to length.**		
2.MD.B.5.	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	Use addition and subtraction through 100 to solve word problems involving lengths that are given in the same units, by using drawings and equations with a symbol for the unknown number to represent the problem.	The example "(such as drawings of rulers)" was removed from the standard as it is an example of instructional support. "...by using drawings and equations with a symbol for the unknown number to represent the problem" was deleted as it indicates instruction.
2.MD.B.6.	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences through 100 on a number line diagram.	"Within" was changed to "through" for consistent language throughout the standards.
Measurement & Data	**Work with time and money.**		
2.MD.C.7.	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	No refinement needed to the existing standard.	No refinement needed on this standard, since it meets criteria for clarity, cognitive demand and measurability. This is a necessary standard in the progression of Measurement and Data.
2.MD.C.8.	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?	Find the value of a collection of coins and dollars. Record the total using $ and ¢ appropriately.	This is a new standard based on public comment to include counting money. It follows the progression of working with money from 1st through 3rd grade. "Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately" was moved to 3rd grade.
Measurement & Data	**Represent and interpret data.**		

Code	2010 Standards	Refinement/Draft	Notes
2.MD.D.9.	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.		No refinement needed to the existing standard, since it meets the criteria for clarity, cognitive demand and measurability.
2.MD.D.10.	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (See Table 1.)	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in **the** graph. (See Table 1.)	"In a bar graph" was changed to "in the graph" to clarify student expectations.
Geometry (G)			
Geometry	**Reason with shapes and their attributes.**		
2.G.A.1.	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (Sizes are compared directly or visually, not compared by measuring.)	**Identify and describe specified attributes of two- and three-dimensional shapes, according to the number and shape of faces, number of angles, and the number of sides and/or vertices. Draw two-dimensional shapes.**	The standard was reworded to clarify the expectation of the standard.
2.G.A.2.	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	No refinement needed to the existing standard.	No refinement needed on this standard, since it meets the criteria for clarity, cognitive demand and measurability. This is a necessary standard in the progression of Geometry.
2.G.A.3.	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	No refinement needed to the existing standard.	No refinement needed on this standard, since it meets the criteria for clarity, cognitive demand and measurability. This is a necessary standard in the progression of Geometry.
2.MP	**Standards for Mathematical Practice**		
2.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they	

Code	2010 Standards	Refinement/Draft	Notes
2.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	
2.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

Code	2010 Standards	Refinement/Draft	Notes
2.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
2.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
2.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	

Code	2010 Standards	Refinement/Draft	Notes
2.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
2.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

3rd Grade Draft Math Standards - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Operations and Algebraic Thinking (OA) *Note: Grade 3 expectations in this domain are limited to multiplication through 10 x 10 and division with both quotients and divisors less than or equal to 10.*			
3.OA.A	**Represent and solve problems involving multiplication and division.**		
3.OA.A.1.	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.	Interpret products of whole numbers as the total number of objects in equal groups. Describe a context in which multiplication can be used to find a total number of objects. (See Table 2)	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The limit of the standard was clarified in the addition of the Domain note.
3.OA.A.2.	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	Interpret quotients of whole numbers by: •determining the number of objects in each share when a total number of objects are partitioned into a given number of equal shares. •determining the number of shares when the total number of objects and the size of each share is given. Describe a context in which division can be used to find the numbers of objects in each share or the number of shares. (See Table 2)	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The limit of the standard was clarified in the addition of the Domain note involving the limit.
3.OA.A.3.	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Table 2.)	Use multiplication and division to solve word problems in situations involving equal groups, arrays, and measurement quantities. (See Table 2)	Based on public comment, the recommended specification of limits (products and dividends limited through 10 x 10) has been added to the domain explanation. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.

Code	2010 Standards	Refinement/Draft	Notes
3.OA.A.4.	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ?÷ 3, 6 × 6 = ?.	Determine the unknown whole number in a multiplication or division equation using properties of operations and/or the relationship between multiplication and division.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The limit of the standard was clarified in the addition of the note for the Domain. Revisions align to laguage use in 2.NBT.B.7
3.OA.B	**Understand properties of multiplication and the relationship between multiplication and division.**		
3.OA.B.5.	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property of multiplication.)	Apply properties of operations as strategies to multiply and divide. This includes use of known facts to solve unknown facts through the application of the commutative, associative, and distributive properties of multiplication. (Students do not need to use the formal terms for these properties)	The edit to the current standard involved deletion of the examples. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.. The limit of the standard was clarified in the addition of the note in the Domain.
3.OA.B.6.	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.	Understand division as an unknown-factor problem. Represent division as a multiplication problem with a missing factor.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The limit of the standard was clarified in the addition of the note in the Domain. Addition of the second sentence provides clarity and raises the level of cognitive demand.
3.OA.C	**Multiply and divide within 100.**	**Multiply and divide through 100.**	The limit of the domain was clarified to align to all standards within the domain.

Code	2010 Standards	Refinement/Draft	Notes
3.OA.C.7.	**change in coding from new 3. NBT.A.5 to 3.OA.7	Demonstrate understanding of multiplication and division through 100 (limited through 10 x 10) using strategies such as the relationship between multiplication and division or properties of operations.	The addition of this standard provides the understanding leading to fluency in 3.OA.C.7. Children benefit from the ability to use a variety of strategies to demonstrate their understanding of multiplication and division as they progress toward fact fluency (CGI, 2015). This aligns with the progression of fluency.
3.OA.C.8.	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Change in coding from 3.OA.7**	Fluently multiply and divide through 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The limit of the standard was clarified in the addition of the Limit included in the note of the Domain. To emphasize the progression of whole number fluency, 3.OA.C7 was divided into two standards, that which is still located here and a new standard 3.NBT.A.3.
3.OA.D	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**		
3.OA.D.9.	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). **Change in coding from 3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order-Order of Operations).	The existing standard guides learning and supports the complexity of reasoning. Minor word changes to make clarity better.

Code	2010 Standards	Refinement/Draft	Notes
3.OA.D.10.	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. **Change in coding from 3.OA.9**	Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. This meets clarity, measureablility and cognitive demand.
Number and Operations in Base Ten (NBT) *Note: A range of algorithms may be used.*			
3.NBT.A	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**		
3.NBT.A.1.	Use place value understanding to round whole numbers to the nearest 10 or 100.	No refinement needed to the existing standard	The existing standard guides learning and supports the complexity of reasoning. It meets the expectations of clarity, cognitive demand and measureability.
3.NBT.A.2.	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Demonstrate understanding of addition and subtraction through 1000 using a variety of strategies such as properties of operations and the relationship between addition and subtraction.	The edit to the current standard differentiates fluency from conceptual understanding. 3.NBT.A2 was written to provide understanding of addition and subtraction while fluency in addition and subtraction in grade 3 is addressed in standards 3.NBT.A.3. This standard is foundational to fluency in 4th grade 4.NBT.B.4.
3.NBT.A.3.	**NEW** 3. NBT.A.3	Fluently add and subtract through 100.	The addition of this standard fills the gap in fluency in addition and subtraction between grades 2 and 4.
3.NBT.A.4	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. **Change in coding from 3.NBT.A.3 to 3.NBT.A.4 for consistency within the cluster	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.

Code	2010 Standards	Refinement/Draft	Notes
Number and Operations-Fractions (NF) *Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.*			
3.NF.A	**Understand fractions as numbers.**		
3.NF.A.1.	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	Understand a unit fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts 1/*b* .	Include the term **unit fraction** to help in clarity.
3.NF.A.2.	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	Understand a fraction as a number on a number line; represent fractions on a number line diagram. a. Represent a unit fraction (1/*b*) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it from 0 into *b* equal parts. b. Represent a fraction *a/b* on a number line diagram by marking off a lengths of unit fractions 1/*b* from 0. Understand that the resulting interval has size a/b and that its endpoint locates the number *a/b* on the number line including values greater than 1.	Include the term unit fraction to help clarify the meaning. 3.NF.A.2b included "values greater than 1" to provide more clarification. "Recognize" has been changed to "understand" for measurability purposes.

Code	2010 Standards	Refinement/Draft	Notes
3.NF.A.3.	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they represent the same size part of the whole, or the same point on a number line. b. Understand and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and understand fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.	Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. "Recognize" has been changed to "understand" for measurability purposes.
Measurement and Data (MD)			
3.MD.A	**Solve problems involving measurement.**		

Code	2010 Standards	Refinement/Draft	Notes
3.MD.A.1.	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. Connections: 3.RI.3; 3.RI.7; ET03-S1C4-01	Tell and write time to the nearest minute and solve word problems involving addition and subtraction of time intervals in minutes.	Refer to notes from small group discussion on the progression of time and money.
3.MD.A.2.	**new**	Solve word problems involving money through $20.00, using symbols $, ¢, and "." as a distinction between dollars and cents.	This is a new standard that adheres to the progression of money in the standards.
~~**3.MD.A.2.**~~	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Table 2).	3.MD.A.3: Measure and estimate liquid volumes and masses of objects using metric and customary units. (Excludes compound units such as cm^3 and finding the geometric volume of a container. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems involving notions of “times as much” (see Table 2).	This was previously standard 3.MD.A 2(clarity) **Replace "standard units of" with metric and customary** to clarify the standard. **Delete** the limiting example units. **Delete** the example of using drawings of beakers because it defines the "how" or instruction of the standard. **Keep** both of the "excludes" statements because they define the limit of the standard. With revisions, it meets the clarity, measureability and cognitive demand requirements.
3.MD.B	**Represent and interpret data.**		
~~**3.MD.B.3.**~~	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	3.MD.B.4: Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step problems using information presented in scaled picture and bar graphs (See Table 1).	To better align to clarity and cognitive demand, replaced "draw" with "create" because draw is too limiting.

Code	2010 Standards	Refinement/Draft	Notes
~~**3.MD.B.4.**~~	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	3.MD. B.5: Generate measurement data by measuring lengths to the nearest quarter inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	Changed the description of the ruler to measure to nearest quarter of an inch to provide clarity.
3.MD.C	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**	**Geometric measurement: Understand concepts of area and perimeter.**	This was changed because cluster D was worded as a standard rather than cluster. Just added perimeter to cluster C and removed cluster D
3.MD.C.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	3.MD.C.6: Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	The development of the understanding of area is crucial for the progression to other standards. Recognize is not measureable and was changed to understand. This aligns to the 5th grade standards involving volume.
~~**3.MD.C.6.**~~	~~Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).~~	**Remove this standard**	This has been removed because they are telling student to count, it is not grade level appropriate and tiliing is found in the new standard 3.MD.C.7 (Find the area of rectangle with whole-number side lengths by tiling it.)

Code	2010 Standards	Refinement/Draft	Notes
3.MD.C.7.	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. **Understand** area as additive by finding the areas of rectilinear figures.	The standard guides learning and supports complexity of reasoning. There were minor areas that indicated the "how" that were removed. Recognize is not measureable and we want kids to understand that area is additive. In Part d, the last part of the old standard is the how, which is a school based decision and not in the scope of our work and thus it was removed.
	New	3.MD.C.8: Understand perimeter as a linear attribute of plane figures and distinguish between linear and area measures.	This is the old Cluster D that is written as a standard and coded as 3.MD.C.8
Cluster	~~**Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**~~	Remove Cluster D	This cluster is removed. Added perimter to Cluster C and made this a standard (3.MD.C.8) since it is written in standard form
3.MD.D.8.	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**3.MD.C.9:** Solve problems utilizing real-world contexts involving perimeters of polygons. (See Table 1-unknown in various positions)	Inclusions of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making. Coding change to align with Cluster C and removal of cluster D. Removed extranious information that is not needed in the standard for clarity.
Geometry (G)			
Geometry	Reason with shapes and their attributes.		

Code	2010 Standards	Refinement/Draft	Notes
3.G.A.1.	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	Understand that shapes in different categories may share attributes and those shared attributes can define a larger category. Draw examples of shapes that do not belong to any of these subcategories.	Removed clutter and unneccessry information from the standard. Examples are not part of standards and were removed. With the changes, it now meets the requirements for clarity.
3.G.A.2.	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction (1/*b*) of the whole. (Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8).	Unit fraction was clarified with the parenthesis. The example is not needed with the clarification of unit fraction. This now aligns with wording in 3.NF.
3.MP	**Standards for Mathematical Practice**		
3.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and	
3.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities,	

Code	2010 Standards	Refinement/Draft	Notes
3.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	
3.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically	

Code	2010 Standards	Refinement/Draft	Notes
3.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
3.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words	
3.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	

Code	2010 Standards	Refinement/Draft	Notes
3.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

4th Grade Draft Math Standards - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Operations and Algebraic Thinking (OA)			
4.OA.A	**Use the four operations with whole numbers to solve problems.**		
4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations. (35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.)	It meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding operations and alegbraic thinking. Slightly rearranged the wording for clarity purposes.
4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Table 2.)	Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison by using models and equations with a symbol for the unknown number to represent the problem. (See Table 2)	Distinguishing multiplicative comparison from additive comparison is a part of the standard, not an example. The example within the standard provides clarification.
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.	No refinement needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding operations and alegbraic thinking.
AZ.4.OA.A.3.1.	Solve a variety of problems based on the multiplication principle of counting. a. Represent a variety of counting problems using arrays, charts, and systematic lists, e.g., tree diagram. b. Analyze relationships among representations and make connections to the multiplication principle of counting.	Remove standard	AZ.4.OA.A.3.1 has been removed. This supports public comments and this does not fit in the progression here.
4.OA.B	**Gain familiarity with factors and multiples.**		

Code	2010 Standards	Refinement/Draft	Notes
4.OA.B.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	No revision needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding operations and alegbraic thinking.
4.OA.C	**Generate and analyze patterns.**		
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	No revision needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding operations and alegbraic thinking.
Number and Operations in Base Ten (NBT) *Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*			
4.NBT.A	**Generalize place value understanding for multi-digit whole numbers.**		
4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.	The inclusion of division in the refinement is to ensure flexibility of thinking. Students should be able to see that, for example, 700 is 10 times greater than 70 by recognizing that 700 ÷ 70 = 10. It now meets criteria for clarity, cognitive demand, and measurability.
4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	No revision needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding number and operations in base ten.

Code	2010 Standards	Refinement/Draft	Notes
4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.	No revision needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding number and operations in base ten.
4.NBT.B	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**		
4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Fluently add and subtract multi-digit whole numbers using a standard algorithm.	The phrase "the standard algorithm" implies a singular efficient, and accurate method to problem solving. It is recommended to change "the standard algorithm" to "a standard algorithm" based on a publication (ED Thoughts What We Know About Mathematics Teaching and Learning) edited by the Mid-Continent Research for Education Learing (McREL): "An algorithm is a precise, step-by-step method or set of rules for solving problems of a particular type...there are algorithms of many types." (p. 82) This matches phrasing from across grade levels in the fluency progression.
4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Demonstrate understanding of multiplication by multiplying whole numbers up to four digits by a one-digit whole number and multiplying two two-digit numbers, using a variety of strategies such as the properties of operations and the relationship between multiplication and division. Illustrate and explain the calculation.	To adhere to the progression of fluency in multiplication, this standard is developing understanding so they will be prepared to become fluent in 5th grade. The language "by using equations, rectangular arrays, and/or area models" has been removed from the standard as it describes "how" to illustrate and explain.
4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation.	The language "by using equations, rectangular arrays, and/or area models" has been removed from the standard as it describes "how" to illustrate and explain.
Number and Operations in Fractions (NF) *Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100*			
4.NF.A	**Extend understanding of fraction equivalence and ordering.**		

Code	2010 Standards	Refinement/Draft	Notes
4.NF.A.1.	Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	Explain why a fraction *a/b* is equivalent to a fraction (*n* x *a*)/(*n* x *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.	The existing standard guides learning and supports the complexity of reasoning, it meets criteria for clarity, cognitive demand, and measurability. "Recognize" has been changed to "understand" for measurability purposes.
4.NF.A.2.	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Compare two fractions with different numerators and different denominators by creating common denominators or numerators and by comparing to a benchmark fraction such as 1/2. Use number sense of fractions to assess the reasonableness of answers. Understand that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.	The edit to the standard reflects consistency across the number and operations in fractions domain (in grades 3-5) as it relates to assessing the reasonableness of answers, it meets criteria for clarity, cognitive demand, and measurability. "Recognize" has been changed to "understand" for measurability purposes.
Number & Operations - Fractions	**Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**		
4.NF.B.3.	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8=1/8+1/8+1/8 ; 3/8=1/8+2/8; 2 1/8=1 + 1+1/8=8/8+8/8 +1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Understand a fraction *a/b* with a > 1 as a sum of unit fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator by recording decompositions using a variety of representations, including equations. Justify decompositions. c. Add and subtract mixed numbers with like denominators by using properties of operations and the relationship between addition and subtraction or by replacing each mixed number with an equivalent fraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators using a variety of representations.	Included the term "unit fraction" to help in clarification. Removed examples that do not provide limits or clarification to the standards

Code	2010 Standards	Refinement/Draft	Notes
4.NF.B.4.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5´(1/4), recording the conclusion by the equation 5/4 = 5´(1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3´(2/5) as 6´(1/5), recognizing this product as 6/5. (In general, n´(a/b)=(n´a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of a unit fraction ($1/b$). (In general, $a/b = a$ x ($1/b$).) b. Understand a multiple of a/b as a multiple of a unit fraction ($1/b$), and use this understanding to multiply a fraction by a whole number. (In general, n x (a/b) = (n x a)/b .) c. Solve word problems involving multiplication of a fraction by a whole number.	Included the term unit fraction in the standard for clarification and maintain consistency across the fraction progression. "Recognizing" has been changed to "understanding" for measurability purposes. Removed examples that do not provide limits or clarification to the standards
4.NF.C	**Understand decimal notation for fractions, and compare decimal fractions.**	**Understand decimal notation for fractions, and compare decimal fractions.**	
4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general, but addition and subtraction with unlike denominators in general is not a requirement at this grade.)	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). (Addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)	Removed examples that do not provide limits or clarification to the standards, it meets criteria for clarity, cognitive demand, and measurability.
4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths), and locate these decimals on a number line.	Removed examples that do not provide limits or clarification to the standards Keep number line to continue progression from the third grade. It meets criteria for clarity, cognitive demand, and measurability.

Code	2010 Standards	Refinement/Draft	Notes
4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	Compare two decimals with tenths and hundredths by reasoning about their size. Use number sense of decimal fractions to assess the reasonableness of answers. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.	Added with tenths and hundredths to clarify that decimal comparisons can include tenths compared to tenths, tenths compared to hundredths, and hundredths compared to hundredths. Added use number sense of decimal fractions to assess the reasonableness of answers to create consistency between NF.A.2 and NF.C.7. "Recognize" has been changed to "understand" for measurability purposes.
Measurement and Data (MD)			
4.MD.A	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**		
4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.	The edit to the current standard invovled the deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	Solve word problems in a real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations that feature a measurement scale.	4.MD.A.2 Include seconds, minutes, hours to address public comments, this aligns with the progession of understanding of time. The term "simple" to describe fractions needed clarification, thus we added "like denominators." Operations of decimals is explicitly taught in 5th grade but introduced in 4th, specifically with addition and subtraction.

Code	2010 Standards	Refinement/Draft	Notes
4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world context including problems with unknown side lengths.	The edit to the current standard invovled the deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. It meets criteria for clarity, cognitive demand, and measurability.
4.MD.B	**Represent and interpret data.**		
4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	The edit to the current standard invovled the deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard, it meets criteria for clarity, cognitive demand, and measurability.
4.MD.C	**Geometric measurement: understand concepts of angle and measure angles.**		
4.MD.C.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	Understand angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	Recognize is not measureable and was changed to understand. In part a, the information after rays (by considering the fraction of the circular arc between the points where the two rays intersect the circle) is all the how and is not appropriate for wording in standards.

Code	2010 Standards	Refinement/Draft	Notes
4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	No revision needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding measurement.
4.MD.C.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	Understand angle measures as additive. Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real world contexts.	Example did not clarify or limit standard. "Recognize" is not measureable and was replaced with understand. It now meets criteria for clarity, cognitive demand, and measurability.
4.G.A	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**		
4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	No revision needed.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding geometry.
4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Understand right triangles as a category, and identify right triangles.	"Recognize" is not measureable and was replaced with understand. It now meets criteria for clarity, cognitive demand, and measurability.
4.G.A.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	Understand a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	"Recognize" is not measureable and was replaced with understand. It now meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding geometry.
4.MP	**Standards for Mathematical Practice**		

Code	2010 Standards	Refinement/Draft	Notes
4.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	
4.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

Code	2010 Standards	Refinement/Draft	Notes
4.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	
4.MP.4	Model with mathematics.	students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual	

Code	2010 Standards	Refinement/Draft	Notes
4.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
4.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
4.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	

Code	2010 Standards	Refinement/Draft	Notes
4.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

5th Grade Draft Math Standards - Technical Review			
Code	**2010 Standards**	**Refinement/Draft**	**Notes**
Operations and Algebraic Thinking (OA)			
5.OA.A	**Write and interpret numerical expressions.**		
5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Use parentheses in numerical expressions, and evaluate expressions with this symbol.	In order to clarify the expectation for 5.OA.A.1, brackets and braces were removed.
5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 ´ (8 + 7). Recognize that 3 ´ (18932 +921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: *For example, express the calculation add 8 and 7, then multiply by 2 as 2 ´ (8 + 7). Recognize that 3 ´ (18932 +921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product* .
5.OA.B	**Analyze patterns and relationships.**		
5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	Generate two numerical patterns using two given rules (i.e. generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	In order to clarify the expectation of 5.OA.B.3 (i.e. generate terms in the resulting sequences) is included in the standard to provide clarity. It is also recommended that "and explain" be inserted to represent the scope of cognitive demand expected in this standard. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*
Number and Operations in Base Ten (NBT)			
5.NBT.A	**Understand the place value system.**		
5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.

5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 ´ 100 + 4 ´ 10 + 7 ´ 1 + 3 ´ (1/10) + 9 ´ (1/100) + 2 ´ (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following has been removed: *e.g., 347.392 = 3 ´ 100 + 4 ´ 10 + 7 ´ 1 + 3 ´ (1/10) + 9 ´ (1/100) + 2 ´ (1/1000)* .
5.NBT.A.4	Use place value understanding to round decimals to any place.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.NBT.B	**Perform operations with multi-digit whole numbers and with decimals to hundredths.**		
5.NBT.B.5.	Fluently multiply multi-digit whole numbers using the standard algorithm.	Fluently multiply multi-digit whole numbers using **a** standard algorithm.	The phrase 'the standard algorithm' implies a singular, efficient, and accurate method to problem solving. It is recommended to change "the" standard algorithm to "a" standard algorithm based on a publication (EDThoughts What We Know About Education and Learning) edited by the Mid-continent Research for Education and Learning (McREL): *An algorithm is a precise, step-by-step method or set of rules for solving problems of a particular type. ... there are algorithms of many types* . (Pg. 82)
5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Apply and extend understanding of division by finding whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using a variety of strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation using a variety of representations including equations and models.	The edit to the current standard involved the deletion of "equations, rectangular arrays, and/or area models" because these address the how to illustrate and explain. These specific methods should be included in the supporting document. The phrase "equations, rectangular arrays and/or area models" was changed to "using a variety of representations including equations and models".
5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; relate the strategy to a written method and explain the reasoning used.	In order to clarify the expectation for 5.NBT.7 "addition and subtraction" was changed to "operations".
Number and Operations - Fractions (NF)			
5.NF.A	**Use equivalent fractions as a strategy to add and subtract fractions.**	**Use equivalent fractions to add and subtract fractions.**	

5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12* .
5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations including equations and models. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2* . The phrase "e.g., by using visual fraction models or equations to represent the problem" was changed to "by using a variety of representations including equations and models". This change was based on research from a publication (EDThoughts What We Know About Mathematics Teaching and Learning) edited by the Mid-continent Research for Education and Learning (McREL): *Representations of mathematical ideas can be visual, including equations, graphs, pictures and charts ... students with well developed understandings of a concept can represent it in a variety of ways* . (Pg. 14)
5.NF.B	**Apply and extend previous understandings of**		
5.NF.B.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ¸ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?	Interpret a fraction as division of the numerator by the denominator *(a/b = a ÷ b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers using a variety of representations including equations and models.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: "noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?" Example(s) should be included in supporting document. In addition, "e.g., by using visual fraction models or equations to represent the problem" was changed to "by using a variety of representations including equations and models". This change was based on research from a publication (EDThoughts What We Know About Mathematics Teaching and Learning) edited by the Mid-continent Research for Education and Learning (McREL): Representations of mathematical ideas can be visual, including equations, graphs, pictures and

5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a.Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd). b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and by a fraction. a. Interpret the product (*a/b*) x *q* as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b* . Use a visual fraction model and create a story context for an equation. b. Interpret the product of a fraction multiplied by a fraction *(a/b) x (c/d)* . Use a visual fraction model and create a story context for an equation. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	The edit to the current standard involved clarity of the progression from 4.NF.B.4 by splitting part a into two parts providing a separate explanation of multiplication of a fraction by a fraction. In addition, deletion of an example occured. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: 2/3 x 4 = 8/3. Examples should be included in supporting document.
5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n´a)/(n´b) to the effect of multiplying a/b by 1.	Interpret multiplication as scaling (resizing) by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b = (n x a)/(n x b)* to the effect of multiplying *a/b* by 1.	Deleted unnecessary wording for clarification purposes. This standard meet the clarity, measureability and cognitive demand expectations.
5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Solve problems in a real-world context involving multiplication of fractions and mixed numbers by using a variety of representations including equations and models.	The phrase "solve real-world problems" was changed to "solve problems in a real-world context" for clarity. Inclusion of real-world problem solving contexts has been clarified in this standard to link classroom mathematics to everyday life, work, and decision-making. The phrase "e.g., by using visual fraction models or equations to represent the problem" was changed to "by using a variety of representations including equations and models". This change was based on research from a publication (EDThoughts What We Know About Mathematics Teaching and Learning) edited by the Mid-continent Research for Education and Learning (McREL): Representations of mathematical ideas can be visual, including equations, graphs, pictures and charts ... students with well developed understandings of a concept can represent it in a variety of ways. (Pg. 14)

5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.) a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) ´ 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4÷(1/5) = 20 because 20 ´ (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions using a variety of representations including equations and models. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. c. Solve problems in a real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.	The phrase "Use a visual fraction model" was changed to "using a variety of representations including equations and models" and added overarching section and removed from letters a-c for conciseness and clarity. This change was added based on research from a publication (EDThoughts What We Know About Mathematics Teaching and Learning) edited by the Mid-continent Research for Education and Learning (McREL): Representations of mathematical ideas can be visual, including equations, graphs, pictures and charts ... students with well developed understandings of a concept can represent it in a variety of ways. (Pg. 14) The edit to the current standard involved deletion of the note. Notes are not included within the standard unless it would provide limits to the standard or clarification to the standard. For this reason the following note has been removed: *(Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.)* . The note
Measurement and Data (MD)			
5.MD.A	**Convert like measurement units within a given measurement system.**		
5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step problems in a real world context.	The phrase "real-world problems" was changed to "problems in a real-world context" for clarity. Inclusion of real-world problem solving contexts has been clarified in this standard to link classroom mathematics to everyday life, work, and decision-making. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: (e.g., convert 5 cm to 0.05 m).
5.MD.B	**Represent and interpret data.**		
5.MD.B.2.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use appropriate grade level fraction operations to solve problems involving information presented in line plots.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.MD.C	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**		

5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas V = l x w ´x h and V = b x h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts to solve real world problems.	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. b. Understand and use the formulas $V = l \bullet w \bullet h$ and $V = B \bullet h$, where in this case B is the area of the base ($B = l \bullet w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in a real world context. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in a real world context.	To accurately symbolize the area of the base, b was changed to B and the variable was defined. "Recognize" was changed to "understand" to adhere to cognitive demand. The phrase "by adding the volumes of the non-overlapping parts" was removed because that is the definition of understanding volume as additive and is redundant. The sentence "Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication." was removed as it did not provide clarity to the standard. The "x" symbolizing multiplication was changed to a "●" symbol. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.
5.G.A	**Graph points on the coordinate plane to solve real-world and mathematical problems.**	**Graph points on the coordinate plane to solve mathematical problems as well as problems in a real-world context.**	
5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	Understand and describe a coordinate system as perpendicular number lines that intersect at the origin (0 , 0). Identify a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number *(x)* indicates the distance traveled on the horizontal axis, and the second number *(y)* indicates the distance traveled on the vertical axis.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: *(e.g., x-axis and x-coordinate, y-axis and y-coordinate).*

5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Represent mathematical problems and problems in real-world context by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.G.B	**Classify two-dimensional figures into categories based on their properties.**		
5.G.B.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. For this reason the following example has been removed: *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **Change:** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding in fifth grade mathematics.
5.MP	**Standards for Mathematical Practice**		
5.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	
5.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

5.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	
5.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
5.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	

5.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
5.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
5.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

6th Grade Draft Math Standards - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Ratio and Proportional Relationships (RP)			
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**		
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ¹ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)	Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Complex fraction notation is not required in the unit rates used in sixth grade.
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Use ratio and rate reasoning to solve mathematical problems and problems in a real-world context. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision making. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
The Number System (NS)			
6.NS.A	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**		

6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	Interpret and compute quotients of fractions to solve mathematical problems and problems in a real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. (In general, *(a/b) ÷ (c/d) = ad/bc* .)	Editing the standard to improve langauge consistency across grade levels. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision making. The edit to the current standard involved deletion of the specific examples. Examples are not included within the standard unless an example would provide limits to the standard or clarification. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**		
6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.	Fluently divide multi-digit numbers using **a** standard algorithm.	The phrase 'the standard algorithm' implies a singular, efficient, and accurate method to problem solving. It is recommended to change 'the' standard algorithm to 'a' standard algorithm based on a publication (EDThoughts: What We Know About Mathematics Teaching and Learning) edited by the Mid-continent Research for Education Learning (McREL): "An alogorithm is a precise, step-by-step method or set of rules for solving problems of a particular type... there are algorithms of many types." (pg. 82) A standard algorithm article found in NCSM Journal by Fuson & Beckmann states that place value decomposition should be the basis for any standard algorithm.
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	Fluently add, subtract, multiply, and divide multi-digit decimals using **a** standard algorithm for each operation.	The phrase 'the standard algorithm' implies a singular, efficient, and accurate method to problem solving. It is recommended to change 'the' standard algorithm to 'a' standard algorithm based on a publication (EDThoughts: What We Know About Mathematics Teaching and Learning) edited by the Mid-continent Research for Education Learning (McREL): "An alogorithm is a precise, step-by-step method or set of rules for solving problems of a particular type... there are algorithms of many types." (pg. 82)
6.NS.B.4	Understand the greatest common factor, understand the least common multiple, and use the distributive property. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9+2).	Understand the greatest common factor, understand the least common multiple, and use the distributive property. a. Find the greatest common factor of two whole numbers less than or equal to 100. b. Find the least common multiple of two whole numbers less than or equal to 12. c. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The standard was clarified by adding a summary heading and breaking the standard into three distinct parts (a., b., c.). It now meets the criteria of clarity, measureability, and cognitive demand.

6.NS.C	**Apply and extend previous understandings of numbers to the system of rational numbers.**		
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. It now meets the criteria of clarity, measureability, and cognitive demand.
6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3 oC > –7 oC to express the fact that –3 oC is warmer than –7 oC. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world contexts. d. Distinguish comparisons of absolute value from statements about order, especially when considering values in context.	Editing the standard to improve language consistency across grade levels. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision making.The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Removed the term "diagram" from "number line diagram" because it is redundant. Change in wording was made to be consistent across grade levels.

6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Solve mathematical problems and problems in a real-world context by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels. It meets the criteria of clarity, measureability, and cognitive demand.
AZ.6.NS.C.9	Convert between expressions for positive rational numbers, including fractions, decimals, and percents.	Delete this standard.	This is written as a performance objective, not as a standard. AZ.6.NS.C.9 as stated does not meet cognitive demands. Standard was removed.
Expressions and Equations (EE)			
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.**		
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	No refinement needed to the existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Sixth Grade Math.
6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as 5 – y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8+7) as a product of two factors; view (8+7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V=s3 and A=6 s2 to find the volume and surface area of a cube with sides of length s=1/2.	Write, read, and evaluate algebraic expressions. a. Write expressions that record operations with numbers and variables. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in a real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	Language was modified to create coherence across grade levels. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
6.EE.A.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.	Apply the properties of operations to generate equivalent expressions.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.

6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.	Identify when two expressions are equivalent.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. It meets the criteria of clarity, measureability, and cognitive demand.
6.EE.B	**Reason about and solve one-variable equations and inequalities.**		
6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Understand solving an equation or inequality as a process of reasoning to find the value(s) which make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Clarified the standard from a question to a statement so it isn't limiting.
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	Use variables to represent numbers and write expressions to solve mathematical problems and problems in a real-world context; understand that a variable can represent an unknown number or any number in a specified set.	Remove the 'depending on the purpose at hand,' due to redundancy in the wording of the standard. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels. It meets the criteria of clarity, measureability, and cognitive demand.
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers	Solve mathematical problems and problems in a real-world context by writing and solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and $p/x = q$ for cases in which p, q and x are all non-negative rational numbers.	Added missing operations by referencing Indiana & Nebraska state standards. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
6.EE.B.8	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in a real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number line.	Added 'greater than or equal to' and 'less than or equal to' forms and deleted redundant statement in last sentence. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels. The term "diagram" is removed from "number line diagram" because it is redundant. It meets the criteria of clarity, measureability, and cognitive demand.
6.EE.C	**Represent and analyze quantitative relationships between dependent and independent variables.**		
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	Use variables to represent two quantities to solve mathematical problems and problems in a real-world context that change in relationship to one another; write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	Editing the standard for clarification of the text. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.

Geometry (G)			
6.G.A	**Solve real-world and mathematical problems involving area, surface area, and volume.**		
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	Find the area of polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in a real-world context.	Editing the standard for redundancy because right triangles, other triangles, & special quadrilaterals are polygons. The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formula $V = B \cdot h$, where in this case, B is the area of the base ($B = l \cdot w$) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in a real-world context.	Suggested phrasing would be redundant and unnecessary, no substantial research found to support change. No content refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Sixth Grade Math. Volume equation V = $b\ h$ was changed to V = $B \cdot h$ to reflect the area of the base using correct formula notation. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context.	No content refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Sixth Grade Math. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
6.G.A.4.	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in a real-world context.	No content refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Sixth Grade Math. Change "solve mathematical problems and problems in a real-world context" was made to keep wording consistent across grade levels.
Statistics and Probability (SP)			
6.SP.A	**Develop understanding of statistical variablity.**		

6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The word "it" was changed to "variability" for clarity of wording in the standard.
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	No refinement needed to existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Sixth Grade Math.
6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a variation measurement uses a single number to describe the spread of the data set.	Clarifying the standard by describing the spread of the data set.
6.SP.B	**Summarize and describe distributions.**		
6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Display and interpret numerical data in plots on a number line including dot plots, histograms, and box plots.	"Display" is a lower cognitive level than "interpret", therefore "interpret" was added to increase the level of rigor. The understanding comes through the interpretation of the data.
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	No refinement needed to existing standard.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Sixth Grade Math.
6.MP	**Standards for Mathematical Practice**		

6.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	
6.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

6.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and	
6.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
6.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	

6.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
6.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
6.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

7th Grade Draft Mathematics Standards - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Ratio and Proportional Relationships (RP)			
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**	Analyze proportional relationships and use them to solve mathematical problems and problems in a real-world context.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.Wording was changed to maintain consitency among the grade levels.
7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.	Compute unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like and different units.	The edit to the current standard involved deletion of example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. The change made to the standards was to focus on the mathematical idea of unit rates without limiting to specific problem contexts while still defining a grade level of rigor.
7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	Use proportional relationships to solve multistep ratio and percent problems. (Limited to: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error)	The edit to the current standard invovled deletion of the word example. Examples are not included, this does however provide limits to the standard or clarification to the standard.

Code	2010 Standards	Refinement/Draft	Notes
The Number System (NS)			
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**		
7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. This wording is taken from the 6th grade standard (6.NS.C.6c) for the purpose of highlighting where the integers are found in the standards.

Code	2010 Standards	Refinement/Draft	Notes
7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide integers and other rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then *–(p/q) = (–p)/q = p/(–q)* . Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.	This wording is taken from the 6th grade standard (6.NS.C.6c) for the purpose of highlighting where the integers are found in the standards.
7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	**Solve mathematical problems and problems in a real-world context** involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.Wording was changed to maintain consitency among the grade levels.
Expressions and Equations (EE)			
7.EE.A	**Use properties of operations to generate equivalent expressions.**		
7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and meausrability. This is a necessary standard in the progression of understanding of 7th Grade Math.

Code	2010 Standards	Refinement/Draft	Notes
7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”	Rewrite an expression in different forms in a problem context and understand the connection between the structures of the different forms.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Using the phrase "shed light on" is not measurable. The cluster says to "use" properties, not just understand them. The example on Page 8 in the Progessions Document is an excellent example.
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**	Solve mathematical problems and problems in a real-world context using numerical and algebraic expressions and equations.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work and decision-making.
7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.	Solve multi-step mathematical problems and problems in a real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate and assess the reasonableness of answers using mental computation and estimation strategies.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. It is implied that "Apply properties of operations to calculate with numbers in any form" is included in solving these multistep problems. By deleting this phrase this standard has become more succinct. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.Wording was changed to maintain consitency among the grade levels.

Code	2010 Standards	Refinement/Draft	Notes
7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px+q>r or px+q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.	Use variables to represent quantities in **mathematical problems and problems in a real-world context**, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form *px+q=r* and *p(x+q)=r* , where *p, q* , and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form *px+q>r* or *px+q < r* , where *p, q* , and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.Wording was changed to maintain consitency among the grade levels.
7.G.A	**Draw, construct, and describe geometrical figures and describe the relationships between them.**		
Geometry (G)			
7.G.A.1	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and meausrability. This is a necessary standard in the progression of understanding of 7th Grade Math.
7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and meausrability. This is a necessary standard in the progression of understanding of 7th Grade Math.

Code	2010 Standards	Refinement/Draft	Notes
7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and meausrability. This is a necessary standard in the progression of understanding of 7th Grade Math.
7.G.B	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**	Solve mathematical problems and problems in a real-world context involving angle measure, area, surface area, and volume.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.Wording was changed to maintain consitency among the grade levels.
7.G.B.4	Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Understand and use the formulas for the area and circumference of a circle to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Understand needs to replace know because formula inclusion within a standard should support the conceptual understanding of the idea and the cognitive demand of the standard as opposed to a lower level or knowledge of the formula (A Research Companion to Principles and Standards for School Mathematics, 2003).
7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and meausrability. This is a necessary standard in the progression of understanding of 7th Grade Math.
7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**Solve mathematical problems and problems in a real-world context** involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.Wording was changed to maintain consitency among the grade levels.
Statistics and Probability (SP)			
7.SP.A	**Use random sampling to draw inferences about a population.**		

Code	2010 Standards	Refinement/Draft	Notes
7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and meausrability. This is a necessary standard in the progression of understanding of 7th Grade Math.
7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
7.SP.B	**Draw informal comparative inferences about two populations.**		
7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (mean absolute deviation).	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard. Mean absolute deviation was added for clarification.
7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.

Code	2010 Standards	Refinement/Draft	Notes
7.SP.C	**Investigate chance processes and develop, use and evaluate probability models.**		
7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.	The edit to the current standard invovled deletion of the explanation. Explanations are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	The edit to the current standard involved deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.

Code	2010 Standards	Refinement/Draft	Notes
7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	**7.SP.C.8. was moved to 8th grade**	7.SP.C.8. was moved to 8th grade to strengthen the Probablity progression across grades 7-Algebra 2. The edit to the current standard invovled deletion of the example. Examples are not included within the standard unless an example would provide limits to the standard or clarification to the standard.
7.MP			
7.MP.1.	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	

Code	2010 Standards	Refinement/Draft	Notes
7.MP.2.	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

Code	2010 Standards	Refinement/Draft	Notes
7.MP.3.	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	
7.MP.4.	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	

Code	2010 Standards	Refinement/Draft	Notes
7.MP.5.	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
7.MP.6.	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
7.MP.7.	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	

Code	2010 Standards	Refinement/Draft	Notes
7.MP.8.	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

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Code	2010 Standards	Refinement/Draft	Notes
The Number System (NS)			
8.NS.A	**Know that there are numbers that are not rational, and approximate them by rational numbers.**	**Understand that there are numbers that are not rational, and approximate them by using rational numbers.**	Changing "know" to "understand" better represents the cognitive demand of the standards within the cluster.
8.NS.A.1	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.	Understand informally that every number has a decimal expansion; rational numbers are those with decimal expansions that terminate in zeros or eventually repeat. Know that other numbers are called irrational.	No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 1. Changed 0 to zero to clarify
8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., p2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
Expressions and Equations (EE)			
8.EE.A	**Work with radicals and integer exponents.**		
8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32´3–5 = 3–3 = 1/33 = 1/27.	Understand and apply the properties of integer exponents to generate equivalent expressions.	This was changed to reflect the numerical and algebraic expressions included in the progressions document. The word "know" was replaced with "understand" to better represent the cognitive demand intended by the standard.
8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. a. Evaluate square roots of perfect squares less than or equal to 225, and rewrite non-perfect squares in equivalent form. b. Evaluate cube roots of perfect cubes less than or equal to 625, and rewrite non-perfect cubes in equivalent form.	"Know that √2 is irrational" was removed because it was already addressed within 8.NS.A.1. The word "small" was removed and replaced with "less than or equal to 225"because this represents the range of the most common values used in Pythagorean triples. The word "small" was removed and replaced with" less than or equal to 625" because this is a common perfect cube used at this grade level. Rewriting "non-perfect squares and cubes in equivalent form" was included because it is a foundational concept that is part of the progression to Algebra.

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Code	2010 Standards	Refinement/Draft	Notes
8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3´108 and the population of the world as 7´109, and determine that the world population is more than 20 times larger.	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times larger or smaller one is than the other.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.EE.A.4.	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**		
8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at (0, b).	The y-intercept was rewritten as (0, b) to accurately represent it as a point on the coordinate plane.
8.EE.C	**Analyze and solve linear equations, and pairs of simultaneous linear equations.**	**Analyze and solve linear equations, inequalities, and pairs of simultaneous linear equations.**	The addition of inequalities to the standard is necessary in the progression of understanding of Algebra 1.

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Code	2010 Standards	Refinement/Draft	Notes
8.EE.C.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	Solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x = a, a = a, or a = b* results (where *a* and *b* are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.	The addition of inequalities to the standard is necessary in the progression of understanding of Algebra 1.
8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
Functions (F)			
8.F.A	**Define, evaluate, and compare functions.**		

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Code	2010 Standards	Refinement/Draft	Notes
8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 1.
8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.F.A.3	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.F.B	**Use functions to model relationships between quantities.**		
8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 1.
8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	Describe qualitatively the functional relationship between two quantities by analyzing linear and nonlinear graphs. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard. Part of the example was included in the standard to indicate that it is referring to both linear and nonlinear graphs.
Geometry (G)			

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Code	2010 Standards	Refinement/Draft	Notes
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**	Understand congruence and similarity.	The tools were removed because they described the instructional techniques and not the limit of the cluster. Support document: include using physical models, transparencies or geometry software as an example.
8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	Verify experimentally the properties of rotations, reflections, and translations. Properties include: line segments to line segments of the same length, angles are taken to angles of the same measure, parallel lines are taken to parallel lines.	Removed "lines are taken to lines" since it did not directly relate to rotations, reflections, and translations. Condensed a, b, c to a single statement that consolidates the types of properties addressed in the standard. The phrase "taken to" is common verbage in transformational geometry which implies movement.
8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates (rotations and dilations about the origin).	In grade 8, this topic is newly introduced and is appropriately limited to figures in the coordinate plane and rotations and dilations about the origin.
8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 1.

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Code	2010 Standards	Refinement/Draft	Notes
8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.G.B	**Understand and apply the Pythagorean Theorem.**		
8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 1.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two dimensions and three dimensions (in regards to slant height).	In grade 8 this topic is newly introduced and is appropriately limited with respect to slant height in 3-dimensional figures.
8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 1.
8.G.C	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**		
8.G.C.9	Know the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	Understand and use given formulas for the volume of cones, cylinders and spheres to solve real-word and mathematical problems.	
Statistics and Probability (SP)			
8.SP.A	**Investigate patterns of association in bivariate data.**		

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Code	2010 Standards	Refinement/Draft	Notes
8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns of association between two quantities.	
8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies to describe a possible association between the two variables.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
8.SP.B.1		**Investigate chance processes and develop, use, and evaluate probability models.**	NEW Cluster-Needed probability standards as a normal progression from 7th to high school.

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Code	2010 Standards	Refinement/Draft	Notes
8.SP.B.1		Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. Identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.	New Standard - moved from 7.SP.C.8 example removed from 7th and current 8th grade standard Standard moved to enhance the Probablity progresson across grades 7-Algebra 2.
8.MP	Standards for Mathematical Practices		
8.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	

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Code	2010 Standards	Refinement/Draft	Notes
8.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	
8.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

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Code	2010 Standards	Refinement/Draft	Notes
8.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
8.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
8.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	

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Code	2010 Standards	Refinement/Draft	Notes
8.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
8.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

High School Mathematics Standards - Algebra 1 Draft- Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Number and Quantity - N			
The Real Number System (N–RN)			
A1.N-RN.B	**Use properties of rational and irrational numbers.**		
A1.N-RN.B.3	Explain why the sum or product of two rational numbers are rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Connection: 9-10.WA1T.1e	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	Minor grammatical correction. No fundamental refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
Quantities (N–Q)			
A1.N-Q.A	**Reason quantitatively and use units to solve problems.**		
A1.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in grapA1 and data displays.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.	Define appropriate quantities for the purpose of descriptive modeling. Include problem solving opportunities utilizing real-world context.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.
A1.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
Algebra - A			
Seeing Structure in Expressions (A–SSE)			
A1.A-SSE.A	**Interpret the structure of expressions.**		

Code	2010 Standards	Refinement/Draft	Notes
A1.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. Connection: 9-10.RST.4 b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)^2 – (y2)^2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).	Use the structure of an expression to identify ways to rewrite it. Focus on numerical expressions, such as recognizing $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the form (53+47)(53-47). Focus on polynomial expressions in one variable, such as seeing an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.A-SSE.B	**Write expressions in equivalent forms to solve problems.**		
A1.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Connections: 9-10.WA1T.1c; 11-12.WA1T.1c (Alg 1 and Alg 2) a. Factor a quadratic expression to reveal the zeros of the function it defines. (Alg 1) b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (Alg 1) c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. (combo Alg1 and Alg 2)	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. Focus on expressions with integer exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
Arithmetic with Polynomials and Rational Expressions (A–APR)			
A1.A-APR.A	**Perform arithmetic operations on polynomials.**		

Code	2010 Standards	Refinement/Draft	Notes
A1.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.A-APR.B	**Understand the relationship between zeros and factors of polynomials.**		
A1.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
Creating Equations (A–CED)			
A1.A-CED.A	**Create equations that describe numbers or relationships.**		
A1.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing a real-world context. Focus on equations and inequalities that are linear, quadratic, or exponential with integer exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	The example was deleted from the standard because it did not promote clarity.

Code	2010 Standards	Refinement/Draft	Notes
A1.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	The example was deleted from the standard because it did not promote clarity
Reasoning with Equations and Inequalities (A–REI)			
A1.A-REI.A	**Understand solving equations as a process of reasoning and explain the reasoning.**		
A1.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from linear to quadratic equations.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.A-REI.B	**Solve equations and inequalities in one variable.**		
A1.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.A-REI.B.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. **(Alg 1)** b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **(both Alg1 and Alg 2)**	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)2 = q$ that has the same solutions. Derive the quadratic formula from this form. (b) Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.A-REI.C	**Solve systems of equations.**		

Code	2010 Standards	Refinement/Draft	Notes
A1.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		PN - No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with grapA1), focusing on pairs of linear equations in two variables.	Solve systems of linear equations exactly and approximately (e.g., with grapA1), focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.	Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.
A1.A-REI.D	**Represent and solve equations and inequalities graphically.**		
A1.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.A-REI.D.11	Explain why the x-coordinates of the points where the grapA1 of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	Explain why the x-coordinates of the points where the grapA1 of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately using technology to graph the functions, make tables of values, or find successive approximations. Focus on cases where f(x) and/or g(x) are linear, absolute value, quadratic and, exponential functions with integer exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
Functions - F			
Interpreting Functions (F–IF)			

Code	2010 Standards	Refinement/Draft	Notes
A1.F-IF.A	**Understand the concept of a function and use function notation.**		
A1.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.F-IF.A.2	Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
A1.F-IF.B	**Interpret functions that arise in applications in terms of the context.**		
A1.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of grapA1 and tables in terms of the quantities, and sketch grapA1 showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Connections: ETA1-S6C2.03; 9-10.RST.7; 11-12.RST.7	For a function that models a relationship between two quantities, interpret key features of grapA1 and tables in terms of the quantities, and sketch grapA1 showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, absolute value, quadratic, and exponential with integer exponents and piecewise-defined functions. (limited to the aforementioned functions).	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.
A1.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Connection: 9-10.WA1T.2f	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.

Code	2010 Standards	Refinement/Draft	Notes
A1.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Connections: ETA1-S1C2-01; 9-10.RST.3	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear, absolute value, quadratic, and exponential with integer exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. Inclusion of real-world problem solving contexts has been added to this standard to link classroom mathematics to everyday life, work, and decision-making.
A1.F-IF.C	**Analyze functions using different representations.**		
A1.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **(Alg 1, Alg2 and +)** a. Graph linear and quadratic functions and show intercepts, maxima, and minima. (Alg 1) b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (Alg 1) ~~c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.~~ (Alg 2) ~~d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.~~ (+) ~~e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.~~ (Alg 2)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, exponential with integer exponents, quadratic, and piecewise-defined functions.	Rewritten to mirror Alg 2 standard. The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.

Code	2010 Standards	Refinement/Draft	Notes
A1.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Connection: 11-12.RST.7 a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Connection: 11-12.RST.7 b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. (Alg 2)	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Connections: ETA1-S6C1-03; ETA1-S6C2-03; 9-10.RST.7	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, absolute value, quadratic, exponential with integer exponents and piecewise-defined functions. (limited to the aforementioned functions)	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
Building Functions (F-BF)			
A1.F-BF.A	**Build a function that models a relationship between two quantities.**		

Code	2010 Standards	Refinement/Draft	Notes
A1.F-BF.A.1	Write a function that describes a relationship between two quantities. (Alg 1 and Alg 2) a. Determine an explicit expression, a recursive process, or steps for calculation from a context. Connections: ETA1-S6C1-03; ETA1-S6C2-03; 9-10.RST.7; 11-12.RST.7 (Alg 1 and Alg 2)	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Focus on linear, absolute value, quadratic, exponential with integer exponents, andpiecewise-defined functions. (limited to the aforementioned functions)	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A1.F-BF.B	**Build new functions from existing functions.**		
A1.F-BF.B.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the grapA1. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their grapA1 and algebraic expressions for them. Connections: ETA1-S6C2-03; 11-12.WA1T.2e	Identify the effect on the graph of replacing f(x) by f(x) + k, k* f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the grapA1. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Focus on linear, absolute value, quadratic, exponential with integer exponents, and piecewise-defined functions (limited to the aforementioned functions).	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
<u>Linear, Quadratic, and Exponential Models (F-LE)</u>			
A1.F-LE.A	**Construct and compare linear, quadratic, and exponential models and solve problems.**		
A1.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.

Code	2010 Standards	Refinement/Draft	Notes
A1.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Connections: ETA1-S6C1-03; ETA1-S6C2-03; 11-12.RST.4; SSA1-S5C5-03	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output pairs.	Use of **two** input-output pairs was deleted from this standard to allow for multiple combinations of input-output pairs. In addition, reading pairs from a table was seen as exclusive of other representation for input-output pairs and was thus removed.
A1.F-LE.A.3	Observe using grapA1 and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	Observe, using grapA1 and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.	Minor grammatical correction. Polynomial function was deleted due to limits created between Alg 1 and 2. Polynomial functions are not introduced until Alg 2 and will cause undue confusion if introduced in this context in Alg 1.
A1.F-LE.B	**Interpret expressions for functions in terms of the situation they model.**		
A1.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context. Connections: ETA1-S6C1-03; ETA1-S6C2-03;SSA1-S5C5-03; 11-12.WA1T.2e	Interpret the parameters in a linear or exponential function with integer exponents in terms of a context.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
Statistics and Probability - S			
Interpreting Categorical and Quantitative Data (S-ID)			
A1.S-ID.A	**Summarize, represent, and interpret data on a single count or measurement variable.**		
A1.S-ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.S-ID.B	**Summarize, represent, and interpret data on two categorical and quantitative variables.**		

Code	2010 Standards	Refinement/Draft	Notes
A1.S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (Alg 1 and Alg 2) a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or chooses a function suggested by the context. Emphasize linear, quadratic, and exponential models. Connection: 11-12.RST.7 (both Alg1 and Alg 2) b. Informally assess the fit of a function by plotting and analyzing residuals. Connections: 11-12.RST.7; 11-12.WA1T.1b-1c c. Fit a linear function for a scatter plot that suggests a linear association. Connection: 11-12.RST.7	Represent data on two quantitative variables on a scatter plot, and describe how the **quantities** are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. b. Informally assess the fit of a function by plotting and analyzing residuals.	The limits of this standard were refined to create a standalone standard that represents Algebra I content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. The example "use given function or choose a function suggested by the context" was deleted from the standard because it did not promote clarity. Part c was removed becuase if was redundant.
A1.S-ID.C	**Interpret linear models.**		
A1.S-ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.
A1.S-ID.C.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.	Compute and interpret the correlation coefficient of a linear fit.	Removed "using technology" since standards address what gets taught not how.
A1.S-ID.C.9	Distinguish between correlation and causation.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra I.

Code	2010 Standards	Refinement/Draft	Notes
Conditional Probability and the Rules of Probability (S-CP)			
A1.S-CP.A	**Understand independence and conditional probability and use them to interpret data.**		New Cluster from Algebra 2
A1.S-CP.A.1		Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	New Standards from Algebra 2
A1.S-CP.A.2		Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	New Standard from Algebra 2
A1.MP	**Standards for Mathematical Practice**		
A1.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	

Code	2010 Standards	Refinement/Draft	Notes
A1.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	
A1.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

Code	2010 Standards	Refinement/Draft	Notes
A1.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
A1.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	
A1.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	

Code	2010 Standards	Refinement/Draft	Notes
A1.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	
A1.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

High School Mathematics Standards - Geometry Draft - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
Geometry -G			
Quantity (N-Q)			
G-N-Q.A	**Reason quantitatively and use units to solve problems.**		
G.N-Q.A.1	Use units as a way to understand problems and to		
G.N-Q.A.2	Define appropriate quantities for the purpose of		
G.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.		
Congruence (G-CO)			
G-G-CO.A	**Experiment with transformations in the plane.**		
G.G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Represent and describe transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.	Clarity: Removed eg from middle of standard and end of standard. Changed 'Represent… describe... to "Represent and Describe"
G.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.

High School Mathematics Standards - Geometry Draft - Technical Review

Code	2010 Standards	Refinement/Draft	Notes
G.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.	Clarity: Removed examples from first sentence
G.G-CO.B	**Understand congruence in terms of rigid motions.**		
G.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Connections: ETG-S1C2-01; 9-10.WGT.1e	Use geometric **definitions** of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Clarity: Change 'descriptions' to 'definitions'
G.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-CO.C	**Prove geometric theorems.**		

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Code	2010 Standards	Refinement/Draft	Notes
G.G-CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	This standard meets criteria for clarity, cognitive demand, and measurability. It is a necessary standard in the progression of understanding of Geometry. A minor refinement was made in adding the word "and" to maintain formatting consistency throughout the Geomtry standards.
G.G-CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.	This standard meets criteria for clarity, cognitive demand, and measurability. It is a necessary standard in the progression of understanding of Geometry. A minor refinement was made in adding the word "and" to maintain formatting consistency throughout the Geomtry standards.
G.G-CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Connection: 9-10.WGT.1a-1e	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.	Clarity: Removed word 'conversely' because it was used incorrectly.
G.G-CO.D	**Make geometric constructions.**		

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Code	2010 Standards	Refinement/Draft	Notes
G.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Connection: ETG-S6C1-03	Make formal geometric constructions with a variety of tools and methods. Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	Clarity: Removed parenthesis - add to support document. Added 'Constructions include:'
G.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
Similarity, Right Triangles, and Trigonometry (G-SRT)			
G.G-SRT.A	**Understand similarity in terms of similarity transformations.**		
G.G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.

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Code	2010 Standards	Refinement/Draft	Notes
G.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-SRT.B	**Prove theorems involving similarity.**		
G.G-SRT.B.4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. Connections: ETG-S1C2-01;9-10.WGT.1a-1e	Prove theorems about triangles. Theorems include: an **interior** line parallel to one side of a triangle divides the other two proportionally; and the Pythagorean Theorem proved using triangle similarity.	Clarity: Added the word 'interior' to clarify that a line can be drawn outside of a triangle that is parallel to one of the triangle's side. This line would not intersect the other two sides of the triangle. Clarity: The word "conversely" was removed because it was used incorrectly.
G.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Connections: ETG-S1C2-01; 9-10.WGT.1a-1e	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing a real-world context.	Clarity: Reversed the order of the standard to emphasize proof as the focus of the standard and to add an emphasis to real-world contexts when solving problems.
G.G-SRT.C	**Define trigonometric ratios and solve problems involving right triangles.**		
G.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.

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Code	2010 Standards	Refinement/Draft	Notes
G.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Connections: ETG-S6C2-03;9-10.RST.7	Use trigonometric ratios **(including inverse trigonometric ratios)** and the Pythagorean Theorem **to find unknown measurements in right triangles** in applied problems.	Clarity: The need to use inverse trigonometric ratios was highlighted. The phrase "solve right triangles" was clarified.
G.G-SRT.D	**Apply trigonometry to general triangles.**		plus
G.G-SRT.D.9	Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		plus - No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
G.G-SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.		plus - No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
G.G-SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). Connections: 11-12.WGT.2c; 11-12.WGT.2e		plus
Circles (G-C)			
G.G-C.A	**Understand and apply theorems about circles.**		
G.G-C.A.1	Prove that all circles are similar.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.

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Code	2010 Standards	Refinement/Draft	Notes
G.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	This standard meets criteria for clarity, cognitive demand, and measurability. It is a necessary standard in the progression of understanding of Geometry. A minor refinement was made in adding the word "and" to maintain formatting consistency throughout the Geomtry standards.
G.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-C.A.4	Construct a tangent line from a point outside a given circle to the circle.		plus - No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
G.G-C.B	**Find arc lengtG and areas of sectors of circles.**		
G.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Connections: ETG-S1C2-01; 11-12.RST.4	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Convert between degrees and radians.**	Clarity - Language was added to highlight the introduction of radians.
Expressing Geometric Properties with Equations (G-GPE)			
G.G-GPE.A	**Translate between the geometric description and the equation for a conic section.**		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.

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Code	2010 Standards	Refinement/Draft	Notes
G.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.	**plus -**	Move to plus standard. Should not be included in Geometry. Add to Algebra II or Plus. Meeting between Algebra II and Geometry - decision to move to Plus standards.
G.G-GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. Connections: ETG-S1C2-01; 11-12.RST.4	plus -	Plus standard
G.G-GPE.A	**Use coordinates to prove simple geometric theorems algebraically.**		
G.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).	Use coordinates to prove or disprove simple geometric theorems algebraically. Theorems include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.	Clarity: Removed examples to remove specific limits on standard; reworded to make standard more general to encompass many possible examples; emphasized proof aspect of standard.
G.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point.	Clarity: changed 'eg' to including to emphasize limit of the standard

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Code	2010 Standards	Refinement/Draft	Notes
G.G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.	Note: Removed modeling star - does not always require problems to have a real-world context Also: removed eg to remove limit on standard
Geometric Measurement and Dimension (G-GMD)			
G.G-GMD.A	**Explain volume formulas and use them to solve problems.**		
G.G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		plus - No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
G.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Connection: 9-10.RST.4	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems **utilizing real-world context.**	Clarity: Standard was rewritten to emphasize real world context. Supporting documentation should emphasize connec tion to G.MG.A.1.
G.G-GMD.B	**Visualize relationships between two-dimensional and three dimensional objects.**		

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Code	2010 Standards	Refinement/Draft	Notes
G.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Geometry.
Modeling with Geometry (G-MG)			
G.G-MG.A	**Apply geometric concepts in modeling situations.**		
G.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	Use geometric shapes, their measures, and their properties to describe objects.	Clarity: Removed 'eg' to remove limits of standard.
G.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	Apply concepts of density based on area and volume in modeling situations.	Clarity: Removed 'eg' to remove limits of standard.
G.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	Apply geometric methods to solve design problems.	Clarity: Removed 'eg' to remove limits of standard.

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Code	2010 Standards	Refinement/Draft	Notes
G.G-MG.A.4		~~Determine and analyze geometric probabilities involving distributions of length, area, and volume.~~	Determine and analyze geometric probabilities involving distributions of length, area, and volume. New Standard created by Workgroup Note 6/6/2016: Group determined to remove this standard because: (1) This standard does not meet the level of rigor for the G standards which focus more on proof - this standard would fit better in the 7th or 8th grade level where students are first applying formulas for area & volume; (2) The standard does not fit in the geometric progression with other volume standards - this would act more as a 'checkbox' standard rather than an integrated part of the standards
G.MP	**Standards for Mathematical Practice**		
G.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	

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Code	2010 Standards	Refinement/Draft	Notes
G.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

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Code	2010 Standards	Refinement/Draft	Notes
G.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

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Code	2010 Standards	Refinement/Draft	Notes
G.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
G.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	

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Code	2010 Standards	Refinement/Draft	Notes
G.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
G.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	

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Code	2010 Standards	Refinement/Draft	Notes
G.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

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Code	2010 Standards	Refinement/Draft	Notes
Number and Quantity- N			
The Real Number System (N-RN)			
A2.N-RN.A	**Extend the properties of exponents to rational exponents.**		
A2.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.	Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
A2.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 2.
Quantities (N-Q)			
A2.N-Q.A	**Reason quantitatively and use units to solve problems.**		
A2-N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in		This standard is integrated throughout Algebra I, Geometry, and Algebra II. No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
A2.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.		This standard is integrated throughout Algebra I, Geometry, and Algebra II. Most notably in modeling tasks. For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude. No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 2.

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A2.N-Q.A.3	Choose a level of accurancy appropriate to limitations on measurement when reporting quantitites.		This standard is integrated throughout Algebra I, Geometry, and Algebra II. No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability.
The Complex Number System N–CN			
A2.N-CN.A	**Perform arithmetic operations with complex numbers.**		
A2.N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form ($a + bi$) with a and b real.	Apply the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form ($a + bi$) with a and b real.	The workgroup combined A2.N-CN.A.1 and A2.N-CN.A.2 to make one standard to provide clarity and emphasize relationship of complex number and i.
~~**A2.N-CN.A.2**~~	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Connection: 11-12.RST.4	combined with A2.N-CN.A.1	The workgroup combined A2.N-CN.A.1 and A2.N-CN.A.2 to make one standard to provide clarity and emphasize relationship of complex number and i.
A2.N-CN.C	**Use complex numbers in polynomial identities and equations.**		
A2.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 2.
Algebra - A			
Seeing Structure in Expressions (A–SSE)			
A2.A-SSE.A	**Interpret the structure of expressions.**		

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A2.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).	Use the structure of an expression to identify ways to rewrite it. Extend polynomial expressions to multivariable expressions. Focus on rational or exponential expressions seeing that $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.A-SSE.B	**Write expressions in equivalent forms to solve problems.**		
A2.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Connections: 9-10.WA2T.1c; 11-12.WA2T.1c a. Factor a quadratic expression to reveal the zeros of the function it defines. (Alg 1) b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (Alg 1) c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem solving opportunities utilizing real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to transform expressions for exponential functions.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.A-SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. Connection: 11-12.RST.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.

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Arithmetic with Polynomials and Rational Expressions A–APR	**Understand the relationship between zeros and factors of polynomials.**		
A2.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a* , the remainder on division by *x – a* is *p(a)* , so *p(a)* = 0 if and only if *(x – a)* is a factor of p(x).		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 2.
A2.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
Arithmetic with Polynomials and Rational Expressions (A–APR)			
A2.A-APR.C	**Use polynomial identities to solve problems.**		
A2.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2+y2)2 = (x2– y2)2 + (2xy)2 can be used to generate Pythagorean triples.	Prove polynomial identities and use them to describe numerical relationships.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
A2.A-APR.D	**Rewrite rational expressions.**		
A2.A-APR.D.6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	Rewrite rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or a computer algebra system.	Removed the word "simple" for clarity.
Creating Equations (A–CED)			
A2.A-CED.A	**Create equations that describe numbers or relationships.**		

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A2.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions with real exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
Reasoning with Equations and Inequalities (A–REI)			
A2.A-REI.A	**Understand solving equations as a process of reasoning and explain the reasoning.**		
A2.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. Removed the word "simple" for clarity.
A2.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.	Removed the word "simple" for clarity.
A2.A-REI.B	**Solve equations and inequalities in one variable.**		

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RE	Solve quadratic equations in one variable. ~~a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form.~~ (Alg 1) b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **(Alg 1 and Alg 2)**	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
A2.A-REI.C	**Solve systems of equations.**		
A2.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with grapA2), focusing on pairs of linear equations in two variables. Connection: ETA2-S6C2-03	removed from algebra 2	The limits of this standard were refined to create a standalone standard that represents **Algebra 1 content only.** This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.	Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard. Removed the word "simple" for clarity.
A2.A-REI.D	**Represent and solve equations and inequalities graphically.**		

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A2.A-REI.D.11	Explain why the x-coordinates of the points where the grapA2 of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	Explain why the x-coordinates of the points where the grapA2 of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately, using technology to graph the functions, tables of values, or successive approximations. Include problem solving opportunities utlizing real-world context. Extend from linear, quadratic, exponential with integer exponents to cases where f(x) and/or g(x) are polynomial, rational, exponential with real exponent, and logarithmic functions.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
Functions -F			
Interpreting Functions (F–IF)			
A2.F-IF.A	**Understand the concept of a function and use function notation.**		
A2.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.	Removed from Algebra 2.	The limits of this standard were refined to create a standalone standard that represents **Algebra 1 content only**. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.F-IF.B	**Interpret functions that arise in applications in terms of the context.**		

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A2.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of grapA2 and tables in terms of the quantities, and sketch grapA2 showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Connections: ETA2-S6C2.03; 9-10.RST.7; 11-12.RST.7	For a function that models a relationship between two quantities, interpret key features of grapA2 and tables in terms of the quantities, and sketch grapA2 showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Connections: ETA2-S1C2-01; 9-10.RST.3	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content only. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.F-IF.C	**Analyze functions using different representations.**		

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A2.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **(Alg 1, Alg2 and +)** ~~a. Graph linear and quadratic functions and show intercepts, maxima, and minima.~~ (Alg 1) ~~b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.~~ (Alg 1) c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ~~d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.~~ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include square root, cube root, polynomial, exponential with real exponents, logarithmic, sine, cosine, tangent, and piecewise-defined functions.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content. this refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Connection: 11-12.RST.7 ~~a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.~~ (Alg 1) b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay. Connection: 11-12.RST.7	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard. The limits of this standard were refined to create a standalone standard that represents Algebra 2 content. this refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.

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A2.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Connections: ETA2-S6C1-03; ETA2-S6C2-03; 9-10.RST.7	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.). Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content. this refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
Building Functions F-BF	**Build a function that models a relationship between two quantities.**		
A2.F-BF.A.1	Write a function that describes a relationship between two quantities. Connections: ETA2-S6C1-03; ETA2-S6C2-03 a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (Alg 2) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. (+)	Write a function that describes a relationship between two quantities. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and all exponential functions with real exponents. Include problem-solving opportunities utilizing real-world context. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations and function composition.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content. this refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses. The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard. Function composition was added to support F-BF.B.4.

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A2.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	Write arithmetic and geometric sequences both recursively and explicitly. Use arithmetic and geometric sequences to model situations, and translate between explicit and recursive forms.	Minor edits made to provide clarity. This is a necessary standard in the progression of understanding of Algebra 2.
Building Functions (F-BF)			
A2.F-BF.B	**Build new functions from existing functions.**		
A2.F-BF.B.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the grapA2. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their grapA2 and algebraic expressions for them. Connections: ETA2-S6C2-03; 11-12.WA2T.2e	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the grapA2. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their grapA2 and algebraic expressions for them. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and exponential functions with real exponents.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content. this refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.F-BF.B.4	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1. ~~b. Verify by composition that one function is the inverse of another.~~ (+) ~~c. Read values of an inverse function from a graph or a table, given that the function has an inverse.~~ (+) ~~d. Produce an invertible function from a non-invertible function by restricting the domain.~~ (+)	Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions f and g are inverse functions if and only if $f(x)=y$ and $g(y)=x$ for all values of x in the domain of f and all values of y in the domain of g . b. Understand that if a function contains a point (a,b) , then the graph of the inverse relation of the function contains the point (b,a) ; the inverse is a reflection over the line $y = x$.	Modified to promote clarity and measurability. This is a necessary standard in the progression of understanding of Algebra 2.

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Linear, Quadratic, and Exponential Models (F–LE)			
A2.F-LE.A	**Construct and compare linear, quadratic, and exponential models and solve problems.**		
A2.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	This is an Algebra 1 Standard.	The limits of this standard were refined to create a standalone standard that represents **Algebra 1 content only**. This refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses.
A2.F-LE.A.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.		No refinement needed on this standard since it meets criteria for clarity, cognitive demand, and measurability. This is a necessary standard in the progression of understanding of Algebra 2.
A2.F-LE.B	**Interpret expressions for functions in terms of the situation they model.**		
A2.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context. Connections: ETA2-S6C1-03; ETA2-S6C2-03;SSA2-S5C5-03; 11-12.WA2T.2e	Interpret the parameters in an exponential function with real exponents in terms of a context.	The limits of this standard were refined to create a standalone standard that represents Algebra 2 content. this refinement addresses many of the public comments received regarding the common standards between Algebra 1 and 2 and the need to address a limit between the two courses
Trigonometric Functions (F–TF)			
A2.F-TF.A	**Extend the domain of trigonometric functions using the unit circle.**		
A2.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.	Wording corrected to be mathematically correct.
A2.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Modified to promote clarity and measurability. This is a necessary standard in the progression of understanding of Algebra 2.

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A2.F-TF.B	**Model periodic phenomena with trigonometric functions.**		
A2.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	Create and interpret trigonometric functions that model periodic phenomena with specified amplitude, frequency, and midline.	Change in cognitive demand of the standard.
A2.F-TF.C	**Prove and apply trigonometric identities.**		
A2.F-TF.C.8	Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	Prove the Pythagorean identity *sin2* (θ) + *cos2* (θ) = 1 and use it to find *sin* (θ), *cos* (θ), or *tan* (θ) given *sin* (θ), *cos* (θ), or *tan* (θ) and the quadrant of the angle.	This was a plus standard, but this content is contained within the ACT, sothis standard should be included in Algebra II.
Geometry - G			
Expressing Geometric Properties with Equations G-GPE			
A2.G-GPE.A	**Translate between the geometric description and the equation for a conic section**	Moved to Plus	Moved to Plus Standards
A2.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.	This is a plus standard.	The topic of conics as a geometric construct should not be split over three years. To promote coherence and progression; ellipses, parabolas, and hyperbolas should be studied together. Circles should remain in geometry. After discussing with the geometry team, we determined that this is most appropriate as a plus standard.
Statistics and Probability - S			
Interpreting Categorical and Quantitative Data S-ID			
A2.S-ID.A	**Summarize, represent, and interpret data on a single count or measurement variable.**		
A2.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	Use the mean and standard deviation of a data set and properties of a normal distribution to approximate a normal curve to estimate population percentages. Recognize and identify data sets for which such a procedure is not appropriate. Consider non-symmetric data sets and presence of outliers.	

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A2.S-ID.B	**Summarize, represent, and interpret data on two categorical and quantitative variables.**		
A2.S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **(Alg 1 and Alg 2)** a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or chooses a function suggested by the context. Emphasize linear, quadratic, and exponential models. **(Alg 1 and Alg 2)** ~~b. Informally assess the fit of a function by plotting and analyzing residuals.~~ (Alg 1) ~~c. Fit a linear function for a scatter plot that suggests a linear association.~~ (Alg 1)	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or chooses a function suggested by the context.	
A2.S-ID.C	**Interpret models.**		NEW Cluster from Algebra 1
A2.S-ID.C.10		Interpret parameters of exponential models.	New Standards for Statistics Progression
<u>Making Inferences and Justifying Conclusions S-IC</u>			
A2.S-IC.A	**Understand and evaluate random processes underlying statistical experiments.**		
A2.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.	

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A2.S-IC.A.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	Explain if a specified model is consistent with results from a given data-generating process.	
A2.S-IC.B	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**		Cluster DELETED - Moved to Plus (+)
A2.S-IC.B.3			Deleted - Moved from Algebra 2 to Plus (+) Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
A2.S-IC.B.4			Deleted - Moved from Algebra 2 to Plus (+) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
A2.S-IC.B.5			Deleted - Moved from Algebra 2 to Plus (+) Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
A2.S-IC.B.6			Deleted - Moved from Algebra 2 to Plus (+) Evaluate reports based on data.
Conditional Probability and the Rules of Probability S-CP			
A2.S-CP.A	**Understand independence and conditional probability and use them to interpret data.**		

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A2.S-CP.A.1			Moved to Algebra 1 - Improves the Probability progression to move to Algebra 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). Connection: 11-12.WA2T.2e
A2.S-CP.A.2			Moved to Algebra 1 - Improves the Probability progression to move to Algebra 1. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
A2.S-CP.A.3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.		

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A2.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to **calculate** conditional probabilities.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
A2.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. Connections: 11-12.RST.4; 11-12.RST.5;11-12.WA2T.1e	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	The example was deleted from the standard because it did not promote clarity and provided too narrow of a focus to encompass the intended scope of the standard.
A2.S-CP.B	**Use the rules of probability to compute probabilities of compound events in a uniform probability model**		
A2.S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.		
A2.S-CP.B.7	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.		

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A2.S-CP.B.8		Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	Moved from Plus (+) to Algebra 2.
A2.MP	**Standards for Mathematical Practice**		
A2.MP.1	Make sense of problems and persevere in solving them.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	

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A2.MP.2	Reason abstractly and quantitatively.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	

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A2.MP.3	Construct viable arguments and critique the reasoning of others.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	

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A2.MP.4	Model with mathematics.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	
A2.MP.5	Use appropriate tools strategically.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	

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A2.MP.6	Attend to precision.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	
A2.MP.7	Look for and make use of structure.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	

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A2.MP.8	Look for and express regularity in repeated reasoning.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	

Background on the Mathematics Workgroup Process from the Executive Summary

From November 2015 through January 2016 the Mathematics Workgroup reviewed, and categorized all public comments that were received including comments that did not directly address the standards. After categorizing public comments, workgroups utilized public comments that directly related to the 2010 standards to inform their decisions regarding the refinement or revision of the 2010 Mathematics Standards.

Using the public comment data, academic research, and three criteria for the review process (clarity, cognitive demand, and measurability), workgroups reviewed all K-12 mathematics standards individually within domains and across domains and grade levels. The Mathematics Workgroups utilized the following definitions when reviewing all standards for clarity, cognitive demand and measurability:

CLARITY: Quality of being easily understood. (Merriamwebster.com)

- The standard is clear and understandable.
- The standard can be used by educators to clearly guide learning for students.
- The standard can be used by educators to build student understanding.
- Examples or parenthesis in/after the standard provide clarification or define the limit of the standard.

COGNITIVE DEMAND: Represents the type of thinking and level of complexity of thought we expect students to engage in when learning. Cognitive Demand is about high levels of reasoning and thinking. Standards are written at different levels/ranges of cognitive demand.

- The standard has complexity of reasoning.
- The cluster contains a range of cognitive demand/complexity of reasoning.

MEASUREABLE: Student progression towards mastery of the standards should be observable and verifiable.

- The standard can be measured through varied modes of assessments.

A final review by progression (purposeful sequencing of teaching and learning expectations across multiple developmental stages, ages, or grade levels) of content within and across grade levels occurred in May and June 2016. After final rounds of revisions were completed, the workgroup conducted an additional review of all public comments to ensure that all data were utilized during the review period. The final meeting included final edits to the introduction, glossary, and standards with a final look at the progressions within and across domains and across grade levels.

The narratives for the Standards for Mathematical Practice do not contain notes. The narratives for the MP were developed first in a small group and then taken to the larger workgroup for feedback and refinement. This process went on for several rounds until no additional refinements were suggested by the full Mathematics Workgroup on June 21, 2016.

The Plus standards did not have any changes to the standards with the exception of a couple deletions to the examples therefore there is not a worksheet that is specific to the Plus standards included with notes for Technical Review.

Vote of support to release the draft for public comment

The final meeting also included an individual survey of all Mathematics Workgroup members present on July 11th to vote in support of the release of the draft Arizona Mathematics Standards for public review. All 40 members present (100%) responded with strongly agree/agree that they supported the release of the current draft of the Arizona Mathematics Standards for public review.