



# Arizona's Draft Standards Mathematics

---

## Algebra 2

ARIZONA DEPARTMENT OF EDUCATION  
HIGH ACADEMIC STANDARDS FOR STUDENTS  
Draft Standards for Public Comment

# Algebra 2 Overview

---

## NUMBER AND QUANTITY - N

### The Real Number System (N-RN)

- Extend the properties of exponents to rational exponents.

### Quantities (N-Q)

- Reason quantitatively and use units to solve problems.

### The Complex Number System (N-CN)

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

## ALGEBRA - A

### Seeing Structure in Expressions (A-SSE)

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

### Arithmetic with Polynomials and Rational Expressions (A-APR)

- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

### Creating Equations (A-CED)

- Create equations that describe numbers or relationships.

### Reasoning with Equations and Inequalities (A-REI)

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

## FUNCTIONS - F

### Interpreting Functions (F-IF)

- Interpret functions that arise in applications in terms of context.
- Analyze functions using different representations.

### Building Functions (F-BF)

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

### Linear, Quadratic, and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

## STATISTICS AND PROBABILITY - S

### Interpreting Categorical and Quantitative Data (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

### Making Inferences and Justifying Conclusions (S-IC)

- Understand and evaluate random processes underlying statistical experiments.

### Conditional Probability and the Rules of Probability (S-CP)

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

## Standards for Mathematical Practices (MP)

## Algebra 2: Critical Areas

---

*For the high school Algebra 2 course, instructional time should focus on four critical areas:*

- 1. Extend the real number system to the complex number system, representing radicals with rational exponents.**
- 2. Solve and interpret solutions to a variety of equations, inequalities, and systems of equations.**
- 3. Demonstrate a fluency graphing and interpreting functions extending from linear, quadratic, and exponential with integer exponents to polynomial, radical, rational, exponential with real exponents, logarithmic, trigonometric functions, and piece-wise defined functions.**
- 4. Extend simple and compound probability calculations to conditional probability.**

(1) Algebra 2 students extend their knowledge of the real number system by working with complex solutions and factors of polynomials. Students expand their experience with polynomial functions, finding complex zeros and interpreting solutions. Students extend properties of exponents to using rational exponents when factoring, solving, and evaluating.

(2) Connections are made between multiplication of polynomials with multiplication of multi-digit integers and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined. Students extend their understanding of solving linear equations, inequalities, and systems to include all the different function types mentioned in the standards.

(3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. Building on their previous work with functions and on knowledge of trigonometric ratios and circles, students now use the coordinate plane to extend trigonometry to model periodic phenomena. Students examine data on two quantitative variables to choose functions and make conclusions in context of the data.

(4) Algebra 2 students build on their foundational probability skills from middle school extending to conditional probability. Students determine independence of events and are able to apply conditional probability to everyday situations.

*The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Mathematical modeling is integrated throughout Algebra 2 course by utilizing real world context.*

|  |   |
|--|---|
| Algebra 2: Mathematics Standards               |   |
| <b>Number and Quantity - N</b>                 |   |
| <b><u>The Real Number System (N-RN)</u></b>    |   |
| <b>A2.N-RN.A</b>                               | <b>Extend the properties of exponents to rational exponents.</b>  |
| A2.N-RN.A.1                                    | Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.                           |
| A2.N-RN.A.2                                    | Rewrite expressions involving radicals and rational exponents using the properties of exponents.  |
| <b><u>Quantities (N-Q)</u></b>                 |   |
| <b>A2.N-Q.A</b>                                | <b>Reason quantitatively and use units to solve problems.</b>   |
| A2.N-Q.A.1                                     | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. |
| A2.N-Q.A.2                                     | Define appropriate quantities for the purpose of descriptive modeling.  |
| A2.N-Q.A.3                                     | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.   |
| <b><u>The Complex Number System (N-CN)</u></b> |   |
| <b>A2.N-CN.A</b>                               | <b>Perform arithmetic operations with complex numbers.</b>  |
| A2.N-CN.A.1                                    | Apply the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form $(a+bi)$ with $a$ and $b$ real.               |
| <b>A2.N-CN.C</b>                               | <b>Use complex numbers in polynomial identities and equations.</b>  |
| A2.N-CN.C.7                                    | Solve quadratic equations with real coefficients that have complex solutions.   |

| <b>Algebra – A</b>  |   |
|---|---|
| <b>Seeing Structure in Expressions (A-SSE)</b>                      |   |
| <b>A2.A-SSE.A</b>   | <b>Interpret the structure of expressions.</b>  |
| A2.A-SSE.A.2  | Use the structure of an expression to identify ways to rewrite it.<br>Extend polynomial expressions to multivariable expressions.<br>Focus on rational or exponential expressions seeing that $(x^2 + 4)/(x^2 + 3)$ as $(x^2 + 3 + 1)/(x^2 + 3)$ , thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$ .                                 |
| <b>A2.A-SSE.B</b>   | <b>Write expressions in equivalent forms to solve problems.</b>   |
| A2.A-SSE.B.3  | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem solving opportunities utilizing real-world context and focus on expressions with rational exponents.<br>c. Use the properties of exponents to transform expressions for exponential functions. |
| A2.A-SSE.B.4  | Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| <b>Arithmetic with Polynomials and Rational Expressions (A-APR)</b> |   |
| <b>A2.A-APR.B</b>   | <b>Understand the relationship between zeros and factors of polynomials.</b>  |
| A2.A-APR.B.2  | Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $(x - a)$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .  |
| A2.A-APR.B.3  | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.<br>Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.   |
| <b>A2.A-APR.C</b>   | <b>Use polynomial identities to solve problems.</b>   |
| A2.A-APR.C.4  | Prove polynomial identities and use them to describe numerical relationships.   |

|   |  |
|---|--|
| <b>A2.A-APR.D</b>   | <b>Rewrite rational expressions.</b>   |
| A2.A-APR.D.6  | Rewrite rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or a computer algebra system.  |
| <b><u>Creating Equations (CED)</u></b>                        |  |
| <b>A2.A-CED.A</b>   | <b>Create equations that describe numbers or relationships.</b>  |
| A2.A-CED.A.1  | Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context.<br>Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions with real exponents.  |
| <b><u>Reasoning with Equations and Inequalities (REI)</u></b> |  |
| <b>A2.A-REI.A</b>   | <b>Understand solving equations as a process of reasoning and explain the reasoning.</b>   |
| A2.A-REI.A.1  | Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.<br>Extend from quadratic equations to rational and radical equations.                              |
| A2.A-REI.A.2  | Solve rational and radical equations in one variable and give examples showing how extraneous solutions may arise.   |
| <b>A2.A-REI.B</b>   | <b>Solve equations and inequalities in one variable.</b>   |
| A2.A-REI.B.4  | Solve quadratic equations in one variable.<br>b. Solve quadratic equations by inspection taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ . |
| <b>A2.A-REI.C</b>   | <b>Solve systems of equations.</b>   |
| A2.A-REI.C.7  | Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |

|   |  |
|---|--|
| <b>A2-A-REI.D</b>                           | <b>Represent and solve equations and inequalities graphically.</b>   |
| A2.A-REI.D.11                               | <p>Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y=f(x)</math> and <math>y=g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately using technology to graph the functions, tables of values, or successive approximations. Include problem solving opportunities utilizing real-world context.</p> <p>Extend from linear, quadratic, and exponential with integer exponents to cases where <math>f(x)</math> and/or <math>g(x)</math> are polynomial, rational, exponential with real exponent, and logarithmic functions.</p>  |
| <b>Functions - F</b>                        |  |
| <b><u>Interpreting Functions (F-IF)</u></b> |  |
| <b>A2.F-IF.B</b>                            | <b>Interpret functions that arise in applications in terms of the context.</b>   |
| A2.F-IF.B.4                                 | <p>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</p> <p>Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.</p> |
| A2.F-IF.B.6                                 | <p>Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context.</p> <p>Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.</p>  |

|                                  |   |
|----------------------------------|---|
| <b>A2.F-IF.C</b>                 | <b>Analyze functions using different representations.</b>   |
| A2.F-IF.C.7                      | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include square root, cube root, polynomial, exponential with real exponents, logarithmic, sine, cosine, tangent, and piecewise-defined functions.   |
| A2.F-IF.C.8                      | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.<br>b. Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.   |
| A2.F-IF.C.9                      | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.).<br>Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.  |
| <b>Building Functions (F-BF)</b> |   |
| <b>A2.F-BF.A</b>                 | <b>Build a function that models a relationship between two quantities.</b>  |
| A2.F-BF.A.1                      | Write a function that describes a relationship between two quantities. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and exponential functions with real exponents.<br>Include problem-solving opportunities utilizing real-world context.<br>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.<br>b. Combine standard function types using arithmetic operations and function composition. |
| A2.F-BF.A.2                      | Write arithmetic and geometric sequences both recursively and explicitly. Use arithmetic and geometric sequences to model situations, and translate between explicit and recursive forms.   |

|   |   |
|---|---|
| <b>A2.F-BF.B</b>  | <b>Build new functions from existing functions.</b>   |
| A2.F-BF.B.3   | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br>Include recognizing even and odd functions from their graphs and algebraic expressions for them. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and exponential functions with real exponents. |
| A2.F-BF.B.4   | Find inverse functions.<br>a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions $f$ and $g$ are inverse functions if and only if $f(x)=y$ and $g(y)=x$ for all values of $x$ in the domain of $f$ and all values of $y$ in the domain of $g$ .<br>b. Understand that if a function contains a point $(a,b)$ , then the graph of the inverse relation of the function contains the point $(b,a)$ ; the inverse is a reflection over the line $y = x$ .   |
| <b>Linear, Quadratic, and Exponential Models (F-LE)</b> |   |
| <b>A2.F-LE.A</b>  | <b>Construct and compare linear, quadratic, and exponential models and solve problems.</b>  |
| A2.F-LE.A.4   | For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.   |
| <b>A2.F-LE.B</b>  | <b>Interpret expressions for functions in terms of the situation they model.</b>  |
| A2.F-LE.B.5   | Interpret the parameters in an exponential function with real exponents in terms of a context.  |
| <b>Trigonometric Functions (F-TF)</b>                   |   |
| <b>A2.F-TF.A</b>  | <b>Extend the domain of trigonometric functions using the unit circle.</b>  |
| A2.F-TF.A.1   | Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.  |
| A2.F-TF.A.2   | Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.   |

|   |   |
|---|---|
| <b>A2.F-TF.B</b>  | <b>Model periodic phenomena with trigonometric functions.</b>   |
| A2.F-TF.B.5   | Create and interpret trigonometric functions that model periodic phenomena with specified amplitude, frequency, and midline.  |
| <b>A2.F-TF.C</b>  | <b>Prove and apply trigonometric identities.</b>  |
| A2.F-TF.C.8   | Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.  |
| <b>Statistics and Probability - S</b>                               |   |
| <b><u>Interpreting Categorical and Quantitative Data (S-ID)</u></b> |   |
| <b>A2.S-ID.A</b>  | <b>Summarize, represent, and interpret data on a single count or measurement variable.</b>  |
| A2.S-ID.A.4   | Use the mean and standard deviation of a data set and properties of a normal distribution to approximate a normal curve to estimate population percentages. Recognize and identify data sets for which such a procedure is not appropriate. Consider non-symmetric data sets and presence of outliers.                                    |
| <b>A2.S-ID.B</b>  | <b>Summarize, represent, and interpret data on two categorical and quantitative variables.</b>  |
| A2.S-ID.B.6   | Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models.<br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. |
| <b>A2.S-ID.C</b>  | <b>Interpret models.</b>  |
| A2.S-ID.C.10  | Interpret parameters of exponential models.   |
| <b><u>Making Inferences and Justifying Conclusions (S-IC)</u></b>   |   |
| <b>A2.S-IC.A</b>  | <b>Understand and evaluate random processes underlying statistical experiments.</b>   |
| A2.S-IC.A.1   | Understand statistics as a process for making inferences about population parameters based on a random sample from that population.   |
| A2.S-IC.A.2   | Explain if a specified model is consistent with results from a given data-generating process.   |

| <b>Conditional Probability and the Rules of Probability (S-CP)</b> |   |
|--|---|
| <b>A2.S-CP.A</b>   | <b>Understand independence and conditional probability and use them to interpret data.</b>  |
| A2.S-CP.A.3  | Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.  |
| A2.S-CP.A.4  | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to calculate conditional probabilities.  |
| A2.S-CP.A.5  | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.  |
| <b>A2.S-CP.B</b>   | <b>Use the rules of probability to compute probabilities of compound events in a uniform probability model</b>  |
| A2.S-CP.B.6  | Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.  |
| A2.S-CP.B.7  | Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.   |
| <b>A2.MP</b>   | <b>Standards for Mathematical Practice</b>  |
| A2.MP.1  | <p>Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate.</p> <p>Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.</p> |

|         |   |
|---------|---|
| A2.MP.2 | <p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>   |
| A2.MP.3 | <p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p> |

|         |   |
|---------|---|
| A2.MP.4 | <p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p> |
| A2.MP.5 | <p>Use appropriate tools strategically.</p> <p>Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.</p>                                    |
| A2.MP.6 | <p>Attend to precision.</p> <p>Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.</p>  |
| A2.MP.7 | <p>Look for and make use of structure.</p> <p>Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.</p>   |

|         |  |
|---------|--|
| A2.MP.8 | <p>Look for and express regularity in repeated reasoning.</p> <p>Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.</p> |
|---------|--|

DRAFT