



Arizona's Draft Standards Mathematics

Algebra 1

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
Draft Standards for Public Comment

Algebra 1 Overview

NUMBER AND QUANTITY - N

The Real Number System (N-RN)

- Use properties of rational and irrational numbers.

Quantities (N-Q)

- Reason quantitatively and use units to solve problems.

ALGEBRA - A

Seeing Structure in Expressions (A-SSE)

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions (A-APR)

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.

Creating Equations (A-CED)

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities (A-REI)

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.

FUNCTIONS - F

Interpreting Functions (F-IF)

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of context.
- Analyze functions using different representations.

Building Functions (F-BF)

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

STATISTICS AND PROBABILITY - S

Interpreting Categorical and Quantitative Data (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Conditional Probability and the Rules of Probability (S-CP)

- Understand independence and conditional probability and use them to interpret data.

Standards for Mathematical Practices (MP)

Algebra 1: Critical Areas

For the high school Algebra I course, instructional time should focus on four critical areas:

- 1. Deepen and extend understanding of linear and exponential relationships.**
- 2. Engage in methods for analyzing, solving, and using quadratic functions.**
- 3. Extend the laws of exponents using integer exponents.**
- 4. Apply linear models to data that exhibit a linear trend.**

(1) In earlier grades, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They can solve linear equations and apply related techniques along with the laws of exponents to solve simple exponential equations.

(2) In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and exponential functions, including sequences, and also explore absolute value, and piecewise-defined functions; they interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students build on and extend their understanding of integer exponents to consider exponential functions. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions.

(3) Students extend the laws of exponents to integer exponents involving square and cube roots and apply this new understanding of number; they strengthen their ability to see structure in and create quadratic and exponential expressions. They identify the real solutions of a function. Students expand their experience with functions to include more specialized functions—absolute value, and those that are piecewise-defined.

(4) Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Mathematical modeling is integrated throughout Algebra 1 by utilizing real world context.

Algebra 1: Mathematics Standards	
Number and Quantity (N)	
<u>The Real Number System (N-RN)</u>	
A1.N-RN.B	Use properties of rational and irrational numbers.
A1.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
<u>Quantities (N-Q)</u>	
A1.N-Q.A	Reason quantitatively and use units to solve problems.
A1.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
A1.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem solving opportunities utilizing real-world context.
A1.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra - A	
<u>Seeing Structure in Expressions (A-SSE)</u>	
A1.A-SSE	Interpret the structure of expressions.
A1.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
A1.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. Focus on numerical expressions, such as recognizing $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the form $(53+47)(53-47)$. Focus on polynomial expressions in one variable, such as seeing an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$.
A1.A-SSE.B	Write expressions in equivalent forms to solve problems.
A1.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. Focus on expressions with integer exponents.
<u>Arithmetic with Polynomials and Rational Expressions (A-APR)</u>	
A1.A-APR.A	Perform arithmetic operations on polynomials.
A1.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A1.A-APR.B	Understand the relationship between zeros and factors of polynomials.
A1.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.
<u>Creating Equations (A-CED)</u>	
A1.A-CED.A	Create equations that describe numbers or relationships.
A1.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities that are linear, quadratic, or exponential with integer exponents.
A1.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A1.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
A1.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
<u>Reasoning with Equations and Inequalities (A-REI)</u>	
A1.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning.
A1.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from linear to quadratic equations.
A1.REI.B	Solve equations and inequalities in one variable.
A1.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A1.A-REI.B.4	<p>Solve quadratic equations in one variable.</p> <p>a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</p> <p>b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.</p> <p>Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.</p>
A1.A-REI.C	Solve systems of equations.
A1.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A1.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.
A1.A-REI.D	Represent and solve equations and inequalities graphically.
A1.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A1.A-REI.D.11	<p>Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately using technology to graph the functions, tables of values, or successive approximations.</p> <p>Focus on cases where $f(x)$ and/or $g(x)$ are linear, absolute value, quadratic and, exponential functions with integer exponents.</p>
A1.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions - F	
<u>Interpreting Functions (F-IF)</u>	
A1.F-IF.A	Understand the concept of a function and use function notation.
A1.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
A1.F-IF.A.2	Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
A1.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
A1.F-IF.B	Interpret functions that arise in applications in terms of the context.
A1.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, absolute value, quadratic, and exponential with integer exponents and piecewise-defined functions (limited to the aforementioned functions).
A1.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
A1.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear, absolute value, quadratic, and exponential functions, all with integer exponents.

A1.F-IF.C	Analyze functions using different representations.
A1.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, exponential with integer exponents, quadratic, and piecewise-defined functions.
A1.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
A1.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Focus on linear, absolute value, quadratic, exponential with integer exponents and piecewise-defined functions. (limited to the aforementioned functions)
Building Functions (F-BF)	
A1.F-BF.A	Build a function that models a relationship between two quantities.
A1.F-BF.A.1	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Focus on linear, absolute value, quadratic, exponential with integer exponents, and piecewise-defined functions (limited to the aforementioned functions).
A1.F-BF.B	Build new functions from existing functions.
A1.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Focus on linear, absolute value, quadratic, exponential with integer exponents, and piecewise-defined functions (limited to the aforementioned functions).

Linear, Quadratic, and Exponential Models (F-LE)	
A1.F-LE.A	Construct and compare linear, quadratic, and exponential models and solve problems.
A1.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
A1.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output pairs.
A1.F-LE.A.3	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
A1.F-LE.B	Interpret expressions for functions in terms of the situation they model.
A1.F-LE.B.5	Interpret the parameters in a linear or exponential function with integer exponents in terms of a context.
Statistics and Probability - S	
Interpreting Categorical and Quantitative Data (S-ID)	
A1.S-ID.A	Summarize, represent, and interpret data on a single count or measurement variable.
A1.S-ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots), for the purpose of comparing statistics between two or more data sets.
A1.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
A1.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets (dot plots, histograms, and box plots), accounting for possible effects of extreme data points (outliers).

A1.S-ID.B	Summarize, represent, and interpret data on two categorical and quantitative variables.
A1.S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
A1.S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. b. Informally assess the fit of a function by plotting and analyzing residuals.
A1.S-ID.C	Interpret linear models.
A1.S-ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
A1.S-ID.C.8	Compute and interpret the correlation coefficient of a linear fit.
A1.S-ID.C.9	Distinguish between correlation and causation.
<u>Conditional Probability and the Rules of Probability (S-CP)</u>	
A1.S-CP.A	Understand independence and conditional probability and use them to interpret data.
A1.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
A1.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

A1.MP	Standards for Mathematical Practice
A1.MP.1	<p>Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate.</p> <p>Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.</p>
A1.MP.2	<p>Reason abstractly and quantitatively.</p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.</p>

A1.MP.3	<p>Construct viable arguments and critique the reasoning of others.</p> <p>Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.</p>
A1.MP.4	<p>Model with mathematics.</p> <p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>
A1.MP.5	<p>Use appropriate tools strategically.</p> <p>Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.</p>

A1.MP.6	<p>Attend to precision.</p> <p>Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.</p>
A1.MP.7	<p>Look for and make use of structure.</p> <p>Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.</p>
A1.MP.8	<p>Look for and express regularity in repeated reasoning.</p> <p>Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.</p>