Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
Counting and Cardinality (CC)			**Carlson-** Consider having a standard related to grouping together objects in group sizes other than 10. With any size groups (including groups of 10), create groups and use the physical act of grouping to support the development of skip counting and foster a conceptual understanding of grouping that supports base ten reasoning. This could also be addressed under the NBT domain. Other than that, the standards are clear and coherent and seem to be measureable and meaningful. **Abercrombie-**Each standard in this domain is clearly stated and describes what students should know and be able to do. Each standard is measurable, has sufficient breadth and cognitive demand, and there are not ambiguous words or phrases included in any of the standards. The standards are written so that they will be unambiguously interpreted across the state. The refinements included in the current draft improve the clarity of the standards. The standards are developmental appropriate. I have no additional feedback on the standards in this domain.	K.CC.B.4c is often addressed in a one to one format with a kindergartener and would be addressed in a support document.	
			Milner-This domain is well covered though I have a concern that teachers may never assess K.CC.B.4c. **Pope-** The majority of the standards in the kindergarten domain of Counting and Cardinality state what students should know and be able to do. B. Almost all of the standards in this domain can be easily measured. Once the term “understand” is defined or operationalized in standards K.CC.B all of the standards should be able to be measured and assessed easily as they will all clearly state the expected student behaviors. The breadth and depth of skill students are required to master for the Counting and Cardinality standards seems developmentally appropriate given the age and skill level of most students in kindergarten. The standards address basic knowledge/recall skills such as being able to count to 100 and write numbers from 0-20 as well as more complex skills such as comparing quantities between two groups. The skills addressed in this strand represent some of the basic concepts key to learning mathematics which students will need to learn and master in order to develop mathematical competence in any area.		
K.CC.A	**Know number names and the count sequence.**				
K.CC.A.1	Count to 100 by ones and by tens.	**Very appropriate for kindergarteners **It would be nice to have specificity as to whether this is rote counting or object-counting. This standard is not specific enough.		No revision	

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K.CC.A.2	Count forward beginning from a given number instead of having to begin at 1.	**Clearer Wording: "Count forward beginning from a given number other than one" **This should provide the limit. Is this also through 100?		Based on public feedback, an example is added and suggested wording is utilized.	Count forward ~~beginning~~ from a given number **other than one, within the known sequence** ~~instead of having to begin at 1.~~ (e.g., "Starting at the number 5, count up to 11.").
K.CC.A.3	Write numbers from 0–20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	**I'm glad you included 0! **"This is purely a reading standards, having nothing to do with Mathematics."		No revision necessary	
K.CC.B	**Count to tell the number of objects.**				
K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.	**This is one of the most important standards in kindergarten! Thank you for including it! I'm concerned with public comments that suggest that the K standards are not developmentally appropriate. This standard alone is the epitome of developmental appropriateness for 5-year-olds! **Dr. Milgram, "This is purely a vocabulary standard. Nothing wrong with it, just don't try to convince teachers that when they teach this, they are teaching 'mathematics.'"	**Pope**- Standard K.CC.B. "Understand the relationship between numbers and quantities" is a bit vague. The word "understand" is used both in the cluster name as well as in parts B and C of standard K.CC.B.4. In neither place is "understand" expanded upon or explained (how are practitioners expected to know if students "understand"? What types of things are students expected to do or demonstrate that show their "understanding"?).	Per Pope's review, specifics are stated in the a and b portions of this standard. no revision necessary	
K.CC.B.5	Count to answer questions about "how many?" when 20 or fewer objects are arranged in a line, a rectangular array, or a circle or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	**Great - developmentally appropriate. I love that you included arrays, circles, and scattered - all serve different purposes. **Why is the word "things" used instead of objects?	**Milner-**K.CC.B.5 contains the common usage, "a number from 1-20", that is much better expressed in formal English as "a number from 1 to 20". **Achieve-**The slight wording change in AZ causes no significant change in the standards' meaning.	Based on Milner's feedback, a minor wording change was made as he stated.	Count to answer questions about "How many?" when 20 or fewer objects are arranged in a line, a rectangular array, or a circle or as many as 10 things in a scattered configuration; given a number from 1 **to** 20, count out that many objects.
K.CC.C	**Compare numbers.**				
K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. (Include groups with up to ten objects.)	Once again, a sign of developmental appropriateness is seen in this standard. I would like to see strategy suggestions such as "using 1:1 correspondence," matching, and counting in an instructional guide for teachers.		No revision necessary	

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K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.	Be more specific about what you mean by compare. If it is greater, less than or equal, etc.	**Wurman-**"between zero and 10" as zero has been already specified and is needed for 10 anyway.	Based on Wurman's feedback, zero is included.	Compare two numbers between ~~1~~ 0 and 10 presented as written numerals.
<u>Operations and Algebraic Thinking (OA)</u>			**Carlson**-This set of standards is clear and coherent with a solid and meaningful progression of ideas across grade levels. **Abercrombie-**The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. **Milner-**This domain would be strengthened by the introduction of the concept of a "unit" or "neutral element" in a binary operation. That allows defining "inverses" and thus understanding subtraction as addition of the additive inverse ("opposite") and division as multiplication by the multiplicative inverse ("reciprocal").		
			Pope-Almost all of the actual standards in this domain clearly state what students are to know and be able to do. Most of the standards clearly state the behaviors that students are to demonstrate even if the Cluster is somewhat ambiguous. For example K.OA.A states that students will "understand addition as putting together and adding to, and understand subtraction as taking apart…" but then the standards that follow are all clearly stated, observable and measureable tasks/behaviors that students would perform indicating their understanding. 1.OA.B.4, 1.OA.D.6, and 3.OA.B.6 all use the term "understand" to describe the student behavior and do not include any further, more specific and clear actions that would demonstrate student understanding. The breadth of the standards in this domain is narrower at the lower grade levels and increasingly more broad, including more skills (such as those related to multiplication and division) with each grade level. The narrower focus in the earlier grade levels makes sense as the focus is on mastering some of the foundational skills needed to be able to perform more complex tasks. The complexity of skills included in this domain increases as well with each successive grade level. In the lower grades students are expected to expand upon basic skills (add and subtract fluently through 10 when in first grade as opposed to through 5 in kindergarten) and are gradually introduced to new, more cognitively challenging skills as well. Presumably as students become more proficient with the basic skills more challenging tasks are introduced.	In response to Pope: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	

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			(cont) While all of the tasks included in the standards seem to follow typical developmental patterns it should be noted that students may struggle in forming the desired deeper conceptual understanding related to some of the skills (such as the inverse relationship between addition and subtraction) even though they are able to reiterate rules that have been taught or follow a sequence of steps.		
K.OA.A	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**		**Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, it is slightly confusing as someone reading the standards that the clusters aren't necessarily related from one grade level to the next. For example, 1.OA.C is "Add and subtract fluently through 10" and 2.OA.C is "Work with groups of objects to gain foundations for multiplication" and 3.OA.C is "Multiply and divide through 100". While all of these standards relate to arithmetic skills there is no consistent or common thread among skills addressed at each grade level in this cluster (OA.C). This is especially confusing given the way the ELA standards are structured with Anchor Standards. It's possible that some practitioners would assume or expect the math standards to follow a similar structure.	In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	
K.0A.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.	**This standards still tells teachers "how to teach" and not "what to teach." This is too prescriptive and does not give the teacher the flexibility to use their own methods. **I appreciate the specificity in this standard. This will help teachers maintain the developmental appropriateness of addition and subtraction with kindergartners. **Algebraic thinking is developmentally inappropriate at this age. Most children cannot use "a variety of strategies" being that they are in the pre-operational phase. They also cannot be expected to use equations to give answers to problems on their own. They need concrete ideas and lots of repetition. This standard also contains prescriptive methods of how a teacher should teach "...with objects, fingers, mental images, drawings, sounds..." etc. Also again equations have no place in K.	**Wurman-**How, exactly, are mental images, sounds, acting out, etc., measurable or clear, as required for the standards? Further, expecting equations is premature in K.	Based on Wurman's feedback as well as public comment, edits were made to remove specifics and just state concretely.	Represent addition and subtraction **concretely**. ~~with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.~~

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K.0A.A.2	Solve addition and subtraction word problems through 10 using a variety of strategies (See Table 1).	Was child development research considered when these standards were developed for 5 year olds. **The standard still includes Table 1 which is too prescriptive. Abstract equations are developmentally inappropriate for Kindergarten. **Since students are expected to fluently add and subtract within 5, then they will need practice to build an understanding of these operations to a level that they will become flexible, accurate, and efficient. Without solving number problems in addition to word problems, they may struggle to reach fluency. Consider changing K.OA.A.2 to include number problems along with word problems as follows: Use addition and subtraction through 10 to solve number problems and word problems involving multiple **K.0A.A.2 Use addition and subtraction through 10 to solve word problems involving multiple problem types (See Table 1), using a variety of strategies. Please specify which problem types should be mastered by the end of Kindergarten.	**Achieve**-CCSS includes more detail about the type of strategies expected at this level. AZ draws attention to Table 1. AZ replaces "within" with "through" to imply a closed interval. However this slight change in wording causes confusion as to the performance expectation. Does "use addition and subtraction through 10" include, for example, 7 + 6? It is not clear what the "multiple problem types" and "a variety of strategies" would be. **Wurman-**Delete "variety of strategies." Insisting on multiplicity of strategies is unnecessary and confuses Kindergartners.	Based on Achieve's and Wurman's and Public feedback, appropriate edits were made to remove a variety of strategies and have within 10. Table 1 is not a "how" but rather an awareness of the different problem types that all childrens should be exposed to.	Solve addition and subtraction word problems and add and subtract ~~through~~ **within** 10. *See Table 1.* ~~using a variety of strategies~~
		(cont.) **This standard would be best if it just stated, "Use addition and subtraction through 10 to solve word problems" and ended it there. The standard stops being a standard and becomes a prescribed method of teaching when it continues with "...multiple problem types (see Table 1), using a variety of strategies." This "standard" does not keep with the promise in the introduction that these are just standards and not methods of teaching. **I would have liked to see the word "situations" included in this standard rather than going back to the "problem types" language from CGI. "Problem situations" is more descriptive of what we ask children to do (e.g., "What's happening in this situation?" leads them to discuss the nature of the action and where the missing number falls. I would never ask a 5-year-old, "What problem type is this?"). In addition, most teachers are not familiar to with the CGI research to know about "problem types."		It is not expected that children know the names of the problem types, this is teacher information only.	

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K.0A.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings**,** and record each decomposition with a drawing or equation.	**Examples need to be provided on a separate document to clarify to teacher what the student should be able to do **Again, this standard is not just a standard, but a prescribed method of teaching, "...by using objects or drawings, and record each decomposition with a drawing or equation". **Awesome standard! **This standard is still too prescriptive and tells the teacher "how to teach" with using objects and drawings. Abstract equations are inappropriate for kindergarten when they do not have a strong background in number sense.	**Wurman-**Actually the "e.g." promoted clarity and the essence that other ways (e.g., fingers, symbols, tally marks) are permissible, while its removal limits the decomposition ONLY to drawings and (concrete?) objects. Further, insistence on equations is premature in K	Wurman's wording was used to edit the standard.	Decompose numbers less than or equal to 10 into pairs in more than one way ~~by using objects or drawings, and record each decomposition with a drawing or equation.~~ (e.g., using fingers, objects, symbols, tally marks, drawings, expressions).
K.0A.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings, and record the answer with a drawing or equation.	This concept is developmentally inappropriate students get frustrated when trying to decompose numbers. Grading on this is very hard when students struggle so much with this! **This standard is still too prescriptive and tells the teacher "how to teach" and not "what to teach." **The standard is overly prescriptive and tell a teacher how to teach not just what the goal is by stating, " ...by using objects or drawings, and record the anwer with a drawing or equation." **This is such an important concept - it lays the groundwork for so much of what will be coming in grades 1 and beyond in regards to base-ten mathematics. This is powerful for students and for teacher awareness.	**Wurman-**Same comment as above regarding "e.g." and "equations."	Wurman's wording was used to edit the standard.	For any number from 1 to 9, find the number that makes 10 when added to the given number ~~by using objects or drawings, and record the answer with a drawing or equation.~~ (e.g., using fingers, objects, symbols, tally marks, drawings, equation).

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K.0A.A.5	Fluently add and subtract through 5.	**(Here is a similar comment I wrote in ELA) Here's a bigger question to consider: Kinder should be able to read 50 high frequency words. I support that. Why then would kinder only have to be fluently adding/subtracting through 5? I think the older standard of 10 was appropriate. **This standard is a keeper. It is simple and non-prescriptive. This should be the example for all of the others. As Leonardo DiVinci said, "Simplicity is the ultimate form of sophistication." **Looks good. **Algebraic thinking is developmentally inappropriate at this age. Most children cannot use "a variety of strategies" being that they are in the preoperational phase. They also cannot be expected to use equations to give answers to problems on their own. They need concrete ideas and lots of repetition. K.OA.A.5 is a good example of what 5 and 6 year old children can do. This specific line is also a good example of clarity. The rest of these "standards" are not really standards at all, they are prescribed methods of how to teach. It would be best to simply state what a child needs to know and learn, not HOW the teacher should teach and what method is to be used.	**Achieve**-AZ replaces "within" with "through" to imply a closed interval. However, this slight change in wording causes confusion as to the performance expectation. Does "use add and subtract through 5" include, for example, 4 + 5? **Wurman**-Limiting to 5 rather than to 10 (see previous 3 standards) is artificial and unnecessary handicap.	Changed through to within based on technical review. "using a variety of strategies" was removed from the standards in K	Fluently add and subtract ~~through~~ within 5.
<u>Number and Operations in Base Ten (NBT)</u>			**Carlson-**Consider having a standard related to grouping together objects in group sizes other than 10. With any size groups (including groups of 10), create groups and use the physical act of grouping to support the development of skip counting and foster a conceptual understanding of grouping that supports base ten reasoning. Asking students to create grouping schemes using a base other than 10 can help support reasoning about the base 10 system and highlight its benefits and historical/biological reasons why humans widely adopted this system. This could also be addressed under the CC domain as well. **Abercrombie**-The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, "Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate.		

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			Pope-Almost all of the standards in this domain clearly state what students are to know and be able to do. The breadth and depth of the standards in this domain seems reasonably appropriate at each grade level in grades K-3. The concepts related to the base ten number system are so crucial to mathematical fluency and to the type of conceptual understanding discussed in the introduction of the standards. It makes sense to begin by introducing students to ideas such as place value in very concrete ways (as with base ten blocks) to illustrate that ten ones also make "a ten" then teach them how to apply these skills in various mathematical contexts (such as rounding and estimating). The progression of the breadth of application of skills related to base ten as well as the complexity of the tasks students are asked to perform based on principles of the base ten number system follow a logical sequence from one grade level to the next.		
K.NBT.A	**Work with numbers 11-19 to gain foundations for place value.**				
K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and additional ones by using objects or drawings and record each composition or decomposition with a drawing or equation.	Overly prescriptive in telling a teacher how to teach the standard: "…by using objects or drawings and record each composition or decompostition with a drawing or equation." "…using a variety of strategies."	**Achieve**-CCSS offers an example of decomposition and requires (explains) understanding of number composition. AZ removed the second example and made the first example part of the standard. They also removed the requirement for understanding place value in terms of compositions. **Wurman--** The same comments as before regarding the incorrect removal of "e.g." thereby limiting options - Same comments as before regarding the wrong-headed insistence on equations in K - The removal of the example 18=10+8 is justified by a misunderstanding. Its purpose was not to limit-- it is already limited by the language -- but rather to illustrate that, for example, 18=9+9 is an incorrect application of this standard.	Based on Technical Review, the e.g. was restored and standard was re-worded for clarity. In regards to public comment, "using a variety of strategies" was removed from the standards in K	Compose and decompose numbers from 11 to 19 into ten ones and additional ones by using objects, **drawings and/or equations.** ~~or drawings and record each composition or decomposition with a drawing or equation.~~ Understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones (e.g., 18 = 10 + 8).
K.NBT.B	**Use place value understanding and properties of operations to add and subtract.**		**Achieve-**This Grade K header and standard have no counterpart in the CCSS at this grade level. These, however, seem redundant to K.OA.2, 3, and 4. This AZ addition is not directly addressed in the CCSS at this grade level. This concept seems to overlap with K.OA.2, 3, and 4, and extends K.OA.5.The distinction between the OA expectations and this header and standard is not clear. It is also not explained how place value understanding would be addressed in a way that is different from 1.NBT.A.1.		

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K.NBT.B.2	Demonstrate conceptual understanding of addition and subtraction through 10 using a variety of strategies.		**Carlson-**K.NBT.B.2: “Demonstrate conceptual understanding of addition and subtraction through 10 using a variety of strategies.” This does not meet the clarity criterion. If you want students to understand something “conceptually”, be explicit about what meanings you want them to develop. “Conceptual understanding of addition and subtraction” is very vague. **Milner-**The new K.NBT.B.2 does not belong in NBT since it does not involve place **value at all. In fact, when talking about the number 10, the conceptualization at this level is only as ten “ones” and not as one “ten”.** **Achieve-**In this standard, students are asked to operate with numbers "through 10." This implies the possibility of adding, for example 8 + 7. Also, how does this standard connect to the new cluster header? The header implies that place value understanding and properties of operations would be required. That is not clear in the standard and may not be appropriate for this level. It might be more realistic to expect decomposing numbers and making 10s as seen in K.OA.A.5 and K.NBT.A.1. It also would be important at this level to inform teachers as to what "a variety of strategies" would entail.	Based on Technical review, conceptual was removed and through was replaced with within. To align with previous change, variety of strategies was also removed.	Demonstrate ~~conceptual~~ understanding of addition and subtraction **within** ~~through~~ 10 using ~~a variety of strategies.~~ p**lace value.**
			Wurman-This seems like a spurious and unnecessary standard adding nothing beyond what K.0A.A.1 already offers. Pope-Standard K.NBT.B.2. does not provide any actual behavior or skill that students are to do. The standard reads that students will “demonstrate their conceptual understanding of addition and subtraction through 10 using a variety of strategies”. There is no clear directive in terms of what types of strategies would accurately show a students’ conceptual understanding. Can students use any strategy to model addition and subtraction and would that count as a demonstration of conceptual understanding for a kindergarten student? There needs to be more information given so that practitioners know what kind of evidence to look for (how can they tell if a student has developed an appropriate conceptual understanding? What does that look like?)	In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	

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Measurement and Data (MD)			**Abercrombie-**The standards are written with clarity, are measurable, and have sufficient breadth and depth. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression **Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, the content within each of the Clusters is again sort of random when looking at the standards in this domain from one grade level to the next. As an entire concept the progression of the skills related to Measurement and Data is logical but there isn't any clear connection of the standards in a Cluster between grade levels. As a whole the skills in the domain build upon one another but the skills addressed by individual standards or clusters do not necessarily relate and build upon one another from one grade to the next.		
K.MD.A	**Describe and compare measurable attributes.**				
K.MD.A.1	Describe several measurable attributes of a single object such as length and weight.		**Carlson**-K.MD.A.1: "Describe several measureable attributes…" and K.MD.A.2: "Directly compare two objects with a measureable attribute in common..." Elsewhere in my feedback I mentioned how the terms "quantities" and "quantitative reasoning" are mentioned several times in the standards but are never defined and explained in any detailed way (which is very problematic since there is a rich body of research related to quantitative reasoning in mathematics education research). This standard is really the starting point for supporting quantitative reasoning, but it is not defined relative to the term "quantitative reasoning" and so any teacher seeking to understand what it means to engage in quantitative reasoning is not supported in seeing how these standards relate to that goal. This continues throughout this strand. You could rename the strand "Measurement, Data, and Quantitative Reasoning, or you could include a detailed description of what the standards writers mean by "quantitative reasoning". **Achieve**-The slight wording change in AZ makes for no significant change in the standards' meaning. **Wurman-**The original standard indicated scaffolding: start with single attribute of a group of objects, then proceed to multiple ones to show that grouping on one may differ from grouping on another. The new standard obscures it by unhelpful generalization.	The "such as" was removed and parenthesis with e.g. was added for consistency within all grade level standards.	Describe several measurable attributes of a single object **(e.g., length and weight).**

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K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference.		**Abercombie-**The standard K.MD.A.2 is developmentally appropriate as long as the attribute being measured presents in a consistent way across cases. For example, the child would be able to compare a measurable attribute such as length for two objects with the same appearance (e.g. two straight lines) but not necessarily when the presentation of the attribute varies across objects (e.g. a straight line and a curved line) as the latter requires cognitive thinking skills that are not typically developed until around age 7. I suggest adding language to specify the equivalence of appearance of the attribute to this standard. **Wurman-**This standard is somewhat unclear and the original example tried to illustrate it. Removing the example doesn't help. Rephrasing it might have helped, such as: Knowing the measured heights of two students, predict which one will be taller. Alternately, directly compare the height of two	Adding "equivalence of appearance of the attribute" would cause more confusion to the meaning and implementation of the standard. An example was added to demonstrate the equivalence of appearance.	Directly compare two objects with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference **(e.g., directly compare the length of 10 cubes to a pencil and describe one as longer or shorter).**
K.MD.B	**Classify objects and count the number of objects in categories.**				
K.MD.B.3	Classify objects or people into given categories; count the number in each category and sort the categories by count. (Note: Limit category counts to be less than or equal to 10.)		**Achieve-**The slight wording change in AZ makes for no significant change in the standards' meaning.	No revision necessary.	
Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength. Removing the list of shapes from the Kindergarten standards is potentially problematic, since there are 2-D and 3-D shapes that are not included in this list (e.g. octagon, icosahedron), and yet the expectation at kindergarten is not for exhaustive knowledge of all 2-D and 3-D shapes. Therefore the scope of the expectations in these standards is left vague and potentially unreasonable for kindergarteners. **Wurman-**I think the original selection of shapes was inappropriate for Kindergarten and shouldn't have included hexagons and cylinders. Removing them all, however, is ill advised as it offers no guidance at what shapes should be included. Arguing that just saying 2-D and 3-D shapes is sufficient is disingenuous -- are rhombi included? Parallelograms? Trapezoids? Pyramids? Toruses?	K.G.A.2 specifically names the shapes in the revised standard.	
K.G.A	**Identify and describe shapes.**				

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K.G.A.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	**This comment is for the K.G.A but you did not provide a comment space: This implies kindergarteners should know ALL 2-D and 3-D shapes since the parameters are no longer defined. **"This standard is just fine. However, there was no place to enter core concepts that are missing in the Kindergarten Math standards. One VERY important concept that needs to be added is PATTERNS and SEQUENCES."		Patterns are addressed throughout the standards and specifically in the math practices. No revision necessary	
K.G.A.2	Correctly name shapes regardless of their orientation or overall size.			Per techincal reviewers comments on Geometry domain, shape names were added to the standard.	Correctly name shapes regardless of their orientation or overall size **(e.g., circle, triangle, square, rectangle, rhombus, trapezoid, hexagon, cube, cone, cylinder, sphere).**
K.G.A.3	Identify shapes as two-dimensional (lying in a plane, flat) or three-dimensional (solid).			No revision necessary	
K.G.B	**Analyze, compare, create, and compose shapes.**				
K.G.B.4	Analyze and compare two-dimensional and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities and differences.	**"in different sizes and orientations" should be kept because it highlights the need to explore various representations of a shape. It sets students up in the future to understand that geometric transformations (excluding dilation) do not alter the defining characteristics of a shape. **The deletion of "similarities, differences, parts, and other attributes" comprehensively changes the nature of the standard. The new standard focuses on position and not attributes. **It would help tremendously to include a list of the speciific two- and three-dimensional shapes that should be included. **Again, overly prescriptive in methodology. “…using informal language to describe their similarities and differences.”	**Wurman**-What resulted from the suggested changes is a completely different standard from the original. Worse, whatever it offers is already present in K.G.A.1 and K.G.A.2 above. In other words, as emasculated it simply duplicates them. Further the use of "environment" seems spurious and unclear -- shouldn't drawn shapes, or shaped blocks, qualify? The original standard aimed at abstracting common and different attributes across a collection of geometric --2D and 3D -- shapes, and their relative position and orientation. All this is lost in the proposed language. I suggest either to eliminate it completely, or leave it as was. **Milner**-The proposed K.G.B.4 is a duplicate of the proposed (and current) K.G.B.1. I recommend to keep the existing K.G.B.4 removing the examples therein.	Per pubic comment and technical review, the original language "parts (e.g. number of sides and vertices/corners), and other attributes (e.g. having sides of equal length).	Analyze and compare two-dimensional and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities and differences, **parts (e.g., number of sides and vertices/corners), and other attributes (e.g. having sides of equal length).**

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K.G.B.5	Model shapes in the world by building and drawing shapes.		**Wurman-**The suggested changes are wrong-headed and the justification doesn't justify them. The "building from components" clarifies that the goal is to assemble pre-existing shapes into more complex shapes, rather than build shapes from clay or Play-Doh. The example clarified it even better.	Based on Wurman's feedback, the clarification points were restored through the example.	Model shapes in the world by building shapes from components and drawing shapes **(e.g., use sticks and clay balls).**
K.G.B.6	Compose simple shapes to form larger shapes.	**While the change has reduced the wordiness and eliminated the example, I think it sacrifices clarity. I know that "model" is defined in the introduction, but I think this looks it the term very liberally. The original standards asks to compose shapes from simpler shapes, which paves the way for the calculation of area of non-standard shapes in the future grades. The new standard is easily interpreted as not saying that. Feel free to remove the example, but the other words should remain the same. **Here's a good example where the elimination of the example made this much tougher to interpret. I would say the same about many of the above standards.	**Wurman**-The suggested changes are wrong-headed and the justification doesn't justify them. The "compose simple shapes to build larger shapes" clarifies that the goal is to assemble pre-existing shapes into more complex shapes, rather than build shapes from clay or Play-Doh. The example clarified this well. The addition of "in the world" corrupts the original meaning that dealt with concrete geometrical shapes -- not even drawings! -- into a duplicate of the previous (and corrupted) standard (K.G.B.5). **Milner**-The proposed K.G.B.6 is a duplicate of the proposed K.G.B.5. I recommend to keep the existing K.G.B.6 removing the example therein.	To give clarity to the standard, based on public comment and Milner, the verb was changed, mathematical language was used, and an example was added.	~~Compose~~ **Use** simple shapes to form ~~larger~~ **composite** shapes **(e.g., "What new shape can we make if we put two squares together with full sides touching?").**
SMP	**Standards for Mathematical Practices**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.		

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K.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				
K.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
K.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can				
K.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
K.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
K.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
K.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
K.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

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Operations and Algebraic Thinking (OA)			**Carlson**-This set of standards is clear and coherent with a solid and meaningful progression of ideas across grade levels. **Abercrombie-**The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. However, the deletion of the mental strategies described in 1.OA.C.6 without reference to the full definition of fluency described in the introduction may alter or limit the cognitive processes engaged away from flexible mathematical thinking and toward rote memorization **Milner**-This domain would be strengthened by the introduction of the concept of a "unit" or "neutral element" in a binary operation. That allows defining "inverses" and thus understanding subtraction as addition of the additive inverse ("opposite") and division as multiplication by the multiplicative inverse ("reciprocal").	In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	
			Pope-Almost all of the actual standards in this domain clearly state what students are to know and be able to do. Most of the standards clearly state the behaviors that students are to demonstrate even if the Cluster is somewhat ambiguous. For example K.OA.A states that students will "understand addition as putting together and adding to, and understand subtraction as taking apart…" but then the standards that follow are all clearly stated, observable and measureable tasks/behaviors that students would perform indicating their understanding. 1.OA.B.4, 1.OA.D.6, and 3.OA.B.6 all use the term "understand" to describe the student behavior and do not include any further, more specific and clear actions that would demonstrate student understanding. The breadth of the standards in this domain is narrower at the lower grade levels and increasingly more broad, including more skills (such as those related to multiplication and division) with each grade level. The narrower focus in the earlier grade levels makes sense as the focus is on mastering some of the foundational skills needed to be able to perform more complex tasks. The complexity of skills included in this domain increases as well with each successive grade level. In the lower grades students are expected to expand upon basic skills (add and subtract fluently through 10 when in first grade as opposed to through 5 in kindergarten) and are gradually introduced to new, more cognitively challenging skills as well. Presumably as students become more proficient with the basic skills more challenging tasks are introduced. While all of the tasks included in	In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	

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1.OA.A	**Represent and solve problems involving addition and subtraction.**		**Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, it is slightly confusing as someone reading the standards that the clusters aren't necessarily related from one grade level to the next. For example, 1.OA.C is "Add and subtract fluently through 10" and 2.OA.C is "Work with groups of objects to gain foundations for multiplication" and 3.OA.C is "Multiply and divide through 100". While all of these standards relate to arithmetic skills there is no consistent or common thread among skills addressed at each grade level in this cluster (OA.C). This is especially confusing given the way the ELA standards are structured with Anchor Standards. It's possible that some practitioners would assume or expect the math standards to follow a similar structure.	Math does not have grade band standards like ELA nor do we have Anchor standards. Generally the understanding of concepts that have to do with place value are in NBT and then the fluency is in OA.	
1.OA.A.1	Use addition and subtraction through 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in change and result unknown problem types using a variety of strategies. (See Table 1.)	The overview on page 1 states, "Add and subtract through 10." Clarify if it is 10 or 20. **Please, please, please, please, PLEASE!!! Stop requiring students to demonstration a "variety" of ways to solve a problem! Having to learn so many different ways to add or subtract numbers is creating a lot of confusion. The students tend to mix the methods up. You can require the teacher teach the all the methods available, but let the student determine what works best for him or her. Once that student has found what works best, let the teacher teach it that way to that studen **Good idea to put the problem types in a separate table. The table is very well organized and gives clear examples of the different problem types. I think it would be a great thing to use as a basis for a poster in my classroom that students can refer to when solving different types of story problems to help them organize their thinking. **This is a standard within a standard and is too prescriptive with Table 1 included. Dr. James Milgram stated about this Common Core Standard, "teaching this standard alone could consume perhaps 80% of time in the first grade! This is a standard within a standard and very unclear as written."	**Achieve**-The CCSS specificity is lost in the "multiple problem types" and "variety of strategies." However these are clarified in the AZ Table 1. NOTE: Table 1 in AZ is part of the Introduction, which is a separate document from the grade level standards.AZ replaces "within" with "through" to imply a closed interval. However this slight change in wording causes confusion as to the performance expectation. Does "Use addition and subtraction through 20" include, for example, 17 + 19?	Edits reflect Achieve's feedback.	Use addition and subtraction ~~through~~ **within** 20 to solve word problems ~~involving situations of adding to, taking from, putting together, taking apart and comparing,~~ **with unknowns in all positions** ~~change and result unknown problem types using a variety of strategies~~ (e.g., by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem). *See Table 1.*

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		**1.OA.A.1 . Please specify which problem types should be mastered by the end of first-grade. **Algebraic thinking is developmentally inappropriate at this age. Most children cannot use “a variety of strategies” being that they are in the preoperational phase. They also cannot be expected to use equations to give answers to problems on their own. They need concrete ideas and lots of repetition. **The word "situations" was a better choice than "problem types." Might you consider returning to that word? It's much more user friendly as a teacher might ask students, "What's happening in this situation?" help them decontextualize the mathematics (SMP2). However, I wouldn't ask a student "What problem type is this?" nor would I suggest that other teachers ask this question of their students. The CGI research is solid, but the associated vocabulary is not common among today's teachers. **Writing equations is not a strategy but a representation. This standard should require students use a variety of strategies to solve but also require an equation be written using a symbol for the unknown number to represent the problem.			

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1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Standard is too prescriptive and tells you "how to teach" and not "what to teach." Some examples were deleted under the heading, but the "variety of strategies" just moved to another place within the standards document- Table 1. **Standards still too prescriptive with "how to's" and not just "what to teach." This is developmentally questionable as a universally achieveable skill in grade 1. Students may be able to via rote memory to recite 10,20, 30, etc., but explaining it is very abstract at age 6 or 7. **Easier to understand. **Again, algebraic thinking is not developmentally appropriate at this age. Children in kindergarten are in the pre-operational phase and need concrete ideas. Equations with unknown factors provide incredible stress on the young pre-operational mind. Please see the developmental stages by psychologist Jean Piaget. Parents are very upset that these inappropriate cognitive demands are being placed on their young children. **"Why is there a difference between the wording of 1.OA.1 and 1.OA.2? They should both end with:""using objects, drawings, and equations with a symbol for the unknown number to represent the problem."""	**Achieve**-AZ removed the example that is included in the CCSS, making the methods listed appear to be the only requirements and that they must be used the same time. **Wurman**-Actually, the new language does not meet criteria for clarity and measurability. First, Table 1 deals with problems calling for addition or subtraction of only two numbers, rather than three like expected here. Further, removing the "e.g." limits the solution only to "objects, drawings, and equations" for no good reason. How about tally marks? How about bar charts? At least restore the original language for clarity and coherence. Ideally restore the language but remove the "and equations with a symbol for the unknown number" based on the same logic as in the previous standard, thereby leaving equations as optional rather than mandatory at this grade.	Based on Achieve's and Wurman's feedback, the standard was restored. Including and/or addresses Wurman's concern about equations being mandatory.	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 (e.g. by using objects, drawings, and/or equations with a symbol for the unknown number to represent the problem).
1.OA.B	**Understand and apply properties of operations and the relationship between addition and subtraction.**				
1.OA.B.3	Apply properties of operations (commutative and associative properties of addition) as strategies to add and subtract through 20. (Students need not use formal terms for these properties.)	**Examples need to be provided on a separate document to clarify for teacher what the student should be able to do **Properties of operations should only be used in the upper grades of elementary school while children are being introduced to pre-Algebra concepts. First grade students need time to learn basic skills of adding and subtracting through repetition (skill & drill). Parents are very upset with the inappropriate early introduction to pre-Algebra.		In response to public comment: As worded in the standard, it is stated that kids do NOT need to know the formal terms for the properties. No revision necessary.	

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1.OA.B.4	Understand subtraction as an unknown-addend problem through 20. (See Table 1).	Standard is still too prescriptive and Table 1 remains which is full of "how to's" and not "what to teach." **The connection from counting and cardinality in Kindergarten to addition and subtraction in first grade is absent from these standards. Suggest reinserting 2010 standard 1.OA.C.5 “Relate counting to addition and subtraction” to maintain coherence among the standards and renumbering draft standard 1.OA.C.5 to 1.OA.C.6 and renumbering all of the following OA standards to correspond.	**Wurman-**As usual, the deletion of the examples seems ill-advised. They illustrate clearly the intention of the standard and make it more accessible. As already discussed, "through 20" is more confusing than "within 20" as it can refer to the addends rather than to the sum. Moreover, Table 1 is irrelevant to treating subtraction as unknown-addend.	Based on Wurman's feedback, through is changed to within, table 1 is removed and example was restored.	Understand subtraction as an unknown-addend problem ~~through~~ within 20 ~~(See Table 1)~~(e.g., subtract 10-8 by finding the number that makes 10 when added to 8).
1.OA.C	**Add and subtract through 10.**		**Achieve**-AZ appears in this cluster header to be lowering the bar for Gr 1 operations. However, the requirement to add and subtract through 20 actually match that of the CCSS. [AZ replaces "within" with "through" to imply a closed interval.]	Based on Achieve's feedback, through was replaced by within.	Add and subtract ~~through~~ **within** 10.
1.OA.C.5				to preserve coding, this originally eliminated standard was restored.	1.OA.C.5 Relate counting to addtion and subtraction.

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1.OA.C.5	Fluently add and subtract through 10.	**New Standard is clear as written and a better standard. **Much clearer and simpler. **The wording of "fluently add and subtract through 10" is too vague. What is the requirement for fluent? How many completed in how much time? Is the speed of completing math facts important or the accuracy? As a teacher of students with special needs - I believe it is more important for a student to be accurate within a time frame that works for them. **I like the flexibility but I wonder if you could put the examples in the third column of examples. **Facts through 10 is very appropriate for 1st graders to be fluent in. **In the critical areas, this states that students should be able to add and subtract through 20. The standard now states that they need to add and subtract through 10. This needs to be consistent and I think it should be stated as it is in the critical area portion. **This is a very low standard for Arizona children. I prefer the old standard of fluently adding/subtracting through 20. Yes, I know those teens are hard, but our kids can do it.	**Achieve**-AZ replaces "within" with "through" to imply a closed interval. However, this slight change in wording causes confusion as to the performance expectation. Does "Use addition and subtraction through 10" include, for example, 7 + 9? **Wurman-**The suggested change lowers even more the already-mediocre requirement of fluent addition and subtraction only to 10. This should be adjusted to fluent addition and subtraction within 20, to be closer to high achieving nations (e.g., in first grade, Singapore expects addition/subtraction within 100). No research to support addition and subtraction to 20, but be fluent only to 10. The elimination of the examples in this standard is justified, as the focus should be on what student can do rather than how they should do it.	Based on Achieve's feedback, through was replaced by within. Fluency within 20 is a 2nd grade expectation, however 1st graders are expected to be fluent within 10 to provide a coherent expectation across grade levels. Changed coding to preserve coding as commented throughout by Achieve.	1.OA.C.6Fluently add and subtract ~~through~~ **within** 10.
		(cont.) **Perhaps revise the cluster to read: "Add and subtract through 20." And revise 1.OA.C.5 to include: Add and subtract through 20. Fluently add and subtract through 10. Having this cluster heading (Add and subtract through 10) listed as a main point in the overview may cause teachers to believe that their work only focuses on fluency to 10 and misses that they are doing significant work with helping students extend beyond ten. **This revision is appropriate			
1.OA.D	**Work with addition and subtraction equations.**				

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1.OA.D.6	Understand the meaning of the equal sign, regardless of its placement within an equation, and determine if equations involving addition and subtraction are true or false.	**the addition of “regardless of its placement” is a strong addition that will enhance understanding the meaning of the equal sign. **Easier to understand. **The addition of "regardless of its placement within an equation" is EXCELLENT because it directly hit the meaning of the equal sign. Keep this change; it is conceptually sound and provides a scaffold from which algebra is built.	**Achieve-**AZ removed the example and added a non-limitation on placement of the equal sign in an equation. (It is not clear what that non-limitation means exactly.) **Wurman**-The proposed language changes the meaning of the standard, lacks clarity, and is partially mathematically wrong. - - what is the meaning of an equal sign "regardless of its placement within an equation" such as: 12=6+6=9+3? What about 12=6+6=9+5? -- Are the examples really "limiting the standard"? The proposed language " determine if equations involving addition and subtraction are true or false" doesn't include identities (doesn't involve addition or subtraction), and one of the examples demonstrates and important general case that is lost by the new language (5+2=2+5). The examples, indeed, do clarify the standard!	Based on Achieve's and Wurman's feedback, the examples were restored to maintain clarity in the standard. Coding preserved	1.OA.D.7 Understand the meaning of the equal sign, ~~regardless of its placement within an equation,~~ and determine if equations involving addition and subtraction are true or false (e.g., Which of the following equations are true and which are false? 6 + 1 = 6 - 1, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2).
1.OA.D.7	Determine the unknown whole number in any position in an addition or subtraction equation relating three whole numbers (see Table 1).	Algebra is not developmentally appropriate for the pre-operational mind that 1st graders have.	**Wurman-**The examples nicely illustrate the standard and should not be removed. The "in any position" is actually more confusing than the examples.	Based on Wurman's feedback, the examples were restored to make the standard more clear. Coding was preserved.	1.OA.D.8 Determine the unknown whole number ~~in any position~~ in an addition or subtraction equation relating three whole numbers ~~(see Table 1).~~ (e.g., determine the unknown number that makes the equation true in each of the equations 8 + o = 11, 5 = o - 3, 6 + 6 = o).

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Number and Operations in Base Ten (NBT)			**Abercrombie-**The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, "Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate. **Pope**-Almost all of the standards in this domain clearly state what students are to know and be able to do. The breadth and depth of the standards in this domain seems reasonably appropriate at each grade level in grades K-3. The concepts related to the base ten number system are so crucial to mathematical fluency and to the type of conceptual understanding discussed in the introduction of the standards. It makes sense to begin by introducing students to ideas such as place value in very concrete ways (as with base ten blocks) to illustrate that ten ones also make "a ten" then teach them how to apply these skills in various mathematical contexts (such as rounding and estimating). The progression of the breadth of application of skills related to base ten as well as the complexity of the tasks students are asked to perform based on principles of the base ten number system follow a logical sequence from one grade level to the next.		
1.NBT.A	**Extend the counting sequence.**				
1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**What is the reasoning of counting to 120 and stopping there? Why not 150? Or 200? According to the Core Knowledge Scope and Sequence and other time proven developmentally appropriate standards, first graders should learn to count to 100 and not beyond.	**Wurman**-This is an example where the standard has a arbitrarily made-up limit of 120, because counting to 1000 is expected in Kindergarten and counting to 100 is expected in grade 2. There is absolutely no reason for 120. It could have been 122 or 144 or 150 with the same justification. In truth, this standard should be eliminated as senseless in this grade and replaced by "Skip count within 100 by 2s and 10s." In fact, "Skip count up and down within 100 by 2s and 10s" would be even better.	Students struggle with the transition of counting past 100.. This is the first time they are experiencing moving from 2-digit to 3-digit numbers and the idea that place value now includes 100's, 10's and 1's. Possible revision: Count to 120 by 1's, 2's, and 10's starting at any number less than 100. In this range, read and write numerals and represent a number of objects with a written numeral.	Count to 120 by 1's, 2's, and 10's starting at any number less than 100. In this range, read and write numerals and represent a number of objects with a written numeral.
1.NBT.B	**Understand place value.**				

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1.NBT.B.2	Understand that the two digits of a two-digit number represent groups of tens and some ones. Understand the following as special cases: a. 10 can be thought of as a group of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).		**Wurman**-The new standard mangled the language and destroyed the precision of the original standard. The digits represent "amounts of" not "groups" or "some" ones. What if the "some" happens to be zero? Does the unit digit represent then a "not some" ones? Restore the original language in toto! **Milner**-1.NBT.B.2 and 2.NBT.A.1 should have consistent language: the former uses "groups" of tens, while the latter uses "amounts". **Pope**-Several of the standards in this domain use the word "understand" without providing any further explanation as to how students are to demonstrate their understanding. In addition to K.NBT.B.2, 1.NBT.B.2, and 2.NBT.A.1. The portion of these last two standards that uses the word "understand" almost seems unnecessary. In reading these standards it appears as though the concepts and skills that these standards address are stated in parts A, B, and C following the statements about "understanding". If verbs were added to parts A, B, and C (show, tell, explain 10 can be represented by a group of ten ones called a "ten") these skills could then easily be measured.	Technical review regarding consistent language was implemented. In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	Understand that the two digits of a two-digit number represent groups of tens and ~~some~~ ones. Understand the following as special cases: a. 10 can be thought of as a group of ten ones — called a "ten". b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.			No revision necessary	
1.NBT.C	**Use place value understanding and properties of operations to add and subtract.**				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard-12/2016
1.NBT.C.4	Add through 100 using models and/or strategies based on place value, properties of operations, and the relationship between addition and subtraction.	This standard is very, very similar to 1.NBT.C.6. Multiples of 10 are numbers through 100, so, I don't think we need to have both standards since they are basically the same. **Standard still has "how to's" and not "what to teach." It mentions using models and/or strategies which implies a teacher must introduce mulitple methods, but it may be that students understand the concept using just one method. **the key idea from the 2010 standard regarding that students are to "Understand that in adding two-digit numbers, one adds tens and tens and ones and ones; and that it is sometimes necessary to compose a ten" is missing from these standards. Consider adding "Recognize that in adding two-digit numbers, one adds tens and tens and ones and ones; and that it is sometimes necessary to compose a ten." **Easier to understand. **Please take a look at the 2nd-grade standard for adding and subtracting through 1000. The verbiage there is far superior and should be included here to provide a beautiful flow. **Standard should read "models and strategies..."	**Achieve**-AZ replaced "within" with "through" to imply a closed interval. It is not clear whether "add through 100" means that the sum cannot be more than 100 or that any two 2-digit numbers are fair game. Would the sum, 78 + 54 be included in the AZ translation? If so, the requirements are different from the CCSS counterpart. Much of the detail in the CCSS was removed in AZ: - AZ replaced "within" with "through" to imply a closed interval. It is not clear whether "add through 100" means that the sum cannot be more than 100 or that any two 2-digit number is fair game. Would the sum, 78 + 54 be included in the AZ translation? If so, the requirements are different from the CCSS counterpart.- The descriptions of the types of addition that are required (e.g. 2-digit and 1-digit) are removed in AZ.Also:- By deleting the adjective "concrete," we lose the distinction between the two uses of the term "model" that is important for teachers to understand. Also deleted is "drawings" as an example.- The description of how students are to relate the strategies to the written method is removed, lowering the rigor from that of the CCSS.- The conceptual understanding of composing a ten is missing in AZ. **Wurman**-The removal of the pedagogy is actually helpful here. Strategies should be understood to include algorithms and not just ad hoc ones.	Changed based on pubic comment and technical review and for consistency across grade levels.	~~Add through~~ ~~**within**~~ ~~100 using models and/or strategies based on place value~~ ~~**(including multiples of 10),**~~ ~~properties of operations, and the relationship between addition and subtraction.~~ Demonstrate understanding of addition within 100, connecting objects or drawings to strategies based on place value **(including multiples of 10),** properties of operations, and/or the relationship between addition and subtraction. **Relate the strategy to a written form.**
1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.			No revision necessary	

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1.NBT.C.6	Add and subtract multiples of 10 through 100 using models and/or strategies based on place value, properties of operations, and the relationship between addition and subtraction.	"Using models and/or strategies..." directs instructional method. The standard should be limited to the "what," e.g., "Add and subtract multiples of 10 through 100." **Still too prescriptive with "how to's" and not just "what to teach." Standard still states using "models and multiple strategies." If a class understands one strategy, which is it required to use other strategies as well? **Standard should read "models and strategies..."	**Achieve**-AZ adds addition to the operations required here and by changing 10-90 to "through 100," they add the 3-digit number to the Gr 1 requirement.Much of the detail included in the CCSS is missing:- The limitation for "positive or zero differences" is missing.- By deleting the adjective "concrete," we lose the distinction between the two uses of the term "model" that is so important for teachers to understand. Also deleted is "drawings" as an example. -The description of how students are to relate the strategies to the written method is removed. **Wurman**-(1) The standard for addition is already present in 1.NBT.C.4 and in this case simply expand it with "Add and subtract" rather than just "Add" and delete this one ; (2) Alternately, leave just subtraction here. Further, this is a good example why the change of "within <boundary>" to "through <boundary>" is wrong-headed. Here the "through 100" is used in the sense of addends and subtrahends each being within 100 rather than their sum, or difference as was in previous standards.	Based on Wurman's feedback, just subtraction is left here and a specific range is used to clarify the wording as suggested in Wurman's feedback as well as Achieve.	~~Add and subtract multiples of 10 through 100 using models and/or strategies based on place value, properties of operations, and the relationship between addition and subtraction.~~ Subtract multiples of 10 in the range of 10-90 (positive or zero differences), using objects ~~concrete models~~ or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form.
1.NBT.C.7	Demonstrate understanding of addition and subtraction through 20 using a variety of place value strategies, properties of operations, and the relationship between addition and subtraction.	"...using a variety of place value strategies..." directs instructional method, that is, the "how," not just the "what." This should be simplified to "Demonstrate understanding of addition and subtraction through 20." **New standard but too prescriptive with "how to's" and not "what to teach" with multiple models and strategies. **Standard should read "through 20 using models and strategies based on place value..."	**Achieve**-This conceptual understanding standard in CCSS 1.OA.6 is partially addressed in NBT in AZ Gr 1. Most examples are removed in AZ. The differences in coding for these two standards will make it difficult for AZ teachers to make national searches for materials aligned to CCSS 1.OA.5 or 1.OA.6 or to 1.NBT.7, which does not exist in the CCSS. **Wurman**-There should not be an artificial separation of fluency with addition and subtraction to 10 or 20 as already mentioned in 1.OA.C.6 above. Consequently, this standard is unnecessary. Demonstrating understanding is already called for by multiple standards (e.g., 1.NBT.C4, 1.NBT.C.6) and no need to repeat ad nauseam.	Per Technical review, this standard is found througout first grade and there is no need to repeat. Deleted	~~Demonstrate understanding of addition and subtraction through 20 using a variety of place value strategies, properties of operations, and the relationship between addition and subtraction.~~ DELETE STANDARD
Measurement and Data (MD)			**Abercrombie**-The standards are written with clarity, are measurable, and have sufficient breadth and depth. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression **Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, the content within each of the Clusters is again sort of random when looking at the standards in this domain from one grade level to the next. As an entire concept the progression of the skills related to Measurement and Data is logical but there isn't any clear connection of the standards in a Cluster between grade levels. As a whole the skills in the domain build upon one another but the skills addressed by individual standards or clusters do not necessarily relate and build upon one another from one grade to the next.		

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard-12/2016
1.MD.A	**Measure lengths indirectly and by iterating length units.**				
1.MD.A.1	Order three objects by length. Compare the lengths of two objects indirectly by using a third object.			No revision necessary.	
1.MD.A.2	Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	I would like to see standard measurement of inches and half inches brought back into the standards. **The part removed is not repetitive and limits the standard. The removed section denotes that the distances measured should be limited to values that when measured using different objects should be a whole number value with no fractional portions. The earlier part of the standard highlights that measuring requires the unit of measure is applied with no gaps or overlaps. Therefore, the deleted portion does have merit because it expressly limits the context of the standard. Please include this.	**Achieve-**AZ did not include the limitation.	Based on Public Comment and Technical Review, the limit in the standard was restored. **Per Dr. Wurman the limit to the standard was restored.**	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. **(Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.)**
1.MD.B	**Work with time and money.**				
1.MD.B.3a	Tell and write time in hours and half-hours using analog and digital clocks.	Adding the standard that addresses money will support the students as they move into 2nd grade. **This is an excellent addition to the 1st grade standards to support continued understanding.		No revision necessary.	

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1.MD.B.3b	Identify coins by name and value (pennies, nickels, dimes and quarters).	**Placing this back in first grade is appropriate. Students need the repeated exposure to coins - especially as society has changed, and children have less and less natural exposure to coins. **I was glad to see that this was added, because students need to be able to Identify something before they can work with it. Knowing what the coins look like and the value will help them be more successful in 2nd and 3rd grade. **Love the addition of money in first **This is an excellent addition to the 1st grade standards. **This is an excellent addition to the 1st grade standards. It helps scaffold the money standards taught in 2nd grade. **Bringing money back to 1st grade is appropriate. This standard is necessary. **Great additional standard. Allows for learning coins prior to mastering add/subtract coins. **This is a good addition to build a scaffold for future work.	**Achieve**-AZ added requirements to identify coins. Note: Inserting this standard caused a difference in coding when comparing the AZ standard to the CCSS. Changing the coding here may cause confusion for teachers who do national searches for materials aligned to 1.MD.4 in the CCSS.	In order to have Arizona standards align to clusters and domains, this is necessary. No revision necessary	
1.MD.C	**Represent and interpret data.**				
1.MD.C.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.		**Achieve**-There is a coding difference here, which may cause confusion for teachers who do national searches for 1.MD.5 in the CCSS.	changed additional money coding to have alignment throughout the rest of the standards as Achieve requested. No revision necessary	
Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.		
1.G.A	**Reason with shapes and their attributes.**				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
1.G.A.1	Distinguish between defining attributes (open, closed, number of sides, vertices) versus non-defining attributes (color, orientation, size) for two-dimensional shapes; build and draw shapes to possess defining attributes.	Probably too much for first grade. If you think otherwise, then you should indicate the research that supports this standard.	**Achieve-**AZ removed the reference to examples in the CCSS, making it appear that only those attributes are required. **Wurman-**The removal of the "e.g." is wrong-headed and mathematically incorrect. First, the list of "defining attributes" in the proposed standard is non-exhaustive. For example, faces of 3D shapes are not mentioned. Further, the original language implied "defining attributes" as geometrical but didn't force the issue, while the new language imposes it rather than implies it. Is the color red "defining" if the task is "select all the shapes colored red"? Clearly it is. The original language was clearer, more correct, and less limiting. **Milner**-1.G.A.1 has awkward wording, "draw shapes to possess defining attributes." I would reword as "draw shapes that possess prescribed (or given) attributes."	Per Wurman the original e.g.s were added and per Milner "to" was changed to "that".	Distinguish between defining attributes (~~open, closed, number of sides, vertice~~ **triangles are closed and 3 sided**~~)~~ versus non-defining attributes (color, orientation, **overall** size) for two-dimensional shapes; build and draw shapes ~~to~~ **that** possess defining attributes.
1.G.A.2	Compose two-dimensional shapes or three-dimensional shapes to create a composite shape and compose new shapes from the composite shape.	**The listed shapes were not examples; they were the parameters....do they need to know ALL 2-D and 3-D shapes **Expected shape names must be included. Just like numerical progressions are specified, so should these. From a testing standpoint, if the AZMerit is going to test with specified shapes, those shapes must be known.	**Achieve-**The details in the CCSS about the types of 2- and 3-D shape and vocabulary requirements are removed in AZ. **Wurman-**Actually, they were unnecessarily pushing the standards such as with right cylinders. In any case, without an example the last part is either wrong of so obscure as to be incoherent: what is the meaning of "compose new shapes from the composite shapes"?	Per Wurman "compose new shapes from the composite shapes" is wrong, obscure and incoherent because there no examples given. Achieve just stated we removed the vocabulary.	Compose two-dimensional shapes or three-dimensional shapes to create a composite shape. ~~and compose new shapes from the composite shape.~~
1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves and fourths. Understand that decomposing into more equal shares creates smaller shares.	So we are no longer having them describe the whole? This is a very important understanding for students to have. i.e. the whole is two of the shares or two halves **The whole is a foundational understanding of fractions, since the numerical representation of a fraction tells us nothing about its actual magnitude unless we know the size of the whole. Consider reinserting: Describe the whole as two of two equal shares or four of four equal shares.	**Achieve**-Additional vocabulary and description requirements are specified in the CCSS. **Wurman**-The omission of "Describe the whole as two of, or four of the shares." takes away a key mathematical element of importance leading to 3.NF.1, 3.NF.2 and 3.NF.3 . Needs to be restored. **Milner-**1.G.A.3 should include "quarters" as a synonym of "fourths". The proposed 2.G.A.1 should end with "Draw two-dimensional shapes having specified attributes."	Per Wurman "Describe the whole as two of, or four of the shares" was restored. Per Milner "quarters" was used in parenthesis as a synonym of "fourths".	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves and fourths **(quarters)**. **Describe the whole as two of, or four of the shares.** Understand that decomposing into more equal shares creates smaller shares.

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SMP	**Standards for Mathematical Practice**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.		
1.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

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1.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard-12/2016
1.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of				
1.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				

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1.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
1.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
1.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				

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1.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

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<u>Operations and Algebraic Thinking (OA)</u>			**Carlson**-This set of standards is clear and coherent with a solid and meaningful progression of ideas across grade levels. **Abercrombie-**The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. **Milner**-This domain would be strengthened by the introduction of the concept of a "unit" or "neutral element" in a binary operation. That allows defining "inverses" and thus understanding subtraction as addition of the additive inverse ("opposite") and division as multiplication by the multiplicative inverse ("reciprocal").	In response to Pope: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	
			Pope-Almost all of the actual standards in this domain clearly state what students are to know and be able to do. Most of the standards clearly state the behaviors that students are to demonstrate even if the Cluster is somewhat ambiguous. For example K.OA.A states that students will "understand addition as putting together and adding to, and understand subtraction as taking apart…" but then the standards that follow are all clearly stated, observable and measureable tasks/behaviors that students would perform indicating their understanding. 1.OA.B.4, 1.OA.D.6, and 3.OA.B.6 all use the term "understand" to describe the student behavior and do not include any further, more specific and clear actions that would demonstrate student understanding. The breadth of the standards in this domain is narrower at the lower grade levels and increasingly more broad, including more skills (such as those related to multiplication and division) with each grade level. The narrower focus in the earlier grade levels makes sense as the focus is on mastering some of the foundational skills needed to be able to perform more complex tasks. The complexity of skills included in this domain increases as well with each successive grade level. In the lower grades students are expected to expand upon basic skills (add and subtract fluently through 10 when in first grade as opposed to through 5 in kindergarten) and are gradually introduced to new, more cognitively challenging skills as well. Presumably as students become more proficient with the basic skills more challenging tasks are introduced. While all of the tasks included in the standards seem to follow typical developmental patterns it should be noted that students may struggle in forming the desired deeper conceptual understanding related to some of the skills (such as the inverse relationship between addition and subtraction) even though they are able to reiterate rules that have been taught or follow a sequence of steps.	In response to Pope: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	
2.OA.A.1	**Represent and solve problems involving addition and subtraction.**				

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2.OA.A.1	Use addition and subtraction through 100 to solve one-step word problems. Use addition to solve two-step word problems using single-digit addends. Represent a word problem as an equation with a symbol for the unknown. (See Table 1)	As a second grade teacher I think although difficult for many students they are possible with repeated practice. I think the third section "Represent a word problem as an equation with a symbol for the unknown" is more than many second grade students will be able to learn without a great deal of practice. Especially in the area where I teach. **This is a standards within a standard and is full of "how to's" with Table 1 included. This needs to be broken down into multiple standards with "what to teach" not "how to's."	**Milgram**-This must have examples to limit it. As stated it is far too vague for second grade students. **Achieve-**AZ limits two-step problems to within 20 and removed the examples of the types of addition and subtraction problems. They limit the strategies to just using an equation. AZ replaces "within" with "through" to imply a closed interval. However, this slight change in wording causes confusion as to the performance expectation. Does "Use addition and subtraction through 100" include, for example, 57 + 79? **Wurman**-The proposed language unnecessarily lowers the expectations for two-step problems to 20, while the original standard expected them to be within 100. Further, it unnecessarily dictates a single representation of word problems by an equation, while in the original language it was not limited and could have also been a bar-chart, for example. Finally, the change of "within" to "through" is wrong-headed as has been already observed multiple times.	Achieve's and Wurman's feedback reflects the edits of the word "within" to the standard. Wurman's feedback is reflected in the re-wording of the standard and is consistent with wording from 1st grade.	Use addition and subtraction ~~through~~ **within** 100 to solve one-step ~~and two-step~~ word problems. **Use addition to solve two-step word problems using single-digit addends. Represent a word problem as an equation with a symbol for the unknown.** *See Table 1.*
2.OA.B	**Add and subtract through 20.**		**Achieve**-AZ replaced "within" with "through" to imply a closed interval, possibly causing specificity issues. **Wurman**-Changing "within" to "through" is ill-advised and introduces lack of clarity.	Achieve's and Wurman's feedback reflects the edits of the word "within" to the standard.	Add and subtract ~~through~~ **within** 20.

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2.OA.B.2	Fluently add and subtract through 20. By end of Grade 2, know from memory all sums of two one-digit numbers.	I feel this is doable with repeated practice. **I like the definition of fluency provided in the executive summary; it will help change the idea that fluency is all about speed. **This is essential so that kids can function in life. They need to be able to add and subtract. **This is a low expectation that only places a bigger burden on 3rd grade. Our kids can at least go through 50 if not 100. **3rd grade is not enough time to gain fluency in multiplication and division math facts. Memorization of a portion of these facts should be required in 2nd grade (ex: multiplication products through 5x5). This allows students more time with the concept of multiplication before expanding into higher numbers and the inverse operation in 3rd grade. **changing within to through makes sense. The changes in this standard are appropriate. **NOW students are capable of understanding all the different strategies without getting them confused. 2nd grade would be a good year to begin requiring them to demonstrate a variety of ways of solving the problems, not Kindergartenor 1st grade. **Not developmentally appropriate to use "mental strategies" in 2nd grade. Will frustrate and confuse student! Where is 1.OA.6- I could not find in red-lines??	**Milgram**-Solid Standard! **Achieve-**AZ removed "mental strategies" as the method of operating fluently. Since the examples for 1.OA.6 were removed in AZ, this CCSS reference has no basis. **Wurman-**The addition of "By end of Grade 2, know from memory all sums of two one-digit numbers" is welcome, even as high achieving countries expect this in grade 1. But the omission of "using mental strategies" shifts the focus to process fluency -- the ability to quickly and routinely calculate sums within 20 -- rather than rely on automaticity and recall that cognitive science shows is necessary. Finally, the regular comment on the wrong-headedness of changing "within" to "through."	Through was changed to within based on TR comments and consistency in the standards.	Fluently add and subtract ~~through~~ **within** 20. By end of Grade 2, know from memory all sums of two one-digit numbers.
		(cont.) **This standard seems confusing since the first sentence includes adding some double digits, but the second does not. Would it be clarifying to add an "and" rather than have two separate sentences? For example: By the end of Grade 2, know from memory all sums of two one-digit numbers, AND fluenty add and subtract through 20. **Thank you for removing "mental strategies"!			

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2.OA.C	**Work with equal groups of objects to gain foundations for multiplication.**		**Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, it is slightly confusing as someone reading the standards that the clusters aren't necessarily related from one grade level to the next. For example, 1.OA.C is "Add and subtract fluently through 10" and 2.OA.C is "Work with groups of objects to gain foundations for multiplication" and 3.OA.C is "Multiply and divide through 100". While all of these standards relate to arithmetic skills there is no consistent or common thread among skills addressed at each grade level in this cluster (OA.C). This is especially confusing given the way the ELA standards are structured with Anchor Standards. It's possible that some practitioners would assume or expect the math standards to follow a similar structure.	In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	
2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members. Write an equation to express an even number as a sum of two equal addends.	Again, doable with repeated practice. **I don't know what the last items is trying to do except confuse; "Write and equation to express an even number as a sum of two equal addends." Re-write or delete!	**Milgram**-Write an equation to express an even number as a sum of two equal addends. This last sentence may well expect too much from second grade students. Most of them, typically, will have little to no understanding of what are and are not equations. **Wurman**-The example was mathematically important to guide teachers to teach the concept of one-to-one correspondence and not just rely on memorization tricks such as "if it ends in 0,2,4,6,8 then it's even." It is important to restore them.	Based on Wurman's feedback, the mathematically important example was restored. Based on Milgram's feedback and looking at the cluster heading, the second part of the standard was removed.	Determine whether a group of objects (up to 20) has an odd or even number of members (e.g., by pairing objects or counting them by 2's). ~~Write an equation to express an even number as a sum of two equal addends.~~
2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays. Write an equation to express the total as a sum of equal addends.	Again, doable with repeated practice. **Please specify the number of column or rows in this standard. Is it still 5? Multiplication is not listed as one of the critical areas in 2nd grade and without limitations on this standard, it could be interpreted as such.	**Achieve**-The notes in standard 2.OA.C.4 mention the addition of parentheses to the standard, though that change did not happen. Arizona should review the notes and changes for consistency. The CCSS limitation of the size of the arrays was removed in the AZ. Note: In the AZ technical review, it states that "parenthesis [sic] were added to define the limit of rectangular arrays used in 2nd grade." Those parentheses are missing.	Changed based on Achieve's feedback.	Use addition to find the total number of objects arranged in rectangular arrays **(with up to 5 rows and 5 columns).** Write an equation to express the total as a sum of equal addends.

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<u>Number and Operations in Base Ten (NBT)</u>			**Abercrombie-**The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, "Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate. **Pope**-Almost all of the standards in this domain clearly state what students are to know and be able to do. The breadth and depth of the standards in this domain seems reasonably appropriate at each grade level in grades K-3. The concepts related to the base ten number system are so crucial to mathematical fluency and to the type of conceptual understanding discussed in the introduction of the standards. It makes sense to begin by introducing students to ideas such as place value in very concrete ways (as with base ten blocks) to illustrate that ten ones also make "a ten" then teach them how to apply these skills in various mathematical contexts (such as rounding and estimating). The progression of the breadth of application of skills related to base ten as well as the complexity of the tasks students are asked to perform based on principles of the base ten number system follow a logical sequence from one grade level to the next.		
2.NBT.A	**Understand place value.**				

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2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the following as special cases: a. 100 can be thought of as a group of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	Again, doable with repeated practice. **By teaching place value students have a BETTER understanding the value of a number and how to add and subtract. Teaching the traditional way to add students are just being robots and doing what they were told. They have no understanding. When you do the place value they understand sooooo much better!	**Milner-**1.NBT.B.2 and 2.NBT.A.1 should have consistent language: the former uses "groups" of tens, while the latter uses "amounts". **Milgram**-The intent of this standard was to have students understand the EXPANDED FORM for three digit whole numbers, and to understand that when one writes such expressions as, e.g. 731 it really means 7 hundreds plus 3 tens plus 1. **Wurman**-There was no harm done by the examples and they clarified the standard, as examples tend to do. In this case the examples were not critical, although their presence contribute to the **overall clarity of the standards.** **Pope-**Several of the standards in this domain use the word "understand" without providing any further explanation as to how students are to demonstrate their understanding. In addition to K.NBT.B.2, 1.NBT.B.2, and 2.NBT.A.1. The portion of these last two standards that uses the word "understand" almost seems unnecessary. In reading these standards it appears as though the concepts and skills that these standards address are stated in parts A, B, and C following the statements about "understanding". If verbs were added to parts A, B, and C (show, tell, explain 10 can be represented by a group of ten ones called a "ten") these skills could then easily be measured.	based on Milner's feedback, both 1st and 2nd grade indicate "groups" for consistency. Based on Wurman's feedback, the example was restored. In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	Understand that the three digits of a three-digit number represent ~~amounts~~ **groups** of hundreds, tens, and ones **(e.g., 706 equals 7 hundreds, 0 tens, and 6 ones and also equals 70 tens and 6 ones).** Understand the following as special cases: a. 100 can be thought of as a group of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NBT.A.2	Count up to 1000 by 1s, 5s, 10s, and 100s from different starting points.	Time consuming to go all the way to 1000, but doable with regular practice. **Why the inclusion of counting by 1's in the 2.NBT.A.2 standard? It seems that counting by ones would be tedious to 1,000. The students have mastered counting by ones in first-grade to 120 and through understanding place value they can extend the counting sequence to larger numbers. Isn't the intention of this standard to help students develop place value understanding and to also prepare them for multiplication in third grade? I suggest revising the standard by removing counting by 1's. **Clearer by defining what "skip counting" applies.	**Milgram-**What happened to the revised standard? **Achieve**-AZ changed "within 1000" to "to 1000." The latter would mean that the requirement is to always count up to 1000 from different starting places but not necessarily to different end places. The CCSS expects counting to different numbers that fall within 1000. AZ also added the requirement to start at different points. Do "points" mean "numbers?" This should be clarified. Note: The AZ technical review states, "parenthesis [sic] were added to clarify that students should skip count starting at different numbers." However, none are here. **Wurman**-First, there are no parentheses. Further, the public comment reflected ignorance as "within 1000" already included different starting points, while the proposed language ("count to") implies always counting up to a 1000. A secondary implication of counting TO 1000 is that it may be misinterpreted as skip-counting by 5 always having to start on a multiple of 5, and skip-counting by 10 having to start on a multiple of 10. The original language was better! If the starting point needed further clarification, simple adding "starting at any number" at the end would be better than what is suggested.	Based on public comment and technical review, to was changed to within and 1's was removed with clarity in wording.	~~Count up to 1000 by 1s, 5s, 10s, and 100s from different starting points.~~ Count within 1000; skip count by 5's, 10's and 100's.

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2.NBT.A.3	Read and write numbers up to 1000 using base-ten numerals, number names, and expanded form.	Again, doable with repeated practice. **In life students need to know how to read a number. It is our job to teach students how to read and write to actually be able to function in life.	**Milgram-**I am concerned that 1000 is a 4 digit number, while the above standards only talked about 3 digit numbers. Would suggest "Read and write whole number LESS THAN 1000 using base-ten numerals and expanded form." (Number names should be deleted.)	No revision necessary.	
2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	doable		No revision necessary.	
2.NBT.B	**Use place value understanding and properties of operations to add and subtract.**				
2.NBT.B.5	Demonstrate understanding of addition and subtraction through 100 based on place value using a variety of strategies such as properties of operations and the relationship between addition and subtraction.	"...using a variety of strategies..." directs instructional technique, the "how." The standard should be limited to the "what," that is, "Demonstrate understanding of addition and subtraction through 100." **Again, doable with repeated practice. **Please add this language to the third-grade standard for adding and subtracting through 1000. There should be a flow. The third-grade standard is far inferior to this one.	**Milner**-In 2.NBT.B.5 fluency should be expected but has been removed. **Milgram-**Marginally ok standard. In my view there is far too much pedagogical material here. Teachers should be responsible only for assuring that students are able to add two numbers less than 100 correctly, and understand what addition is. How they do it should be left to their best judgement. **Achieve**-The requirement for fluency is removed in this AZ standard (and moved to Grade 3). AZ replaced "within" with "through" to imply a closed interval, possibly causing specificity issues. **Wurman**-The elimination of process fluency requirement here, and its replacement with "understanding" is wrong-headed. The understanding has already been developed in grade 1 (e.g., 1.OA.A.1, 1.OA.B.3, 1.OA.C.6). Here it should be about fluency rather than delaying it even further. And the unnecessary and problematic change of "within" to "through."	Technical review comments were all taken into consideration and this standard was restored based on Public, Milner, Milgram, Achieve, and Wurman.	~~Demonstrate understanding of addition and subtraction Fluently add and subtract within through 100 based on place value using a variety of strategies such as properties of operations and the relationship between addition and subtraction.~~ Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.	"...using strategies..." directs instructional technique, (how). Should be "Add up to four two-digit numbers." **Again, doable with repeated practice.	**Milgram**-I would say that adding 4 numbers is too many. Three would be adequate.	Based on Milgram's feedback, 3 numbers is adequate and standard was revised.	Add up to ~~four~~ three two-digit numbers using strategies based on place value and properties of operations.

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2.NBT.B.7	Demonstrate understanding of addition and subtraction through 1000, connecting concrete models or drawings to strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form.	Again, doable with repeated practice. **I really like the changes here. It just makes so much sense!! **Easier to understand. **beautiful!!!!!!	**Milgram**-This is both entirely redundant and incoherent. 2.NBT.B.5 covers the same material but is entirely coherent. As far as I can tell, this is a standard that appears in the Common Core document, and is one that I use to illustrate the major problems with the common core document. My strong advice is to delete this "standard." **Achieve-**AZ put the emphasis only on demonstration of conceptual understanding of the two operations, while the CCSS primarily expects students to understand and also to perform the operations using the described strategies. The example of "understanding" offered in the CCSS emphasizes and defines operations based on place value. AZ removed the reference to composition or decomposition of 10s or 100s.AZ replaced "within" with "through" to imply a closed interval, possibly causing specificity issues.	2.NBT.B.5 is fluency in addition and subtraction through 100, this standard is to 1000. This is foundational for 3rd grade.	Demonstrate understanding of addition and subtraction within 1000, connecting objects or drawings to strategies based on place value **(including multiples of 10),** properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written form.
2.NBT.B.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900 from different starting points.	Again, doable with repeated practice. **This allows students to see there are several ways to add and subtract. By teaching this the students gain confidence that not everyone has to solve a problem the same way. **These two are exactly the same...different starting points are not added....I even checked the draft? **In the comparison document, it stated that	**Milner-**For 2.NBT.B.8 the draft does not show any difference with the 2010 standard. **Milgram-**What do you mean by 100-900 here? If you mean BETWEEN 100 and 900, would it not be much clearer to say this? **Achieve**-In the AZ technical review there is a note about adding "different starting points," based on public comments. This did not make it into the AZ standard.Note: In the AZ technical review, there is a mention of using different starting points. This clarification does not appear in this draft. **Wurman-**I don't see any clarification change. The original standard, however, is rather clear as to its starting point expectations.	Wording was changed based on Technical Review feedback.	~~Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900 from different starting points~~ Mentally add 10 or 100 to a given number ~~between~~ in the range of 100 and 900, and mentally subtract 10 or 100 from a given number ~~between~~ in the range of 100 and 900.
2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)	More difficult, but, doable with repeated practice. **While it doesn't specify I have to assume this section expects students to use composing/decomposing/making tens to solve equations b/c the kindergarten standard clearly stated it.I would like these practices removed,they are EVER so CONFUSING.A number or sum can't change therefore I think we dont need 101 ways to explore what 60-3 is.these practices fall under NY Engage, College & Career Ready, Eureka Math, all Common Core.Exactly what families don't want.		Not actionable No revision necessary.	

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Measurement and Data (MD)			**Abercrombie**-The standards are written with clarity, are measurable, and have sufficient breadth and depth. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression Pope-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, the content within each of the Clusters is again sort of random when looking at the standards in this domain from one grade level to the next. As an entire concept the progression of the skills related to Measurement and Data is logical but there isn't any clear connection of the standards in a Cluster between grade levels. As a whole the skills in the domain build upon one another but the skills addressed by individual standards or clusters do not necessarily relate and build upon one another from one grade to the next.		
2.MD.A	**Measure and estimate lengths in standard units.**				
2.MD.A.1	Measure the length of an object by selecting and using appropriate tools.	Again, doable with repeated practice.	**Wurman**-Would a laser-based measuring tape be acceptable? Would a chain of paper clips be acceptable? This standard is about using standard-length measuring tools, and the examples are critical. As written this is meaningless.	Examples of standard-length measuring tools were restored to the standard based on Wurman's technical review.	Measure the length of an object by selecting and using appropriate tools **(e.g., ruler, meter stick, yardstick, measuring tape).**
2.MD.A.2	Understand that the length of an object does not change regardless of the units used. Measure the length of an object twice, using different standard length units for the two measurements; describe how the two measurements relate to the size of the unit chosen.	Again, doable with repeated practice. **I believe this really gets at the heart of the standard more. **I like the clarification **Might be pretty sophisticated in second grade. Show the research that demonstrates that this is appropriate, or move up to third or even fourth grade.	**Achieve**-The AZ requirement to "understand" in the first part of their standard is a partial description that is required in the last part of the CCSS. **Wurman**-The problem with the "clarification" is that the goal of the original standard is NOT to point out that the length of an object does not change. The point was to illustrate that depending on the size of the unit, the number of units for the same length vary. With the "clarification" this goal is now obscured.	The first statement was deleted and last statement added based on Wurman's technical review.	~~Understand that the length of an object does not change regardless of the units used.~~ Measure the length of an object twice, using different standard length units for the two measurements; describe how the two measurements relate to the size of the unit chosen. **Understand that depending on the size of the unit, the number of units for the same length vary.**
2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.	Again, doable with repeated practice.		No revision is necessary.	
2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	Again, doable with repeated practice.	**Milgram-**In the previous standard you use "estimate" (which is correct when dealing with physical measurements), but here you use "determine" which is technically IMPOSSIBLE when physically measuring lengths. I would suggest replacing "determine" by estimate.	No revision is necessary. We appreciate Milgram's review but feel the use of the word "determine" is appropriate for 2nd grade students.	
2.MD.B	**Relate addition and subtraction to length.**				

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2.MD.B.5	Use addition and subtraction through 100 to solve word problems involving lengths that are given in the same unit.	More difficult, but, doable with repeated practice.	**Achieve**-AZ replaced "within" with "through" to imply a closed interval, possibly causing specificity issues. Removing the "e.g." may lead to the implication that only drawings and equations are required. **Wurman**-The removal of the example is not disastrous, but is really unnecessary here as it reduces the overall clarity of the standards. And the usual observation of the wrong-headedness of replacing "within" by "through." And, clearly, the explanation is incorrect with regard to what it actually removed.	"Through" was restored to "within" based on the technical review.	Use addition and subtraction ~~through~~ **within** 100 to solve word problems involving lengths that are given in the same unit.
2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences through 100 on a number line diagram.	Again, doable with repeated practice.	**Milgram**-One question: here you add and subtract through 100, whereas earlier in this document you add and subtract through 1000. Wouldn't more consistency be helpful? (My own view is that second graders do not need to be able to handle more that hundreds.) **Achieve**-AZ replaced "within" with "through" to imply a closed interval, possibly causing specificity issues. **Wurman**-The usual observation of the wrong-headedness of replacing "within" by "through."	"Through" was restored to "within" based on the technical review.	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences ~~through~~ **within** 100 on a number line diagram.
2.MD.C	**Work with time and money.**				
2.MD.C.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	Again, doable with repeated practice.		No revision is necessary.	

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2.MD.C.8	Find the value of a collection of coins and dollars. Record the total using $ and ¢ appropriately.	Again, doable with repeated practice. **This is appropriate, given that students will be exposed to coins in first grade. However, I would like to see the parameters defined. Will students be required to count coins with a total greater than a dollar? Are they expected to add dollars to 100? Please consider providing clear parameters. **Changing the progression of money was a good idea and appropriate.	**Milner**-2.MD.C.8 needs to specify the coins because other countries have coins of different denominations than the US. Moreover, the "dollars" in the draft allow for dollar bills of any denomination, which is not appropriate for this grade. 2.MD.C.8 and 3.MD.A.2 have inconsistent notation for cents. The latter needs better explanation of the decimal point. When we write $12.00 we are using the decimal point but there are no cents. What is probably meant is "1 ₵ = $0.01". **Milgram**-I think that it should read "using $ and Cent symbols appropriately." **Achieve**-The CCSS requires word problems here, while AZ moved this requirement to Grade 3. This is arguably a lower expectation at this grade level in AZ. Also the CCSS specifies which bills and coins are required. AZ removed the CCSS example. **Wurman**-The goal to enhance the use of money in the early curriculum is correct, yet the suggested changes are counterproductive. The purpose was not to push knowledge of nickels dimes and quarters to 3rd grade, but rather to ascertain the totals are also expected at this level. The goal could have been easily ascertained by adding "sums and differences of" in the original language such as: Solve word problems involving sums and differences of dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?	The specific coins were restored in this standard based on technical review.To directly address Achieve's concerns, word problems were restored. This also addresses Wurman's comment about finding sums and differences.	~~Find the value of a~~ Solve word problems in volving collections of ~~coins and dollars.~~ **money, inlcuding dollar bills, quarters, dimes, nickels, and pennies.** Record the total using $ and ¢ appropriately.
2.MD.D	**Represent and interpret data.**				
2.MD.D.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	Again, doable with repeated practice.		No revision is necessary.	
2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in the graph. *See Table 1.*	Again, doable with repeated practice. **Standard is still too prescriptive and full of "how to's" if Table 1 remains!	**Achieve**-AZ removed the reference to a "bar graph" in the last sentence, making both picture and bar graphs part of the problem solving requirement	No revision is necessary.	

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Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.		
2.G.A	**Reason with shapes and their attributes.**				
2.G.A.1	Identify and describe specified attributes of two-dimensional and three-dimensional shapes, according to the number and shape of faces, number of angles, and the number of sides and/or vertices. Draw two-dimensional shapes.	Again, doable with repeated practice. **This is a very different standard than the original; wording change did not clarify, it changed it. The parameters were also removed so are they to know ALL 2-D and 3-D shapes	**Achieve-**AZ replaced "recognize" with "identify and describe." Identification is required in the CCSS for specific shapes, which are not specified in AZ. (Teachers may need help with the limitations at this grade level.) In removing the specific shapes listed in the CCSS, AZ opens the door to any and all 2- and 3-dimensional shapes. Drawing in AZ is restricted to 2-dimensional shapes. The CCSS suggestion and limitation for comparing size is removed in AZ. **Wurman**-Mangled meaning, different from the original. The original expects students to draw to a specification of attributes (5 sides, 7 angles), the proposed speaks of drawing some abstract and unspecified 2D shapes. Further, the original included 3D shapes (it used "faces" to indicate that), while the new expects drawing only 2D shapes. As modified, it reflects no growth from K and grade 1. **Milner-**The proposed 2.G.A.1 should end with "Draw two-dimensional shapes having specified attributes."	The last statement was revised and example restored based on technical review.	Identify and describe specified attributes of two-dimensional and three-dimensional shapes, according to the number and shape of faces, number of angles, and the number of sides and/or vertices. Draw two-dimensional shapes **based on the specified attributes (e.g. triangles, quadrilaterals, pentagons, and hexagons).**
2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of squares.	More difficult, but, doable with repeated practice.	**Milgram**-.In order to do this in any sensible way, one has to limit the types of lengths for the sides of the rectangle. If the lengths are both whole numbers, then this is pretty straightforward, but if, say, one of them is PI, and the other is the square root of 2, then such a partition is extremely difficult, and far beyond the capacity of even the strongest second grade students. **Wurman**-Actually, properly the original language should be modified to partition a rectangle into ... "same-sized rectangles" to be correct.	"Squares" was replaced with "rectangles" based on Wurman's technical review. We appreciate Milgram's technical review but feel this level of precision is not an expectation for 2nd grade students.	Partition a rectangle into rows and columns of same-size ~~squares~~**rectangles** and count to find the total number of ~~squares~~ **rectangles**.
2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	Again, doable with repeated practice.	**Milgram-**See my comments on 2.G.A.2 above.	No revision is necessary. We appreciate Milgram's technical review but feel this level of precision is not an expectation for 2nd grade students.	

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SMP	**Standards for Mathematical Practice**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.		
2.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

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2.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

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2.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others,				

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2.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
2.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				

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2.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
2.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				

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2.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

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Operations & Algebraic Thinking (OA) *Note: Grade 3 expectations in this domain are limited to multiplication through 10 x 10 and division with both quotients and divisors less than or equal to 10.*			**Carlson**-This set of standards is clear and coherent with a solid and meaningful progression of ideas across grade levels. **Abercrombie-**The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. **Milner**-This domain would be strengthened by the introduction of the concept of a "unit" or "neutral element" in a binary operation. That allows defining "inverses" and thus understanding subtraction as addition of the additive inverse ("opposite") and division as multiplication by the multiplicative inverse ("reciprocal"). **Wurman**-Add "whole-number" to become "Grade 3 expectations in this domain are limited to whole-number multiplication …" Incidentally, here "through" is appropriate as it refers to the operands only.	Based on Wurman's feedback, whole number was added before multiplication and division to provide clarification. inverse explicitly stated in 3.OA.B	Operations & Algebraic Thinking (OA) *Note: Grade 3 expectations in this domain are limited to* ***whole number*** *multiplication through 10 x 10 and* ***whole number*** *division with both quotients and divisors less than or equal to 10.*	
			10	In response to Pope: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.		
3.OA.A	**Represent and solve problems involving multiplication and division.**		**Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, it is slightly confusing as someone reading the standards that the clusters aren't necessarily related from one grade level to the next. For example, 1.OA.C is "Add and subtract fluently through 10" and 2.OA.C is "Work with groups of objects to gain foundations for multiplication" and 3.OA.C is "Multiply and divide through 100". While all of these standards relate to arithmetic skills there is no consistent or common thread among skills addressed at each grade level in this cluster (OA.C). This is especially confusing given the way the ELA standards are structured with Anchor Standards. It's possible that some practitioners would assume or expect the math standards to follow a similar structure.	Based on Wurman's feedback, whole number was added before multiplication and division to provide clarification. In response to Pope's comment: When we think about measuring understanding at the classroom level with revised Blooms - can students explain ideas or concepts, this happens naturally, formatively and summatively throughout learning.	Represent and solve problems involving **whole number** multiplication and division.	
3.OA.A.1	Interpret products of whole numbers as the total number of objects in equal groups. Describe a context in which multiplication can be used to find a total number of objects. (See Table 2)	I like that the example is removed and it is more concise. **Too prescriptive as written with "how to's" especially with Table 2 included.	**Milgram-**BE VERY CAREFUL HERE. The "group" representation of multiplication is non-symmetric: 5 groups of 4 elements each are DIFFERENT than 4 groups of 5 elements each. So when talking about interpreting multiplication in these terms, need to specify which of 5 AND 7 IN THE EXPRESSION 5 x 7 is the size of the group and which is the number of groups. HENCE THE REASON FOR THE EXAMPLE. Put it back! **Achieve**-AZ used the CCSS example as part of the requirement.	Based on Milgram and Achieve's feedback, the example was restored.	Interpret products of whole numbers as the total number of objects in equal groups ~~Describe a context in which multiplication can be used to find a total number of objects.~~ **(e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each).** ~~(See Table 2)~~	
3.OA.A.2	Interpret quotients of whole numbers by: •determining the number of objects in each share when a total number of objects are partitioned into a given number of equal shares. • determining the number of shares when the total number of objects and the size of each share is given. Describe a context in which division can be used to find the numbers of objects in each share or the number of shares. (See Table 2)	Written much more clearly, so the examples are not needed. **"-Should capitalize the D's at the beginning of each bullet. -Second bullet has an extra space at the beginning." **The bullet points help clarify. **Excellent changes **Too prescriptive as written with "how to's" especially with Table 2 included.	**Milgram-**See my comments on 3.0A.A.2. The lack of symmetry needs to be dealt with. **Achieve**-AZ does not specify if the quotients are also whole numbers. AZ does not specify that the quotients are also whole numbers. CCSS examples are included as part of the AZ requirement. In rewriting the standard, AZ did not make it clear whether the final sentence is intended to be a bullet, is part of the stem statement for the standard, or is meant to be an example. Clarity is needed. **Wurman-**•determining the number of objects in each share when a given number of objects is partitioned into a given number of equal shares. •determining the number of shares when the total number of objects and the size of each share are given. "Describe contexts" rather than "describe a context." The original was an example, not the whole domain.	In response to Milgram, Achieve, and Wurman's feedback, the examples were restored to provide consistency with 3.OA.A.1 as well as clarity within the standard. As much as we appreciate Mr. Wurman's rephrasing of the bullets, the example was retored with the same end in mind.	Interpret **whole number** quotients of whole numbers (See Table 2 - e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each). ~~•determining the number of objects in each share when a total number of objects are partitioned into a given number of equal shares.~~ ~~• determining the number of shares when the total number of objects and the size of each share is given.~~ ~~Describe a context in which division can be used to find the numbers of objects in each share or the number of shares.~~	
3.OA.A.3	Use multiplication and division to solve word problems in situations involving equal groups, arrays, and measurement quantities (See Table 2).	To prescriptive as written with "how to's" especially with Table 2 included.	**Achieve**-AZ removed the limitation and the CCSS example, possibly leaving the specificity for this standard open to interpretation. However, since the domain explanation includes a limitation of multiplication and division through 10x10, this limitation is a match.	Table 2 is not a "how" but rather an awareness of the different problem types that all childrens should be exposed to. Added limit of within 100 to provide clarification.	Use multiplication and division **within 100** to solve word problems in situations involving equal groups, arrays, and measurement quantities. *See Table 2.*	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
3.OA.A.4	Determine the unknown whole number in a multiplication or division equation using properties of operations and/or the relationship between multiplication and division.		**Achieve**-It is not clear in the AZ version that the equations relate three whole numbers. AZ removed the example, which might have accomplished that. They add the requirement to use the properties (already required in 3.OA.B.5) and relationship between multiplication and division. **Wurman**-Here , again, the proposed new language prohibits early use of strategies other than properties of operations and relationship between multiplication and division. In other words, using any type of arrays, written algorithms -- whether standard or not -- or recall of the multiplication table are explicitly forbidden. This is wrong-headed, it imposes a preferred pedagogy, and will not promote fluency in higher grades. The original language was more flexible in that it allowed for teacher choices. in effect, it now duplicates 3.OA.B.5 and can be simply eliminated. Revert to the original!	In response to Achieve and Wurman's feedback, this has standard has been restored.	Determine the unknown whole number in a multiplication or division equation ~~using properties of operations and/or the relationship between multiplication and division.~~ relating three whole numbers (e.g., 8 x o = 48, 5 = o ÷ 3, 6 x 6 = o). *See Table 2.*	
3.OA.B	**Understand properties of multiplication and the relationship between multiplication and division.**					
3.OA.B.5	Apply properties of operations as strategies to multiply and divide. This includes use of known facts to solve unknown facts through the application of the commutative, associative, and distributive properties of multiplication. (Students do not need to use the formal terms for these properties.)	Yes, students should use the formal terms. These are not complicated words and we want students to use correct terminology. Also, what about zero property and identity property. Those aren't hard concept and many students enter higher graders where those terms are used and don't know what they mean. Will these 2 terms be introduced in other grades? "Students do not need to use the formal terms for these properties" is excellent. This puts the focus on noticing and applying different patterns/conjectures. The formal name of the properties is not that important and can come later.	**Achieve**-It is not clear what "use of known facts to solve unknown facts" means. There may be a word missing. **Achieve-**AZ removed the CCSS examples. It is not clear what "use of known facts to solve unknown facts" means. There may be a word missing. **Wurman-**The deleted examples were highly clarifying and their deletion is unfortunate. Further, it duplicates the previous standard (3.OA.A.4) in that one's newly proposed form.	The deleted examples will be included in the glossary (Table 4) in response to Wurman's feedback. In response to Achieve, the phrase "use of known facts to solve unknown facts" has been removed. The focus of the standard in grade 3 is the application of the properties of operations as strategies to solve problems. While the terms can and should be used by the teacher, the focus is not memorization of this terminology.	~~Apply properties of operations as strategies to multiply and divide. This includes use of known facts to solve unknown facts through the application of the commutative, associative, and distributive properties of multiplication.~~ ~~(Students do not need to use the formal terms for these properties.)~~ Apply properites of operations as strategies to multiply and divide. Properties include commutative and associative properties of multiplication and the distributive property. (Students do not need to use the formal terms for these properties.)	
3.OA.B.6	Understand division as an unknown-factor problem. Represent division as a multiplication problem with a missing factor.		**Achieve**-The second part of this AZ standard repeats the first. AZ removed the CCSS example. The new sentence seems to repeat the first. However, it is not completely clear what is meant by, "Represent division as a multiplication problem with a missing factor." **Wurman**-The deleted example was much clearer than the new one. In any case, this standard is already included in 3.OA.B.4, is redundant, and can be safely eliminated. Treating division as an "unknown factor" problem is precisely "using properties of operations and/or the relationship between multiplication and division"	Based on Wurman's and Achieve's feedback, the example was restored and eliminated the need for the second sentence per Achieve's feedback.	Understand division as an unknown-factor problem ~~Represent division as a multiplication problem with a missing factor.~~ (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
3.OA.C	**Multiply and divide through 100.**		**Wurman**-Here again we have the inconsistent and confusing use of "through" -- does it relate to the operand of multiplication, or to the product?	Based on Wurman's feedback, the term is related to the product and therefore needs to be within 100 referring to the set that is inclusive of 100.	Multiply and divide ~~through~~ **within** 100.	
3.OA.C.7	Demonstrate understanding of multiplication and division through 100 (limited through 10 x 10) using strategies such as the relationship between multiplication and division or properties of operations.	Do we only want students to understand them, or be fluent. Those are 2 very different ideas. A student can draw a picture of 6 x 7, but not know the answer from memory. I know 3.OA.C.8 talks about fluency, but maybe we need to make sure we want students to show it (this standard) and know their facts (3.OA.C.8). That should be clear to teachers. I agree with this standard and think it is very appropriate for 3rd grade; however, I want to emphasize the notion that students are just beginning to learn the basics of multiplication and division. they are stull using strategies to solve and understand the concepts. This is important to understand when looking at 3.OA.C.8. **Point D. - There is an inconsistent application of the removal of "how" elements in the standards. Breaking down rectilinear shapes into non-overlapping rectangles is an important strategy to solving these types of problems. While it does stray into the "how," it does not expressly say how to accomplish this. This is a crucial element of this standard. Plus, in later standards, this type of geometric decomposition (polygons into rectangles and triangles) remains in the standards.	**Achieve**-AZ does not specify that the products to be memorized are 1-digit but limits to 10x10 instead. This possibly adds only the product of 10x10 to the CCSS requirement. They also add memorization of quotients related to those multiplication facts. Notes: Changing the coding from that of the CCSS may cause problems for teachers who search nationally for materials aligned to the CCSS's 3.OA.8. Adding "multiplication" to products is redundant and unnecessary. **Wurman**-This seems unnecessary standard as it duplicates 3.OA.A.4 and contributes nothing here -- understanding was already required there, being inherent in using the properties of operations and the inverse nature of multiplication and division.	This new standard was deleted based on Wurman's comments. Standards that assist in developing understanding of multiplication and division to assist with gaining fluency include most of the standards in OA specifically 3.OA.A.4, 3.OA.B.5, 3.OA.B.6. Deletion of this standard also addresses Achieve's concerns with the coding issues as it is no longer an issue and will continue with the further coding issues in this domain. Deleted Standard: ~~Demonstrate understanding of multiplication and division through 100 (limited through 10 x 10) using strategies such as the relationship between multiplication and division or properties of operations.~~ 3.OA.C.7 Fluently multiply and divide through within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	~~Demonstrate understanding of multiplication and division through 100 (limited through 10 x 10) using strategies such as the relationship between multiplication and division or properties of operations.~~	
3.OA.C.8	Fluently multiply and divide through 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	This is too much to ask students to memorize in only one year of instruction. My students struggle just to learn their multiplication facts. Division facts should be done in fourth grade instead. **I don't see the reasoning to separate this from 3.OA.7. The new 3.OA.7 in which students just demonstrate understanding of multiplication/division through 100 should be taking place in 3.OA.1 and 2 and does not need to be stated again. **If you keep the 2016 standards as is for K-2 I recommend moving this to grade 4. Here's why. Third graders are under the gun and stress of MOWR. Third graders must also learn cursive according to the ELA Draft Standards. Third graders must be able to write/spell the top 500 words on a list. And then comes math -- memorize multiplication/division to 100. Ouch! Have mercy somewhere. Recommendation 2: Higher expectations in K-2 math, remove cursive. Thanks. **This standard seems a little high for third grade. I think that third graders should understand multiplication and be able to make models for the facts. However, I think that they should only be required to memorize facts 0-5.	**Milgram-**It might be worth while to make this stronger by requiring that the multiplication facts be learned to AUTOMATICY. This is the expectation in the high achieving countries.	Instruction in the domain of Operations and Algebraic Thinking should begin in August and continue throughout the year in order for fluency to be obtained by the end of the school year. The domain is not a unit, chapter or module but rather an entire year of learning. In regards to Milgram's comment, Arizona's definition of fluency includes efficient, accurate, flexible, and appropriate use of procedures. All of which, if present, would indicate that a student has automatic recall based on understanding.	3.OA.C.7 Fluently multiply and divide ~~through~~ **within** 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
		(cont.)**In 3.OA.C.8 The students are just beginning to learn multiplication and division, and it has not been taught yet in 2nd grade. 2nd graders are taught arrays; however, they are not taught multiplication and division strategies, With that being said, the students are just beginning to learn multiplication and division, so they should no be expected to be also fluent in these areas as well. 4th grade should have the multiplication/fluency expectation after 3rd has learned the strategies. **Students progressively work from K-2 on fluently knowing addition and subtraction facts, yet they have to learn and become fluent in 3rd grade for multiplication and division facts. **Please remove know from memory all division quotients. Division fluency has never been a 3rd grade standard! Our Title 1 students have trouble memorizing the multiplication facts by the end of 3rd grade much less division! More than half do not have their addition and subtraction facts memorized when they leave 2nd grade, so we have to work on those too! Division should stay a 4th grade fluency standard.		(cont.)In response to public comment, the relationship between multiplication and division is emphasized in 3.OA.C.6 & 3.OA.A.4 The coding was changed to c.7 due to the deletion of the previous standard: 3.OA.C.7 Fluently multiply and divide through within 100. By the end of Grade 3, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.		
3.OA.D	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**					
3.OA.D.9	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order-Order of Operations).	You do realize these are many different types of problem solving. A teacher would need to teach each as individual lessons (plural, not one lesson). Please make sure everyone (not just teachers) realize this is complex and should not all be done in one lesson. Excellent clarification **Is this developmentally appropriate to "assess reasonableness and estimation strategies in 3rd grade?" Show research to back this up!	**Milner-** In 3.OA.D.9 "…perform operations in the conventional order when there are no parentheses to specify a particular order-Order of Operations" needs to be rewritten as "…perform operations in the conventional order–Order of Operations–when there are no parentheses to specify a particular order" **Milgram-**This is a confusing amalgam of DIFFERENT standards. Separate them out and limit each as necessary. **Achieve**-Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to the CCSS's 3.OA.8 or 3.OA.9. **Wurman**-The first clause in parentheses ("Limited to problems posed with whole numbers and having whole-number answers") seems unnecessary in view of the Domain Note and should be deleted.	The time designated to teach a standard is not within the scope of the standards. Strategy type is outlined in 3.OA.C.5, 3.OA.C.7, 3.NBT.A.2 Milner's wording was utilized to revise the order of operations wording for clarity. Wurman's feedback was utilized in removing the redundancy with whole numbers since it is stated in the domain note. Milgram's feedback was validated and an additional standard is added to address using mental computation and estimation strategies including rounding.	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. ~~Assess the reasonableness of answers using mental computation and estimation strategies including rounding.~~ (~~Limited to problems posed with whole numbers and having whole-number answers;~~ ~~students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order-Order of Operations~~ **Utilize understanding of the Order of Operations when there are no parentheses.**	
3.OA.D.10	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.		**Milner**-3.OA.D.10 needs the example that was deleted **Milgram**-.I have no idea what "arithmetic patterns" might be. It is not in the glossary, and the glossary definition of pattern does not help. As best I can determine, using the definition of pattern in the glossary this standard should be revised to read "Identify patterns in the addition table and the multiplication table and explain them using properties of operations." The the example should be put back. PLEASE, PLEASE, PLEASE make these changes. **Achieve**-AZ removed the CCSS example. Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 3.OA.10, since this standard does not exist in the CCSS. **Wurman**-The proposed language makes this into essentially a meaningless standard because it does not illustrate the type of pattern expected. Is a fixed-difference (i.e., arithmetic) series "a pattern"? Is the Fibonacci series an appropriate "pattern" here? The examples are critical in this standard, in that the standard is intended to build towards deeper retention and understanding the structure of the memorized addition and multiplication tables, not just any "arithmetic pattern." If at all, this standard should be illustrated with more examples!	Based on Milner's, Milgram's, and Wurman's feedback, the example was restored. We also used Milgram's wording for the standard to help clarify. The workgroup also makes mention that this is 3rd grade and inappropriate to include the fibonacci series at this level.	3.OA.D.9 ~~Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.~~ Identify patterns in the addition table and the multiplication table and explain them using properties of operations (e.g. observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends).	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
				This is added to address Milgram's previous concerns of rounding truly being an estimation strategy and it will address the need for students to determine reasonableness.	3.OA.D.10 When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
<u>Number and Operations in Base Ten (NBT)</u> *Note: A range of algorithms may be used.*			**Abercrombie**-The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, "Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate. **Wurman**-Indeed, multiple algorithms may be used. Yet teachers should be reminded that the eventual goal is for the student to be fluent with the standard algorithms, and hence choose early algorithms to scaffold that final expectation of fluency. For example, either eliminate this unnecessary Note, or expand it along the lines of: Note: A range of algorithms may be used, with the eventual goal of instilling fluency with the standard algorithms. **Pope**-Almost all of the standards in this domain clearly state what students are to know and be able to do. The breadth and depth of the standards in this domain seems reasonably appropriate at each grade level in grades K-3. The concepts related to the base ten number system are so crucial to mathematical fluency and to the type of conceptual understanding discussed in the introduction of the standards. It makes sense to begin by introducing students to ideas such as place value in very concrete ways (as with base ten blocks) to illustrate that ten ones also make "a ten" then teach them how to apply these skills in various mathematical contexts (such as rounding and estimating). The progression of the breadth of application of skills related to base ten as well as the complexity of the tasks students are asked to perform based on principles of the base ten number system follow a logical sequence from one grade level to the next.	A standard algorithm is valuing all students and what they bring to the classroom of Arizona students. Guidance will be given to educators in support documents that focus on Wurman's comments.		
3.NBT.A	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**					
3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.	I would like clarity here. Are we rounding only two digit numbers to the nearest 10? Only 3 digit numbers to the nearest hundred? Should students be able to round 236 to the nearest 10 as well? What about 2236 to the nearest hundred or ten? Again, various people/companies/programs have conflicting views on what this standard is asking for. I've seen programs that say they are linking to this standard ask students to round 4 and 5 digit numbers to the nearest 10 or 100, is this correct?		No revision necessary		
3.NBT.A.2	Demonstrate understanding of addition and subtraction through 1000 using a variety of strategies such as properties of operations and the relationship between addition and subtraction.	It should be made clear that these strategies do not include the standard algorithm. **I like that the fluency piece was taken out. Making it a separate standard helps with scaffolding. **"Why was algorithms removed? This was the progression. 2nd grade used strategies, 3rd used strategies and algorithms (plural), 4th grade used the standard algorithm Also, what happened to ""based on place value""? this is critical" **This was so, so much better in the previous standards! Please, please put the notion of "place-value-based strategies" back in!	**Achieve-**These two AZ standards show separation between calculations to 1000 and fluency. At this grade level, AZ only requires fluency with addition and subtraction through 100 and understanding to 1000. AZ replaced "within" with "through" to imply a closed interval, possibly causing specificity issues. **Wurman**-The splitting of the original standard serves no purpose other that silently eliminate algorithms as an expected method of solution in this grade under the guise of creating another "understanding" standards that grade 2 is already full of. Is "understanding" of addition and subtraction to 1,000 any different from "understanding" addition and subtraction to 100 grade 2 (2.NBT.B.5-9)? This fluency standard has been turned on its head!	Based on Technical Review and public comment, this standard includes algorithms and restored as per Wurman's feedback.	~~Demonstrate understanding of addition and subtraction through 1000 using a variety of strategies such as properties of operations and the relationship between addition and subtraction.~~ Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
	Fluently add two addends with a sum up to and including 100. Fluently find the difference between two numbers less than and including 100.	Take this out and put it back in second grade where is belong. Learning these many fact (ALL addition, ALL subtraction, ALL multiplication, and ALL division) is TOO MUCH for a third grader. They have to have mastery of addition and subtraction before multiplication and division. It takes a FULL YEAR to for students to master multiplication and division. I can use half the year on addition and subtraction. I like that the fluency piece was taken out. Making it a separate standard helps with scaffolding. **Adding this standard clarifies fluency requirement for third grade students. **This sounds like standard algorithm all the way around. I don't think that's your intention is it? This does not flow well with the 4th-grade standard for using the standard algorithm. This standard is definitely deficient.	**Milner-**3.NBT.A.3 should require fluency to 1000 not to 100. **Achieve-**This is a requirement in Grade 2 of the CCSS (2.NBT.5). Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to the CCSS's 3.NBT.3. **Wurman**-Restore the original limit of 1,000 with, or without, the "using strategies and algorithms ..."	Based on Technical Review and public comment, this is deleted. ~~Fluently add two addends with a sum up to and including 100.~~ ~~Fluently find the difference between two numbers less than and including 100.~~	DELETE	
3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations.		**Milgram**-Put the example back. **Achieve**-Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 3.NBT.4, since this standard does not exist in the CCSS. **Wurman**-Removal of the example is uncalled for, but in this case it makes little difference as the standard is pretty clear.	Coding has been addressed with previous deletion and based on Technical Review of Milgram and Wurman, the example was restored.	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations (e.g., 9 x 80, 5 x 60).	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
Number and Operations-Fractions (NF) *Note: Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.*			**Abercrombie-**The standards are measurable, clear, and contain breadth and depth of the content. The developmental progression is clear and apparent across grade levels. The clarification of the link between the standards and real world problem solving is an improvement.			
3.NF.A	**Understand fractions as numbers.**					
3.NF.A.1	Understand a unit fraction (1/b) as the quantity formed by one part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts 1/b.	Sorry still confusing and not sure what needs to be done. Thank you for adding the term unit fraction.	**Milgram-**.I would strongly suggest this standard be deleted. It is mathematically incorrect and leads to horrible misunderstandings. **Achieve-**Removing "of size" may lead to misunderstanding the quantitative reasoning used in the CCSS, and therefore, some - if not all - of the need for recognizing fractions as numbers, with the denominator used to indicate the size of the part. The expression "a parts 1/b" is not clear. **Wurman**-Pedagogically the change to "unit fraction" makes little sense and is probably counter-productive at this point. The purpose of this standard is to define a fraction. Unit fraction is a special kind of fraction that will be dealt with later, but changing an initial general definition into a specific definition at this point seems unnecessary and ill-advised. The parentheses around 1/b may be helpful but in this case insert them also around (a), (b), (a/b) and (1/b).	Based on Technical review it is understood that a unit fraction is a special type of fraction. The understanding of "fraction" needs to precede the understanding of "unit fraction" This standard was restored for clarity purposes based on technical review.	Understand a ~~unit~~ fraction (1/*b*) as the quantity formed by one part when a whole is partitioned into *b* equal parts; understand a fraction a/*b* as the quantity formed by *a* parts of size 1/*b*.	
3.NF.A.2	Understand a fraction as a number on a number line; represent fractions on a number line diagram. a. Represent a unit fraction (1/*b*) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it from 0 into b equal parts. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths of unit fractions 1/*b* from 0. Understand that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line including values greater than 1.	What fractions are we talking about? Simple, mixed, improper? This is not clear in this standard and could be interrupted many ways. You don't want a teacher only doing simple and another doing mixed. Third graders should only be doing simple and that needs to be clear to all. How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure.	**Milner-**In regards to Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. The deletion in 3.NF.A.2 is ill-guided. As proposed it never defines the location of 1/b on the number line. **Milgram**-This entire standard is pedagogy. It is not a standard, and serious thought is needed to see how to handle it. **Achieve**-3.NF.2.aPartitioning "from zero" does not make sense. **Achieve**-3.NF.2b-The "including values…" reads as if the number line should include values greater than 1. In its current form, it is grammatically awkward and mathematically unnecessary. If the intent is for a/b to include values greater than 1, It might be that a comma is needed after "number line." However, it would be more clear to clearly state, "including values for a/b that are greater than 1," or "including values where a > b." **Wurman-**The important part of (a.) saying that "each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line." has mysteriously disappeared from the proposed language. Restore. "unit fraction" -- see comment to the previous standard. Being overly pedantic at this point seems counter-productive.	The scope of grade 3 fraction work is not limited to values within one. See part B of the standard.	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a ~~unit~~ fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it ~~from 0~~ into b equal parts. Understand that each part has size 1/*b* and that the end point of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths ~~of unit fractions~~ 1/*b* from 0. Understand that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line including values greater than 1. c. Understand a fraction 1/*b* as a special type of fraction can be referred to as a unit fraction (e.g. 1/2, 1/4).	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they represent the same size part of the whole, or the same point on a number line. b. Understand and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and understand fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.	Again, what fractions are we talking about, simple, mixed, improper? Again, only simple fractions should be taught at this grade level and that needs to be told in the standard. **There are conflicting viewpoints on if this standard means students at this grade should understand fractions greater than 1 whole that are not entire whole numbers, such as 5/2. 3.NF.3c leads me to believe it should only be greater than one whole if it involves fractions equivalent to whole numbers, but should they also be able to recognize, name, represent, and find equivalent fractions for fractions like 5/2, 9/6, etc.? Some clarity would be nice. How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure. **Ever since equivalency was added to 3rd grade with Common Core, 3rd graders have struggled with this! They can understand "a" using pictures or the same point on a number line, but developmentally they cannot generate simple equivalent fractions on their own with understanding. We would have to teach other algorithms that are not included in our standards in order for them to do this, but they do not have any conceptual understanding of what they are doing. Please remove "b" for 3rd grade!	**Carlson** -3.NF.A.3: Part (a) “Understand two fractions as equivalent if they represent the same size part of the whole, or the same point on a number line.” There are numerous ways to interpret fractions, and “part to whole” is only one way (so you are specifically emphasizing one interpretation), and in many research studies is a way of thinking that leads to students thinking that fractions have a value less than one (the part is smaller than the whole). I suggest rewording this standard, perhaps something like “Understand two fractions as equivalent if they have the same relative size compared to 1 whole.” Essentially the wording should support interpretations that foster flexibility in applying the reasoning to numbers less than 1 and greater than 1. **Milner**-In 3.NF.A.3c it is important to “recognize” fractions that are whole numbers as well as understanding them. **Achieve**-3.NF.A.3c The AZ decision to replace the verb "recognize" with "understand" in the AZ standards sometimes causes reduced clarity and sometimes increases the rigor. In this case it is less correct/clear to say, "understand fractions that are equivalent to whole numbers." Here using "recognize" is more accurate, more measurable, and more clear. **Achieve**-AZ changed "recognize" to "understand," possibly increasing rigor, and removed the CCSS example. AZ changed recognition to understanding and removed the example. The apparent AZ decision to replace the verb "recognize" with "understand" in the AZ standards sometimes causes reduced clarity and sometimes increases the rigor. In this case it is less correct/clear to say, "understand fractions that are equivalent to whole numbers." Here, using "recognize" is more accurate, measurable, and clear. **Wurman**--- (a) What exactly is "the same size part of the whole"? Sounds like gibberish. (b) "Understand" is rather meaningless when it comes to recognizing fractions. Unless there is some Zen in certain fractions that needs to be understood. The removal of examples significantly impacts the clarity of expectations.	Carlson's note has validity, please pay particular attention to her feedback. Revised language using Carlson's exact verbage. The scope of grade 3 fraction work is not limited to values within one. The exception is in Part B of the standard which specifies that students generate simple equivalent fractions. Retained language of recognize for 3.NF.3b. Explaining shows understanding. Retained language of recognize for 3.NF.3c. per several comments from TR	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent if they ~~represent the same size part of the whole, or the same point on a number line~~ **have the same relative size compared to 1 whole**. b. ~~Understand~~ **Recognize** and generate simple equivalent fractions. Explain why the fractions are equivalent. c. Express whole numbers as fractions, and ~~understand~~ **recognize** fractions that are equivalent to whole numbers. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Understand that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with the symbols >, =, or <, and justify conclusions.	
Measurement and Data (MD)			**Abercrombie-**The standards are written with clarity, are measurable, and have sufficient breadth and depth. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression **Pope**-On the whole the skills represented by the standards in grades K-3 in this domain follow a logical progression from one grade level to the next. However, the content within each of the Clusters is again sort of random when looking at the standards in this domain from one grade level to the next. As an entire concept the progression of the skills related to Measurement and Data is logical but there isn’t any clear connection of the standards in a Cluster between grade levels. As a whole the skills in the domain build upon one another but the skills addressed by individual standards or clusters do not necessarily relate and build upon one another from one grade to the next.			
3.MD.A	**Solve problems involving measurement.**					
3.MD.A.1a	Tell and write time to the nearest minute and solve word problems involving addition and subtraction of time intervals in minutes.	Does this still involve measuring/calculating elapsed time? Or does it only include problems like the movie started at 12:30 and lasted 47 minutes, when did the movie end, etc? **time intervals and word problems seems like a big jump from 2nd grade. **Students must be able to "measure time intervals" in order to add and subtract time intervals so the wording should be included. For example: Jazmyn started her homework on Monday at 3:45 and finished at 4:10. On Tuesday, she started at 4:15 and finished at 4:50. How many minutes did she spend on homework these two days? How do they add the time intervals if they don't know how to measure them? **Where are group notes in the red-line that are referenced? It appears example was just removed?	**Wurman**-The actual measurement of time has silently disappeared from the suggested version. The example of using number line for time calculation is non-obvious and should have been retained	Time intervals and elaped time are synonomous. The word problems are the contextual version of time intervals. Measuring time intervals added back in, per public comment & TR. TR - Actual measurement of time readded. Restored example of using number line for calculation.	Tell and write time to the nearest minute **and measure time intervals in minutes.** ~~and~~ **S**olve word problems involving addition and subtraction of time intervals in minutes **(e.g., representing the problem on a number line diagram).**	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
3.MD.A.1b	Solve word problems involving money through $20.00, using symbols $, ¢, and "." as a separator between dollars and cents.	I do think we should do this in 3rd grade, but students should really be doing most of this in lower grades and reviewing in 3rd grade. Also you can students to use the 'cent' symbol, but that not used anymore in computer programs or on keyboards. Though they can write it out, we should also be encouraging students to use the 'dollar' symbol with zero and a decimal. Ex., $0.52 **I understand the purpose of this standard, but should we expect students to be able to solve word problems, that will inevitably include at least addition/subtraction if not multiplication/division as well, with this many place values? They are only expected to add/subtract within 1000 in the NBT standards. This adds two new place values which complicates it more than just adding/subtracting within 1000. I'm not sure it fits in this grade level when decimals are really introduced in 4th grade. Glad to see that money is spread out through more grade levels than just 2nd **This is an appropriate place for word problems involving money. Defining the parameters "through $20" is an important piece of this standard.**This is good. Before it was only implied that money as a unit should be taught. **This is an excellent clarification and addition to the 3rd grade standards.**Clarifies what specifically third grade students must be able to solve in regard to money. **It seems that this standards should be included in 3.NBT.A.2 rather than in measurement and data.	**Milner**-2.MD.C.8 and 3.MD.A.2 have inconsistent notation for cents. The latter needs better explanation of the decimal point. When we write $12.00 we are using the decimal point but there are no cents. What is probably meant is "1 ¢ = $0.01". **Achieve**-AZ added this standard addressing problems involving money. This is addressed in Grade 2 in the CCSS but without the $20 limit and without the reference to the decimal point. Since students at this grade have not been introduced to decimal numbers, requiring the use of a decimal point in their notation is beyond the reach of students in this grade level. **Wurman**-Good. I would leave the "as a distinction between dollars and cents" out. The standard is clear as it is, and this is poor language to clarify.	Students identify coins and their values in 1.MD.B.4. Students then work with a collection of money using $ and cent signs in 2.MD.C.8 The use of decimals to represent monetary amounts is already included in the standard and may have been misunderstood due to the formatting in the public comments document. Thanks for the positive feedback. While this standard is related to 3.NBT.2, there are not specified contexts in any of the NBT standards - those are outlined in the MD domain across the grade levels, so for consistency, addition & subtraction with monetary values will remani in MD. TR - Removed distinction between dollars and cents, per Wurman, which also addresses Milner's concerns. While Achieve is correct, in that decimal notation is not included in 3rd grade, since decimal notation in monetary values is a social convention, it will stay.	Solve word problems involving money through $20.00, using symbols $, ¢, and "." ~~as a separator between dollars and cents.~~	
				(cont.)Coding swapped with measurement standard to preserve original coding and based on feedback from Achieve. 3.MD.A.1b - Solve word problems involving money through $20.00, using symbols $, ¢, and "." as a separator between dollars and cents. 3.MD.A.2 - Understand and apply capacity and mass of objects. a. Measure and estimate liquid volumes and capacitiy and masses of objects using metric and customary units. (Excludes compound units such as cm3 and finding the geometric volume of a container.) b. Add, subtract, multiply, or divide to solve one-step word problems involving capacity or masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much".) (see Table 2)		

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
3.MD.A.2	Measure and estimate liquid volumes and masses of objects using metric and customary units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Table 2).	Take out the compound, too complex for 3rd. Why are multiplicative comparison problems in this standard? Doesn't make sense. When both customary and metric was included for liquid volume and mass in the old standards it was always confusing for students to switch or understand why there is both systems. Also, I notice that measuring length in 3.MD.B.5 is still only measured in customary units, not metric. So shouldn't we keep it consistant and only include customary for liquid and mass in 3rd grade, and add metric in 4th grade?	**Achieve**-The specific units used to measure liquid volume were removed in AZ. Either a closing parenthesis is missing after "container" or the parenthesis before the first "excludes" should be removed. Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 3.NMD.3. **Wurman-**The original standard dealt with only with metric units of mass and volume. The new one deals with both metric and customary units. Grade 4 (4.MD.A.1) already deals with customary units.	Standard EXCLUDES both compond units and multiplicative comparisons to limit the scope to what is appropriate for grade 3. Ensure formatting of (). Using the term "volume" implied the use of cubic units which was not the intent of the standrd. Changes reflect necessary clarification. 2nd grade introduces students to measuring length in both US Customary & Metric systems, so the progression supports using both systems in 3rd grade. 3. MD. B.5 is limited to inches since the standard connects to fractional notation as well - it would be difficult for 3rd graders to represent 1/4 of a cm. TR - Coding swapped with money standard to align with national standards' coding, per suggestion from Achieve.	**Understand and apply capacity and mass of objects.** **a.** Measure and estimate liquid volumes and capacitiy and masses of objects using metric and customary units. (Excludes compound units such as cm^3 and finding the geometric volume of a container.) **b.** Add, subtract, multiply, or divide to solve one-step word problems involving capacity or masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much".) *See Table 2.*	
				(cont.)Already addressed parenthesis issue. 2nd grade introduces students to measuring length in both US Customary & Metric systems, so the progression supports using both systens in 3rd grade. Since 4th grade moves into expressing measuremnets in larger sizes, the introduction to both measurements in 3rd grade lessens their load. 3.MD.A.3 - Solve word problems involving money through $20.00, using symbols $, ¢, and "." as a separator between dollars and cents. 3.MD.A.2 - Understand and apply capacity and mass of objects. a. Measure and estimate liquid volumes and capacitiy and masses of objects using metric and customary units. (Excludes compound units such as cm3 and finding the geometric volume of a container.) b. Add, subtract, multiply, or divide to solve one-step word problems involving capacity or masses or volumes that are given in the same units. Excludes multiplicative comparison problems (problems involving notions of "times as much".) (see Table 2)		

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3.MD.B	Represent and interpret data.					
3.MD.B.3	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step problems using information presented in scaled bar graphs (See table 1).	I like the change from draw to create. It allows students to use multi-media to create their graphs.	**Achieve**-AZ removed the CCSS examples.Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 3.MD.4. **Wurman**-The examples were helpful and illustrative, and their deletion reduced standards' clarity. Further, the eliminated words were specific about the type of problems expected by this standard (how many less, how many more). The generic reference to Table 1 is unhelpful in that the table includes some 15 "types" of problems, of which at most 6 are expected by this standard.	TR - coding was adjusted to preserve Retained "how many more" and "how many less" langauge per Wurman.	Create a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step **"how many more" and "how many less"** problems using information presented in scaled bar graphs. *See Table 1.*	
3.MD.B.4	Generate measurement data by measuring lengths to the nearest quarter inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	This removed the parameter of the marks on the ruler students are to use...which means they may be required to use a ruler with markings to the 16th inch.	**Milner**-In 3.MD. B.5, "quarter inch" needs to be hyphenated. **Achieve**-AZ changed the CCSS description of the ruler to a measurement precision requirement that may not be appropriate for this grade level.Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 3.MD.5. **Wurman**-The new language is unclear whether the measurement to quarter of an inch requires a ruler marked down to quarters. For example, a ruler marked in half plus estimating in between may be sufficient.	Per public comment and TR, retained the specification of using rulers marked with halves and quarters.	Generate measurement data by measuring lengths **using rulers marked with halves and fourths of an inch** to the nearest **quarter-inch.** Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	
3.MD.C	Geometric measurement: Understand concepts of area and perimeter.					
3.MD.C.5	Understand area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure.	**Wurman**-Perhaps this is the place to observe that "understand" and "recognize" are both almost equally unmeasurable? The measurable operative words are "do" or "perform" (an their specific castings such as "add," "multiply," etc. If at all, "recognize" is more measurable than "understand" as it is lower on cognitive hierarchy and somewhat easier to measure. But we only directly measure what students DO and then we draw CONCLUSIONS -- whether justified or not -- about what caused students to do what they did.	No action taken		
3.MD.C.6				Although the workgroup feels this is subsumed in other standards within this cluster, it was put back in to preserve coding as requested througout.	**3.MD.C.6 Measure areas by counting unit squares (e.g., square cm, square m, square in, square ft, and improvised units).**	
3.MD.C.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Understand area as additive by finding the areas of rectilinear figures.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure. **Third grade students do not understand "real world" problems nor "mathematical reasoning." Not developmentally appropriate for a 3rd grader!	**Carlson-** 3.MD.C.7: Part (d): "Understand area as additive by finding the areas of rectilinear figures." Since additive and multiplicative reasoning are well-defined concepts in mathematics education, and area calculations are multiplicative comparisons to a unit, the wording of this standard is problematic. My interpretation is that you want students to understand that they can break up plane figures, find the area of each part, and sum these areas to find the area of the original figure. If that is the case, consider rewording this standard to make this clearer as it was in the original standards. Perhaps something like "Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure." This is an understanding and is not really a "how to teach it" directive (you specifically talk about decomposition and composition skills with shapes in the Geometry standards, so this is clearly within the realm of reasoning skills you want students to develop and not a prescriptive teaching method – see 6.G.A for an excellent example of this that seems at odds with your motivation for changing part (d) of this standard). The current wording does not capture the mathematical idea that you intend. **Milner**-In 3.MD.C.7b the last part, "and represent whole-number products as rectangular areas in mathematical reasoning", was removed but it should really stay. It is an important example of "creating" as opposed to "applying". Moreover, it naturally leads to the commutativity of multiplication.	Addressed concerns in technical review with suggested additions/exact wording.	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems**, and represent whole-number products as rectangular areas in mathematical reasoning**. c. Use tiling to show that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. ~~Understand area as additive by finding the areas of rectilinear figures.~~ **Understand that rectilinear figures can be decomposed into non-overlapping rectangles and that the sum of the areas of these rectangles is identical to the area of the original rectilinear figure. Apply this technique to solve problems in real-world contexts.**	

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			Achieve-AZ changed "recognize" to "understand," possibly increasing rigor but also making it less easily measured. The emphasis of the AZ standard is on the concept, area as additive, as opposed to finding the area. In addition by removing the example for how to find area, AZ further distances itself from the computation.			
3.MD.C.8	Understand perimeter as an attribute of plane figures and distinguish between linear and area measures.	How would this be assessed? I am struggling to understand why this can't just be paired together with 3.MD.C.9 in which this standard is the stepping stone to solving problems utilizing real word contexts involving perimeters of polygons. I can see assessing this formatively in the classroom to make sure students understand the differences and purposes of area and perimeter, but that has always been a "given" sub-objective to me that just makes sense and doesn't need to be a standalone standard. **I approve of removing cluster D and separating it into to standards 8 and 9 **This makes it clear that perimeter needs to be taught. It was often overlooked. Before it was buried in with area. **Third grade students do not understand "real world" problems nor "mathematical reasoning." Not developmentally appropriate for a 3rd grader! **Fits nicely under cluster C instead of creating a new cluster.	**Achieve**-This CCSS cluster header was removed in AZ and the CCSS cluster title became a standard. AZ changed recognize to understand, which makes sense in this context. **Wurman**-This new standard is completely inappropriate at this level. Recognizing perimeter as an attribute and understanding the difference in nature of area and perimeter -- the original section title -- is very different from recognizing that perimeter is a linear attribute and distinguishing between it and (non-linear) area measures. The latter is more appropriate for a 6th-7th grade, not a 3rd grade. This is ridiculous! In the context of of a title, "measures" carries a generic notion of a quantity. In the context of an actual standard, "measures" implies understanding the different kind of the measure (linear, square) and how they grow with linear dimension.	3.MD.C8 focuses on understanding the difference between area and perimeter (and can be assessed) while 3.MD.C.9 requires them to solve real world problems related to perimeter (which is also assessable). Revision to 3.MD.C.9 required as real world application was not included in student work with area but was included in student work with perimeter. Thank you for positive feedback on the modifications to the cluster/standards. 3rd graders (and all students) can understand real world problems, and research shows they actually are more proficient with contextual situations than "number-crunching" (CGI). When students have the ability to reason mathematically they are more successful mathematicians. TR - Clarified the focus of the standard	~~Understand perimeter as an attribute of plane figures and distinguish between linear and area measures.~~ Solve real-world and mathematical problems involving perimeters of plane figures and areas of rectangles, including finding the perimeter given the side lengths, finding an unknown side length, and ~~exhibiting~~ represent rectangles with the same perimemter and different areas or with the same area and different perimeters.	
3.MD.C.9	Solve problems utilizing real-world contexts involving perimeters of polygons. (See Table 1- unknown in various positions)		**Milner-**In 3.MD.C.9 "mathematical problems" should not be removed. Moreover, the end of the old standard, "exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters", is very important and should not be removed (Table 1 does not include such problems that are at a higher cognitive level). In the Notes, the word extraneous is misspelled. **Achieve**-AZ removed the CCSS examples of problem types. Pointing to the table is less clear in AZ than in the CCSS.Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 3.MD.9, which does not exist in the CCSS. **Wurman-**- The removal of examples makes this standard so general as to be meaningless - The elimination of the examples with same perimeter and different areas (or vice versa) loses a critical point that the standard attempted to make.	TR - Retained language per unanimous agreement among technical reviewers.-now C.8	~~3.MD.C.9~~ ~~Solve **real world and mathematical** problems utilizing real-world contexts involving perimeters of **plane figures** and areas of rectangles. (See Table 1 unknown in various positions)~~ Solve problems utilizing real-world contexts involving perimeters of polygons and areas of rectangles with unknown in various positions. Understand the distinction between perimeter and area as an attribute of plane figures. and distinguish between linear and area measures.	
Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.			
3.G.A	**Reason with shapes and their attributes.**					
3.G.A.1	Understand that shapes in different categories may share attributes and those shared attributes can define a larger category. Draw examples of shapes that do not belong to any of these subcategories.	I wish we could get even more clarity on what this standard is asking. There are conflicting viewpoints on what categories should be explicitly explored. **The parameters were what is called "clutter"? The original standard focused on quadrilaterals...Now it is open to ALL shapes	**Achieve**-AZ removed the specificity in this CCSS, regarding the types of shapes that are required and the attributes they share. Recognition of the shapes and drawing examples of, specifically, quadrilaterals is not required in this AZ standard. **Wurman-**Without examples, the standard is so general and unclear as to be meaningless.	Examples are not included within the standard but may be included in the support documents. Per public and TR comments, retained language specifying quadrilaterals	Understand that **quadrilaterals** in different categories may share attributes and those shared attributes can define a larger category. Draw examples of **quadrilaterals** that do not belong to any of these subcategories.	
3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction (1/*b*) of the whole. (Grade 3 expectations are limited to fractions with denominators: 2,3,4,6,8.)		**Milner-**3.G.A.2 should read "Partition shapes into b parts with equal areas. Express the area of each part as a unit fraction 1/b of the whole. (Grade 3 expectations are limited to fractions with denominators b = 2,3,4,6,8)."	Clarity of b as representative of denominator, retained per Milner's technical review comment.	Partition shapes into ***b*** parts with equal areas. Express the area of each part as a unit fraction 1/*b* of the whole. (Grade 3 expectations are limited to fractions with denominators ***b*** **=** 2,3,4,6,8.)	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
SMP	**Standards for Mathematical Practices**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.			
3.MP.1.	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.					
3.MP.2.	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.					

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3.MP.3.	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.					
3.MP.4.	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.					
3.MP.5.	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.					

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016	
3.MP.6.	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.					
3.MP.7.	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.					
3.MP.8.	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.					

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
Operations and Algebraic Thinking (OA)			**Carlson**-This set of standards is clear and coherent with a solid and meaningful progression of ideas across grade levels. **Abercrombie**-The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. **Milner**-This domain would be strengthened by the introduction of the concept of a "unit" or "neutral element" in a binary operation. That allows defining "inverses" and thus understanding subtraction as addition of the additive inverse ("opposite") and division as multiplication by the multiplicative inverse ("reciprocal").		
4.OA.A	**Use the four operations with whole numbers to solve problems.**				
4.OA.A.1	Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations. (35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.)	In the Critical Areas section, the Introduction line states there are three critical areas and there are actually four listed.	**Milgram**-This is total nonsense! The correct statement is that 35 is the number of elements in 5 sets, each containing 7 elements, and is also the number of elements in 7 sets, each containing 5 elements. It is frankly scary that nobody in the writing group noticed this.	Based on Milgram's feedback, the recommending wording was used as suggested. Support documents will also contain guidance in multiplicative comparison situations.	Represent verbal statements of multiplicative comparisons as multiplication equations. Interpret a multiplication equation as a comparison ~~(35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.)~~ **(e.g., 35 is the number of elements in 5 sets, each containing 7 elements, and is also the number of elements in 7 sets, each containing 5 elements).**

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4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison by using models and equations with a symbol for the unknown number to represent the problem. (See Table 2.)	Prescriptive examples or "how to's" still remain with Table 2 included.	**Milgram**-I am not familiar with the the terms "multiplicative comparison" and "additive comparison." I would strongly suggest that this standard either be deleted or revised so that it can be understood by a person who understands mathematics, but not necessarily "educationese jargon." **Achieve**-AZ moved this CCSS example so that it appears to be exemplifying distinguishing between multiplicative and additive comparisons rather than how the problems are solved. AZ also implies that every word problem should include distinguishing multiplicative from additive comparison. **Wurman**-As is common whenever an "e.g." in the original is replaced by an actual list of the examples, this here narrows the standard. While the original gave an example of "drawings or equations" but allowed anything else (e.g., bar-charts, graphs) the new wording allows ONLY "models or equation." Luckily "models" can mean anything and everything so little harm was done.	Based on Technical Review, e.g was restored. A limit of within 1000 was provided for clarification purposes.	Multiply or divide **within 1000** to solve word problems involving multiplicative comparison (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison). *See Table 2.* ~~by using models and equations with a symbol for the unknown number to represent the problem.~~
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.	Is this standard developmentally appropriate and research-based to use "mental computation" and to think algebraically? **Because there is no other place to do so, I am commenting on 4.OA.A.3.1 here. This standard was removed, but I did not see it added back in. Is this combinatorics standards removed completely? (I only reviewed up through 6th grade.)	**Milgram**-This is NOT a single standard but an almost indigestable mix of three or more separate standards some of which are very important. This "standard" should be decomposed into its separate parts, and examples should be given. **Wurman**-Actually, the original standard is flawed because it requires that the problems will have whole-number answers, yet at the same breath is allows problems "in which reminders must be interpreted." These reminders must have resulted from division that produced fractional rather than whole number result. This needs to be clarified.	"having whole number answers" removed per Wurman technical review. The last sentence was added in response to Wurman's technical review. "Assess the reasonableness of answers…" was removed as it is found in the new 4.OA.C.6 PC: AZ.4.OA.3.1 was removed completely.	Solve multistep word problems ~~posed with whole numbers and having whole-number answers~~ using the four operations, including problems in which remainders must be interpreted. **Understand how the remainder is a fraction of the divisor.** Represent these problems using equations with a letter standing for the unknown quantity.~~Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.~~
4.OA.B	**Gain familiarity with factors and multiples.**				
4.OA.B.4	Find all factor pairs for a whole number in the range 1–100. Understand that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	This is a standard within a standard. Can it be separated out?	**Wurman**-The original language is flawed in that: - It is the only place primes are even mentioned, so the standard needs a preamble along the lines of "understand that primes have only two factors: 1 and the number itself" - Determination whether a given number between 1 and 100 is prime is not a trivial task, unless the primes are memorized by rote, a foolish task. A good standard would require learning how to decompose numbers into prime factors, allowing a meaningful way to address this standard. If necessary, this could then be moved to grade 5.	Based on Wurman's feedback, his wording was used in the 5th grade standard that addresses prime factors	Find all factor pairs for a whole number in the range 1–100 and understand that a whole number is a multiple of each of its factors. ~~Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.~~

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4.OA.C	**Generate and analyze patterns.**				
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.			Minor wording changes for consistency across grades.	Generate a number pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself and explain the pattern informally (e.g., given the rule “**a**dd 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers). ~~Explain informally why the numbers will continue to alternate in this way.~~
				To provide coherence from 3rd grade this was added.	When solving problems, assess the reasonableness of answers using mental computation and estimation strategies including rounding.
<u>Number and Operations in Base Ten (NBT)</u> *Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*			**Abercrombie-**The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, “Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate.		
4.NBT.A	**Generalize place value understanding for multi-digit whole numbers.**				

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4.NBT.A.1	Apply concepts of place value, multiplication, and division to understand that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.		**Milgram-**This revised standard is, in fact, much better than the original. **Achieve**-How a student would "apply concepts" in order "to understand" is unclear, as is how a teacher would measure the understanding of place value through application of place value and operations. AZ changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured.How a student would "apply concepts" in order "to understand" is unclear, as is how a teacher would measure the understanding of place value through application of place value and operations. **Wurman**-As usual, the new language is so general as to be opaque. The original examples helped, while without them this standards' meaning is a needless brain teaser.	Based on Milgram's feedback, no edits are necessary	
4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.		**Milgram**-.I don't agree. For example, what are "number names?" This is NOT STANDARD in mathematics, though it might be supposed to mean something in educationese. It is my view and that of the math educators in the high achieving countries that the KEY concept here is that of the expanded form.A much better version would be "Understand multi-digit whole numbers as the number given by the expanded form. Compare two multi-digit numbers using their expanded forms."	For consistency across grade levels, support documents will emphasize the importance of standard form in multi digit numbers. No revision.	
4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.	This should be clarified to up to the millions place.	**Milgram**-.I agree, provided the above standard is revised as I suggested.	There is a limit in the domain. No revision necessary	
4.NBT.B	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**				

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4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using a standard algorithm.	This was the progression. 2nd grade used models and strategies, 3rd used strategies and algorithms (plural), 4th grade used the standard algorithm There are many different algorithms...but only one STANDARD algorithm. **Important word change from "the" to "a". **I LOVE the change from "the" to "a." This small change reflects a bigger understanding we are trying to push! **I understand the intention, but this standard is not distinct enough from the third-grade standard which states that students should fluently add and subtract whole numbers. The third-grade standard needs work!	**Milgram**-This is nonsense, and classic educationese. It is based on a complete misunderstanding of what algorithms actually are, and starts a process in which students in this country gradually lose the capacity to do the advanced mathematics that is essential in going into STEM and related areas. **Wurman-**The removal of the "the" is a gross mistake. The national Mathematics Advisory Panel, certainly a much bigger authority than McREL, purposely inserted the "the" into its recommendations to teach "the standard algorithms." While there are many possible algorithms for arithmetic, only a single set is "standard" and it deserves to have the definite article. All around the world people use the four standard (arithmetic) algorithms and the few differences one see across the world are cosmetic, trivial, and non-essential. Pretending that there are multiple standard algorithms for the four arithmetic operations is mathematically ignorant or intentionally misleading.	No revision necessary. A standard algorithm is valuing all students and what they bring to the classroom as recognized by the public comments.	
4.NBT.B.5	Demonstrate understanding of multiplication by multiplying whole numbers up to four digits by a one-digit whole number, and multiply two two-digit numbers, using a variety of strategies such as the properties of operations and the relationship between multiplication and division. Illustrate and explain the calculation.	"...using a variety of strategies..." directs instructional technique, the "how." Should be limited to "Demonstrate understanding of multiplication by ... two two-digit numbers. Illustrate and explain the calculation." **I believe that students need to be flexible when working with numbers and teaching them a variety of strategies will increase the pass rate of students because they do not have to be only required to learn one way. **"by using equations, rectangular...." needs to be included in a supporting document if removed from here	**Achieve**-The product of two 2-dgit numbers is not specifically required in AZ. AZ changed the intent of this CCSS by asking for multiplication of whole numbers of up to four digit by one digit as a way of demonstrating understanding of the operation. The CCSS asks for the calculations and an explanation of the solution. **Wurman**-The original standard is very specific in the expected strategies: only those based on place-value and properties of operations are acceptable. Others may be used to illustrate and explain, but are not the expected ones to actually do the multiplication. The proposed language makes those two just examples and allows other undefined "variety of strategies." This is a major defocusing of the original standard that was focused on doing the multiplication, with explanations if and when needed while building fluency. In contrast, the new standard focuses on understanding and explaining multiplication by any possible means, which belongs to a lower grade.	Based on technical review, edits were made. The strategies show demonstrate understanding.	~~Demonstrate understanding of multiplication by multiplying~~ Multiply a whole number of up to four digits by a one-digit whole number, and multiply (2) two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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4.NBT.B.6	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation.	Eliminate "using strategies based on ...and division." This directs instructional technique, beyond the "what' of the standard. **"by using equations, rectangular...." needs to be included in a supporting document if removed from here	Milgram-.I strongly recommend DELETING the phrase that I've colored blue above. This phrase determines PEDAGOGY, not understanding of actual mathematics, and, comparing with what is actually done in the schools in the high achieving countries, it is very poor pedagogy at that. (using a variety of strategies such as the properties of operations and the relationship between multiplication and division) **Achieve**-AZ changed the intent of this CCSS by asking for division of whole numbers of up to four digit by one digit as a way to "demonstrate understanding" of the operation. Both versions expect students to be able to illustrate and explain their calculation, making the "demonstrate understanding" a double requirement in the AZ version but without the basic requirement to do the division problems **Wurman**-Like in the previous standard, there is a clear shift of focus from doing the division in the original language to understanding it in the new language. As an aside, there is little logic in requiring to "illustrate and explain" when the standard expects demonstrating "understanding" -- there is no difference between the two.	Based on feedback from technical review, the strategies were removed as it is redundant to demonstrating understanding.	Demonstrate understanding of division by finding whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. ~~using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation.~~
Number and Operations - Fractions (NF) *Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.*			**Abercrombie**-The standards are measurable, clear, and contain breadth and depth of the content. The developmental progression is clear and apparent across grade levels. The clarification of the link between the standards and real world problem solving is an improvement.		
4.NF.A	**Extend understanding of fraction equivalence and ordering.**				
4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to understand and generate equivalent fractions.	Thank you for adding parameters (number range and type) for the fractions. **"They need to be able to recognize which fractions are equivalent. How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure."	**Milgram**-Before one can do this in any sensible way students have to know what is meant by "a fraction." **Achieve**-AZ changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured. **Wurman**-Actually, in those context changing "recognize" to "understand" changes the standard. The original asks students to recognize that say, 1/3 is the same as 6/18. Understanding the principle does not necessarily lead to quick recognition that is necessary to build fluency to operate on, and simplify, more complex fractions.	Understand is used here because within the standard it asks students to "explain why" fractions are equivalent. This requires understanding rather than recognition. No revision necessary	

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4.NF.A.2	Compare two fractions with different numerators and different denominators by creating common denominators or numerators, and by comparing to a benchmark fraction such as 1/2. Use number sense of fractions to estimate mentally and assess the reasonableness of answers. Understand that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure.	**Carlson**-4.NF.A.2 – This standard is fairly dense (and seems to contain multiple ideas that could be assessed independently). Consider writing it with subparts (a), (b), etc. **Achieve**-AZ's deletion of "e.g." in the first part of the standard makes it seem that this example is the only method required. They also changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured.AZ added a requirement to assess the reasonableness of results **Wurman**-Both standards are deeply incorrect in that they require the understanding that "comparisons are valid only when the two fractions refer to the same whole." Yet this standard treats fractions as numbers rather than parts of some "whole," not that different from whole numbers. Do we require students to understand that 4 is greater than 3 "only when the two [numbers] refer to the same whole"? So why we insist on this for 4/1 and 3/1? We are talking fractions here, not pie slices! "Use number sense of fractions to asses the reasonableness of answers" is a rather meaningless exhortation without specifics.	Per Carlson's feedback, the standard was broken into subparts. Per Achieve's feedback, the "e.g." was added. Per Wurman's feedback, the "number sense" sentence was removed.	Compare two fractions with different numerators and different denominators **(e.g.,** by creating common denominators or numerators and by comparing to a benchmark fraction). ~~such as 1/2~~. ~~Use number sense of fractions to estimate mentally and assess the reasonableness of answers.~~ **a.** Understand that comparisons are valid only when the two fractions refer to the same sized whole. **b.** Record the results of comparisons with symbols >, =, or <, and justify the conclusions.
4.NF.B	**Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.**				**Apply and extend previous understandings of multiplication to multiply a whole number by a fraction.** **~~Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.~~**

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4.NF.B.3	Understand a fraction a/b with a > 1 as a sum of unit fractions (1/b). a. Decompose a fraction into a sum of fractions with the same denominator by recording decompositions using a variety of representations, including equations. Justify decompositions. b. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators using a variety of representations . c. Add and subtract mixed numbers with like denominators by using properties of operations and the relationship between addition and subtraction or by replacing each mixed number with an equivalent fraction.	examples need to be included in a supporting document if removed from here	**Milgram-**a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.What do these words mean? I really can't figure out what the authors are trying to say. In fact the only things that I think they can mean are incorrect. The first phrase, "Decompose a fraction into a sum of fractions with the same denominator" is entirely reasonable as a standard, the rest of (b) SHOULD BE DELETED. c. Add and subtract mixed numbers with like denominatorsby using properties of operations and the relationship between addition and subtraction or by replacing each mixed number with an equivalent fraction.The material in blue should be deleted.d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators using a variety of representations.Again, the material in blue should be deleted. **Achieve**-AZ removed the "e.g." making it seem that these methods are the only ones required.It appears that AZ attempted to include a generic version of the CCSS example in the standard. However, it is not clear what is meant by "using a variety of representations" in the context of word problems involving addition and subtraction of fractions. This should be specified. **Wurman**- In this case, decomposing fractions (b) in the original specifically requested the decomposition to be done via equations. In the rewrite, anything goes. The examples were used precisely to limit the standard, but they have been spuriously removed and justified by the fact they "do not provide limits ... to the standards"	Per Milgram's feedback the remainder of subcategory for b was removed. Per Achieve's feedback "using a variety of representations" was removed. Working group determined properties of operations is critical in understanding addition and subtraction of mixed numbers and marked them as e.g. for clarification.	Understand a fraction *a/b* with *a* > 1 as a sum of unit fractions (1/*b*). a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator **in more than one way** ~~by recording decompositions using a variety of representations, including equations. Justify decompositions.~~ **(e.g., 3/8 = 1/8 + 1/8+1/8; 3/8 = 2/8 + 1/8; 2 1/8 = 1 + 1 + 1/8 + or 2 1/8 = 8/8 + 8/8 + 1/8.** c. Add and subtract mixed numbers with like denominators **(e.g.** by using properties of operations and the relationship between addition and subtraction and/or by replacing each mixed number with an equivalent fraction.) d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators using ~~a variety of representations.~~ **more than one way.**

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4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of a unit fraction (1/b). (In general, a/b = a x (1/b).) b. Understand a multiple of a/b as a multiple of a unit fraction (1/b), and use this understanding to multiply a fraction by a whole number. (In general, n x (a/b)=(n x a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number.		**Milgram-**Reasonable standard! **Achieve-**Here, the practice of making the CCSS example part of the AZ standard (e.g. 4.NF.3d) was not followed. Is there a reason for including the example strategy as part of the statement in one place and not the other? **Wurman-**The examples did provide clarification of the standard. Without some examples the standard is much less clear and more difficult to read.	Based on Worman's comment, the examples was included in part c. Based on technical review comments, additional examples will be included in support documents.	~~Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.~~ **~~whole number by a fraction~~** **Build fracitons from unit fractions.** a. Understand a fraction *a/b* as a multiple of a unit fraction (1/*b*). (In general, *a/b* = *a* x (1/*b*).) b. Understand a multiple of *a/b* as a multiple of a unit fraction (1/b), and use this understanding to multiply a **whole number by a fraction**. ~~fraction by a whole number~~. (In general, *n* x (*a/b*)=(*n* x *a*)/*b* .) c. Solve word problems involving multiplication of a **whole number by a fraction**. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* ~~fraction by a whole number.~~
4.NF.C	**Understand decimal notation for fractions, and compare decimal fractions.**				
4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). (Addition and subtraction with unlike denominators, in general, is not a requirement at this grade.)		**Milgram**-Reasonable standard! **Wurman**-Yet again, the eliminated examples were helpful and made the standard more accessible.	No revision is necessary.	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 (tenths) and 100 (hundredths). (Addition and subtraction with unlike denominators, ~~in general,~~ is not a requirement at this grade.)
4.NF.C.6	Use decimal notation for fractions with denominators 10 (tenths) or 100 (hundredths) and locate these decimals on a number line.		**Milgram**-Reasonable standard! **Wurman**-The original standard called for conversion of common fractions to decimals and vice versa. This was clear from the example. Now that the example was removed, the modified standard expects only common to decimal conversion.	No revision is necessary.	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
4.NF.C.7	Compare two decimals with tenths and hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure. **How is the reasonableness of answers determined?	**Milner**-In 4.NF.C.7 the concept of decimal fractions is used but has not been introduced. Also, “decimals with tenths and hundredths” is not what is intended since they may be lacking one or the other. The 2010 wording “decimals to hundredths” is better, albeit not best. **Milgram**-What a confused mess. The second sentence is nonsense as written, since the first sentence talks only about comparing two fractions, but these are NUMBERS and we know how to compare them! There is nothing there that involves “the same whole!” My recommendation is that this horrible mess be entirely removed. **Achieve**-AZ changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured. They also added the requirement to assess the reasonableness of answers. **Wurman**-Same comment as before: Decimal fractions are treated here as numbers. We don't qualify that 4>3 "only when two decimals refer to the same whole," so why do we need to make this qualification for 4.0>3.0 or even for 0.4>0.3? Both original and rewritten standards are incorrect.	Used Milner's wording based on his feedback.	Compare two decimals to ~~with tenths and~~ hundredths by reasoning about their size. Understand that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <.
Measurement and Data (MD)			**Abercrombie**-The standards are written with clarity, are measurable, and have sufficient breadth and depth. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression		
4.MD.A	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**				

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard-12/2016
4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.		**Milner**-In 4.MD.A.1 the Notes have the word "involved" misspelled. **Milgram-**Why not also ask students to express smaller units in terms of larger ones? After all, students are supposed to know something about fractions is fourth grade, and this particular exercise is a very good application and even justification of fractions. **Wurman**-Yet again the rewrite doesn't understand the power of examples and, in the rush to get rid of them, discarded an important part of the standard: "Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ..." This part serves to train students in fluently converting between these two very important everyday conversions and demonstrates an important linear relationship pattern.	Based on Milgrams comments, edits were made. Based on Wurman's comments, example was included in standard.	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit **and in a smaller unit in terms of a larger unit**. ~~Record measurement equivalents in a two-column table.~~***For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), 2,24), (3,36).***
4.MD.A.2	Solve word problems in real-world contexts involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations that feature a measurement scale.		**Achieve**-AZ is less specific than the CCSS in their change form "diagrams such as number line diagrams" to the less specific, "a variety of representations." It is not clear what the "variety" would include. **Wurman**-OK for the changes and additions, except: - The original language steered representations towards the number line, while the modified one leaves it wide open. In general, the original standards attempted to use the number line as much as possible in their quest for uniform representation of numbers. The new language ignores this preference. - Additionally, the original did not insist only on real-world problem, while the new language unwisely does so.	Based on Achieves comments, examples will be included in support documents.	**Use the four opeartions to** solve word problems **and problems in** real-world context involving distances, intervals of time (hr, min, sec), liquid volumes, masses of objects, and money, including decimals and problems involving fractions with like denominators, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using a variety of representations, **including number lines,** that feature a measurement scale.
4.MD.A.3	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world context including problems with unknown side lengths.		**Milner-**In 4.MD.A.3 the end of the proposed standard, "including problems with unknown side lengths", should rather specify "see Tables 1 and 2" for consistency with other standards. **Milgram-**Where did this come from? It needs considerable preparation, and, typically, there is very little discussion of area and perimeter before fourth grade. **Achieve-**AZ removed the CCSS example problem and added a more generic type of problem to the standard. However, it is not clear whether other types of problems would be required. Would unknown areas or unknown perimeters be included? Perhaps in this case, generically blending the CCSS example into the standard may make the AZ standard less clear. **Wurman-**The original was much crisper and clearer, even if one removes the example.	Per Milner's feedback, we added reference to "Tables 1 and 2." Per Worman's comment original wording was added back in. Examples will be included in support documents.	Apply the area and perimeter formulas for rectangles in mathematical problems and problems in real-world contexts including problems with unknown side lengths. ***See Tables 1 and 2.***

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4.MD.B	**Represent and interpret data.**				
4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.		**Achieve**-AZ deleted the defining statement for angle measurement. Without that statement the next sentence, about a commonly misunderstood concept, is less clear. The technical notes indicate that the statement was removed because it was, "all the how... and not appropriate for wording in standards." The deleted statement, however, is not about "how" but is rather a key part of the understanding of what one should attend to when measuring an angle. **Wurman**-The examples were clear and illustrative and the clarity of the proposed wording suffers by their removal.	No revision necessary.	
4.MD.C	**Geometric measurement: understand concepts of angle and measure angles.**				
4.MD.C.5	Understand angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure.	**Milgram**-.In practice angles are never constructed using rays, since they go on forever (in one direction!). Instead, it would be much better to replace rays by line segments. **Achieve**-AZ deleted the defining statement for angle measurement. Without that statement the next sentence, about a commonly misunderstood concept, is less clear. The technical notes indicate that the statement was removed because it was "all the how.. and not appropriate for wording in standards." The deleted statement, however, is not about "how" but is rather a key part of the understanding of what one should attend to when measuring an angle. **Wurman**-Calling the use of the fraction of circular arc the "how" is mathematically incoherent. Students at this point are familiar with lengths but not with angles. Writing "An angle that turns through 1/360 of a circle is called a “one-degree angle” is an empty circular definition, and that is why arc fragment is needed. Further, both versions imply that an angle must have only integer values. This should be clarified, e.g., along the lines of adding "n does not need to be a whole number."	Per Achieve and Wurman's feedback, the original wording was restored.	~~Understand~~ **Recognize** angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, **by considering the fraction of the circular arc between the points where the two rays intersect the circle.** An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
4.MD.C.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.		**Milgram**-.It would probably be much better for students to understand that physical measurements are virtually never precise, always having small errors, and somehow understand that the protractor measurements will always have small errors and just be approximations.	No revision is necessary.	

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4.MD.C.7	Understand angle measures as additive. Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real world contexts.		**Milgram**-This must have examples to limit it. As written it is far too vague for fourth grade. **Achieve-**AZ changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured. They also removed the example and deleted the explanation of additive for angles. In making "measure" plural, AZ appears to be thinking of the individual measurements rather than the concept. **Wurman**-While the original standard was reasonably clear, the proposed change makes it read like a gobbledygook.	Per Milgram, Achieve and Wurman's feedback, some of the original wording was restored.	Understand angle measures as additive. **(When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.)** Solve addition and subtraction problems to find unknown angles on a diagram within mathematical problems as well as problems in real-world contexts.
Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.		
4.G.A	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**				
4.G.A.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.		**Milgram-**This is a very low level standard, asking nothing more than that students understand the words, “points,,” “lines,” “line segments,” etc. There is also a problem than needs to be thought about: two line segments that are very close to parallel, but not parallel cannot really be distinguished from parallel lines without more than visual data. Similarly for close to equal angles, etc.	No revision is necessary.	
4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Understand right triangles as a category, and identify right triangles.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure.	**Milgram**-Frankly, it would be better to remove 4.Ġ.A.1 above, and replace it with this standard, though I'm doubtful here about the phrase “recognize right triangles as a category.” What does this mean in fourth grade? **Achieve-**AZ changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured.	Per Milgram's feedback, right triangles is used as an e.g.	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size **(e.g.,** understand right triangles as a category, and identify right triangles).

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4.G.A.3	Understand a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure.	**Milgram-**Envision two congruent, disjoint circles in the plane, with each OUTSIDE the other. This figure will have a line of symmetry, but it will be DISJOINT from the figure itself. It will not be a line "across" the figure, so, technically, this standard is nonsense. REAL CARE IS NEEDED IN CONSTRUCTING STANDARDS, AND VERY, VERY FEW EDUCATORS ARE REALLY ABLE TO DO IT PROPERLY. **Achieve**-AZ changed "recognize" to "understand," increasing the rigor but making the AZ standard less easily measured.		~~Understand~~ **Recognize** a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
SMP	**Standards for Mathematical Practices**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.		
4.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

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4.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

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4.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and				

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4.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
4.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				

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4.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
4.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				

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4.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

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Operations and Algebraic Thinking (OA)			**Carlson**-This set of standards is clear and coherent with a solid and meaningful progression of ideas across grade levels. **Abercrombie-**The standards in this domain are clear, measurable, have sufficient breadth and depth, and are unambiguous. In general, the changes made, such as removing the examples and clarifying the language are sound and do not affect the interpretability or measurability of the standards. **Milner**-This domain would be strengthened by the introduction of the concept of a “unit” or “neutral element” in a binary operation. That allows defining “inverses” and thus understanding subtraction as addition of the additive inverse (“opposite”) and division as multiplication by the multiplicative inverse (“reciprocal”).		
5.OA.A	**Write and interpret numerical expressions.**				
5.OA.A.1	Use parentheses in numerical expressions, and evaluate expressions with this symbol.	So 5th grade students will no longer have problems containing brackets and braces? **This standard was unchanged, as are the vast majority of standards contained within the 5th grade math standards. Any changes found throughout the standards, 5.OA.A.1 through 5.MD.B.2, indicate there was little or no good faith effort to improve standards for education in Arizona.	**Milner-**5.OA.A.1 With the removal of brackets and braces, is the intention that those never be used? When (if so) will they be introduced? **Milgram-**If you are going to do things this way, then you NEED a second sentence saying something like “Generally, brackets or braces in numerical expressions are used in exactly the same way as parentheses, but they often MAKE THE EXPRESSION MUCH EASIER TO READ. (For example, replacing the expression ((((3 + 6)*6) + 11)*33) with the expression {[((3+6)*6)+11]*33} makes it much easier to parse.)” **Achieve**-AZ excludes other symbols of inclusion other than parentheses. It is not clear how removing brackets and braces clarifies the expectation as claimed in the AZ Technical Review. **Wurman-**This strikes me as ill advised. Using brackets and braces for nested expressions is much easier and less error-prone than using nested parentheses. This is already 5th grade!	Based on public feedback in the fall of 2015, brackets and braces were removed. Based on public comment and technical review, brackets were reinstated but braces are kept out of this 5th grade standard.	Use parentheses and **brackets** in numerical expressions, and evaluate expressions with ~~this~~ **these** symbols (Order of Operations).
5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Examples need to be provided in a supporting document	**Milgram-**This standard is simply far too general as stated. What kinds of problems are appropriate to test it here in fifth grade? For example, you clearly do not want a question about (3 + 2^(11))^(4/7) in fifth grade. My best advice would be to PUT THE EXAMPLE BACK. **Wurman**-Removing the example is wrong-headed and makes the standard opaque and unclear.	Based on Milgram and Wurman's feedback, the example was restored.	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them **(e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product).**
5.OA.B	**Analyze patterns and relationships.**				

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5.OA.B.3	Generate two numerical patterns using two given rules (i.e. generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	Examples need to be provided in a supporting document **Removing examples does not constitute a good faith effort to make real change in the standards.	**Milgram**-See my comment directly above. You have to include limiting examples or other limiting information. In this case, since the second sentence is totally impossible to make mathematical sense of in full generality (for two such patterns there is almost always absolutely no real relationship between the corresponding terms), one could also include a limitation such as "use only rules involving addition and multiplication by fixed numbers." **Achieve**-AZ added an explanation of "rules. "They also increased the rigor for this standard by expecting students to "explain" the relationships between corresponding terms. AZ removed the CCSS example. **Wurman**-Without the example the standard is unclear.	Based on Milgram, Achieve, Wurman, and Public feedback, the example was restored.	Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane **(e.g. given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).**
5.OA.B.4	NEW	.	In response to Wurman's comment from 4th Grade 4.OA.B.4: .Wurman-The original language is flawed in that: - It is the only place primes are even mentioned, so the standard needs a preamble along the lines of "understand that primes have only two factors: 1 and the number itself" - Determination whether a given number between 1 and 100 is prime is not a trivial task, unless the primes are memorized by rote, a foolish task. A good standard would require learning how to decompose numbers into prime factors, allowing a meaningful way to address this standard. If necessary, this could then be moved to grade 5.	Based on Wurman's response to 4.OA.B.4 and public comment on that standard, it was split and a portion moved here.	Understand primes have only two factors and decompose numbers into prime factors.
Number and Operations in Base Ten (NBT)			**Abercrombie**-The standards in this domain are clear, measurable and have sufficient breadth and depth. The additional standards added to this domain support the domain knowledge. The phrase, "Use of a standard algorithm is a 4th Grade standard, see 4.NBT.B. 4), added to standard 2.NBT.B.6 may confuse rather than clarify the interpretation of standardard 2.NBT.B.6. Overall, the standards in this domain are developmentally appropriate.	Statement was removed from 2.NBT.B.6.	
5.NBT.A	**Understand the place value system.**				
5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.		**Milgram**-What, exactly, do the authors mean by "recognize," and how do you write a question to test it? A least one explicit example is really needed here to clarify things. **Achieve**-While "recognize" has fairly consistently been replaced with "understand" in the AZ standards, it is left here. Is that intentional? Or an oversight? **Wurman**-This standard effectively repeats the 4th grade 4.NBT.A.1 and should be eliminated here.	Based on Milgram and Achieve's comments, edits were made and it is in alignment with the previous grade level standard.	~~Recognize that in a~~ **Apply concepts of place value, multiplication, and division to understand that in a** multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.		**Milgram-**Both the original standard in column B and its "revision" in this column are extremely problematic. After all, suppose the the original number has lots of zeros, such as 304500678000.3754. The critical thing students need to understand is that when you multiply by 10 you move the decimal point (If present) one place to the right, and if the decimal point is not present you add a single 0 on the right. Why can't the standard be revised to say that students should understand this is what happens when multiplying by 10? **Wurman**-This is the first time exponents show up, without any preparation. Exponents are expected only in the 6th grade standards. Insisting on exponents here seems ill-advised.	Based on Wurman's feedback, the exponent requirement was removed. Milgram's comment doesn't apply to 5th graders.	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. ~~Use whole-number exponents to denote powers of 10.~~
5.NBT.A.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Removing examples does not make substantive change. This standard is deemed unchanged from 2010.	**Milgram-**"Number names" are things in English, not mathematics. To be consistent, and not introduce irrelevancies, I strongly recommend deleting "number names" here. **Wurman**-Here the deletion of the example is not very damaging, but it is also unnecessary.	After careful consideration, the workgroup chose to keep number names since it is appropriate for a 5th grade standard. No revisions necessary. Based on Wurman's comments, the example will be included in the support documents.	
5.NBT.A.4	Use place value understanding to round decimals to any place.		**Milgram-**Actually, this is not true! It should be rephrased as something like "Use place value understanding and explicit rounding rules to round decimals to any place to the right of the decimal point."	After careful consideration, the workgroup chose not to make any further revisions.	
5.NBT.B	**Perform operations with multi-digit whole numbers and with decimals to hundredths.**				
5.NBT.B.5	Fluently multiply multi-digit whole numbers using a standard algorithm.	There are many different algorithms...but only one STANDARD algorithm. **Substituting "a" for "the" is not substantive change and does not constitute improvement.	**Milgram-**I wish that at least one REAL mathematician, such as Eric Milnor at Arizona State had been involved in this revision. THERE IS SUCH A THING AS THE STANDARD ALGORITHM. What do they think is that the "standard algorithm" is simply one of its REPRESENTATIONS using compressed forms of numbers in base ten form. The actual standard algorithm is defined as follows: Take two whole numbers A and B. Write the second number in base ten expanded form Bn times 10^n + B(n-1) times 10^(n-1) + ... Then write the product in the form A*B(n) times 10^n + A*B(n-1)*10^(n-1) + ... and perform the indicated multiplications and additions. THIS IS THE STANDARD STAIR-STEP MULTIPLICATION ALGORITHM. It always works. **Achieve**-By changing the article from "the" to "a," AZ opens the door to there being multiple standard algorithms.	Dr. Milgram was giving examples of exponents but this standard is based on whole numbers without the use of exponents. The workgroup found it appropriate. No revisions necessary	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
5.NBT.B.6	Applying and extending understanding of division by finding whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using a variety of strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.	What supporting document is referred to here?	**Milgram**-.I would suggest that the last sentence is pedagogy, not math and should not be present in the standards. Likewise, the first phrase “Apply and extend understanding of division by finding” is unreasonable for testing. REPLACE BY THE ORIGINAL “FIND.” **Achieve**-AZ requires using the operation to extend understanding of itself. They also limit the "variety of representations" by not mentioning arrays or area models. Clarity: It is not clear how finding quotients applies and extends understanding of division. Instead of "apply and extend understanding...by finding" (which is awkward) perhaps match previously used AZ language "apply and extend understanding to find" (See 4.NF.B.4). **Wurman**-Essentially OK except that the change of "rectangular arrays, and/or area models" to "models" is wrong-headed. The standard purposely specifies the two types of models it expects rather than any undefined model.	Based on Technical review, appropriate changes were made. Based on Milgram's comment, the original "find" was restored and the last sentence was removed.	Apply and extend understanding of division **to find** whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. ~~using a variety of strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.~~
5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; relate the strategy to a written method and explain the reasoning used.	Prescriptive language or "how to's" still in the standard calling our models and drawings to be used instead of just "what to teach." **Changed from "addition and subtraction" to "operations." This is not a substantive change and the standard is not improved.	**Milner**-In 5.NBT.B.7 the change introduced is ill-conceived: even when multiplying or dividing two decimals, the relationship needed in a standard algorithm is between addition and subtraction. **Milgram**-Huge confusion between numbers and their representations. The “concrete models” are representations of numbers, not the numbers themselves. **Achieve**-The CCSS requires only the relationship between addition and subtraction, while the AZ counterpart appears to be addressing the relationships between all four operations.	based on technical review, concrete models was removed and the last phrase was removed.	Add, subtract, multiply, and divide decimals to hundredths, ~~using concrete models or~~ connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations; relate the strategy to a written form. ~~method and explain the reasoning used.~~
<u>Number and Operations - Fractions (NF)</u>			**Abercrombie**-The standards are measurable, clear, and contain breadth and depth of the content. The developmental progression is clear and apparent across grade levels. The clarification of the link between the standards and real world problem solving is an improvement.		
5.NF.A	**Use equivalent fractions to add and subtract fractions.**				
5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	This comment stands for every standard that follows. The process of changing, revising, rewriting standards involves doing something other than simply removing examples, which is what the SDC has done. The ONLY thing the committee has done. Commenting further is a pointless endeavor....much like the job with which the committee was tasked. A pointless effort. Rebranding these standards, yet again, will only create more unrest among parents and stakeholders.	**Milgram**-Are you sure you do not want limitations here? Do you really want questions such as “Determine the single fraction in reduced form that is equal to the sum 7536/471 + 19/37” appearing in fifth grade exams? Also, what earthly reason would you have for deleting the EXTREMELY IMPORTANT last phrase “(In general, a/b + c/d = (ad + bc)/bd.)” from the original standard? **Wurman**-The removal of the example detracts.	Based on technical review, example was restored. The cluster heading states to use equivalent fractions as a strategy in which the e.g. assists with.	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators **(e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12).**

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations including equations and models. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	How is a student going to be measured on "assessing the reasonable of their answers?" **Removing examples does not constitute review/revisions. The standard remains unchanged from 2010.	**Milgram**-Are you sure you do not want limitations here? Do you really want questions such as "Determine the single fraction in reduced form that is equal to the sum 7536/471 + 19/37" appearing in fifth grade exams? Also, what earthly reason would you have for deleting the EXTREMELY IMPORTANT last phrase "(In general, a/b + c/d = (ad + bc)/bd.)" from the original standard? **Achieve**-Reading the AZ standard is awkward with "including" used twice in one sentence. Also, in AZ, one of the suggested "variety of representations" is given as "models" rather than visual fraction models. Teachers may not understand that "models" does not refer to modeling with mathematics, as required in MP.4. **Wurman**-The change seems to be driven by misunderstanding the advantage of being specific rather than generic. The original purposely specified two specific models. The "improvement" replaces them with generic "models" offering no guidance which models make sense or are expected. The same can be also said about the deletion of the very illustrative example.	Examples are reinstated as suggested by technical reviewers.	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators **by using a variety of representations, ~~including~~ equations, and visual models** to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers **(e.g. *recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*).**
5.NF.B	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**			Since apply and extend is used within the standards in this cluster, it was removed from the cluster heading to eliminate redundancy.	**~~Apply and extend~~ Use previous understandings of multiplication and division to multiply and divide fractions.**
5.NF.B.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers using a variety of representations.	Examples removed. No substantive change. Standard remains unchanged from 2010.	**Carlson**-5.NF.B.3: "Interpret a fraction as division of the numerator by the denominator (a/b = a divided by b)..." This does not seem quite right to me. a/b is a number. It is the result of dividing a by b. a/b and a "divided by" b are not just two equivalent ways to write the same operation. They mean different things. a/b represents how many times as large a is compared to b, which is calculated by dividing a by b. We should be encouraging students to flexibly see fractions as numbers (a/b is a number that is a times as large as 1/b) as you have called for elsewhere, not as a command to calculate something that encourages them to see a/b as an a, and a bar, and a b, instead of seeing a/b as a number that could be interpreted as the result of a calculation. **Milgram**-The original standard is very confusing, but the revision has the same confusion and needs examples. There are really two situations here. The first refers to the original standard "Interpret a fraction as .." This standard has been badly misstated in the first sentence in both the original and the "corrected" version. IT SHOULD READ SOMETHING LIKE "Interpret a fraction as the NUMBER that results from dividing the whole number numerator by the whole number denominator." Then PART (B) of the standard should start with "Solve word problems ..." AND PUT BACK THE EXAMPLES IN THE ORIGINAL. **Achieve**-AZ replaced the specific CCSS wording with the less specific, "using a variety of representations." In this standard, again, "visual fraction models" is changed to just "models." Clarity is needed for teachers to know what "models" are included. **Wurman**-The original standard is unclear, and the rewording is not any better. What is "interpret a fraction as a division of the numerator by a denominator"? Is there any other way? The purpose of this standard is unclear.	Milgram's wording from his feedback was included, which also helps clarify Carlson's and Wurman's concerns. Workgroup decided to put some examples back in.	~~Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers using a variety of representations.~~ Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. ***For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?***

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and by a fraction. a. Interpret the product of a fraction multiplied by a whole number (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. Use a visual fraction model and create a story context for this equation. b. Interpret the product of a fraction multiplied by a fraction (a/b) x (c/d). Use a visual fraction model and create a story context for this equation. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Examples need to be provided in a supporting document **Removing examples does not constitute a revision to the standard. The standard remains unchanged from 2010; common core rebranded.	**Carlson-** You mention “1 whole” many times, but there doesn’t appear to be a standard explicitly tied to reasoning about fractions related to a whole that is not thought of as “1” in some other unit. For example, if there is a bag of apples, students can visually represent (using number line reasoning or similar visualizations) how to interpret 4/5 of the bag of apples. If they are later told that the bag had 30 apples in it, then (4/5)(30) also represents 4/5 of “1 whole” but in units of “apples” now instead of “bags of apples”. It’s possible that this is already included, and maybe you intend for this reasoning to be supported in 5.NF.B.4, but this flexibility in understanding and moving between “1 whole” (that is, the value of a quantity using its own magnitude as the measurement unit” and the size of this whole (and any multiplicative comparisons to this whole) using other measurement units is extremely important and should be specifically highlighted and encouraged in the standards (and is a measureable standard). **Milgram**-Both to original and the revision are mathematically incoherent, mixing numbers and their (possible) representations in various contexts into an indigestible mess.	Based on Wurman's feedback,wording was changed to add clarity. Based on TR, explicit examples were added back in with additional assistance to readers in the form of the "in general" statement	~~Apply and extend previous understandings of multiplication to multiply a fraction or a whole number by a fraction.~~ **Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction.** a. ~~Interpret the product of a fraction multiplied by a whole number (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. Use a visual fraction model and create a story context for this equation.~~ a. Interpret the product (*a/b*) x *q* as a parts of a partition of *q* into *b* equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.*
			(cont.) **Achieve-**Attention to clarity is needed here. There is a slight word order change in the stem part of the standard (5.NF.4): The required operations "whole number by a fraction" is changed to "fraction by a whole number." The difference is subtle but not insignificant. In this case, part a asks for a fraction by a whole number, which is the reverse of the AZ stem standard. It should be noted that in other AZ standards (e.g. 5.NF.B.7) the difference between the two orders is attended to by including both. AZ split part a into two parts. The CCSS example was removed.The new AZ part b comes from the example in part a of the CCSS. The support for understanding the product of a fraction by a fraction is not included in this additional AZ standard. AZ will need to make sure to identify the standards that have the codes changed to avoid confusion when teachers match their standards with materials that are shared across states. Wurman-- The first part of the rewording mistakenly used "and" instead of an "or" in "and by a fraction." - sub-standard (a) is unclear in both variants		*(cont.)* b. Interpret the product of a fraction multiplied by a fraction (*a/b*) x (*c/d*). Use a visual fraction model and create a story context for this equation. *For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* (In general, (*a/b*) x (*c/d*) = *ac/bd*). c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

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5.NF.B.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n x a)/(n x b) to the effect of multiplying a/b by 1.	Minimal alterations to wording, simply a semantic adjustment. Standard remains substantially unchanged from 2010.	**Milgram-**The original standard here “Interpret multiplication as scaling (resizing)” makes absolutely no sense to me. I KNOW HOW TO USE MULTIPLICATION BY POSITIVE NUMBERS TO SCALE THINGS. BUT I DON'T EVEN BELIEVE IT IS POSSIBLE TO TAKE SCALING AS PRIMITIVE AND MAKE MULTIPLICATION INTO A SPECIAL CASE OF IT. IF WE DID THIS WE COULD NOT MULTIPLY NEGATIVE NUMBERS OR COMPLEX NUMBERS ETC.	There is a great need for support documents on this standard. It is important to understand that scaling is not limited to only fractions. This allows students to interpret the multiplication operation as a form of changing the magnitude of the size of the original entity. With scaling, we are putting a physical meaning to the operation of multiplication. The forthcoming support documents will address Mr. Milgram's comments.	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
5.NF.B.6	Solve problems in a real-world context involving multiplication of fractions and mixed numbers by using a variety of representations including equations and models.	It seems that rectangular arrays were removed, but it is a helpful strategy to keep. **No substantive change. The standard remains the same as the 2010 standard. It seems unlikely any of the "changes" that follow will in any way move Arizona away from one size fits all education.	**Milgram**-.I think things would be much clearer if this were rephrased as follows: “Solve problems arising in a real-world context that involve multiplication of fractions.” For what it is worth, mixed numbers are really fractions. What is happening is that when we write 3 5/7, what we really mean is 3 + 5/7 or 3/1 + 5/7 or 21/7 + 5/7 or 26/7. **Achieve**-AZ replaced the CCSS examples with the generic, "a variety of representations." Using the general term "models" here, rather than "visual fraction models," might lead to the conclusion that MP.4 is at play.	Specific strategies were removed from the standards to allow teachers/ school dsitricts to determine the "how". Examples can be included in a supporting document.	Solve problems in real-world contexts involving multiplication of fractions, ~~and~~ including mixed numbers, by using a variety of representations including equations and models.

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5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. c. Solve problems in real-world contexts involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.	No substantive change. The standard remains a 2010 common core standard.	**Milner-**In 5.NF.B.7 it is desirable to keep "create a story context for a whole number divided by a fraction". It cognitively more difficult to create the problem than to solve it. **Milgram**-As seems to be usual in this area there is a tremendous confusion between NUMBERS, in this case fractions, and representations of these numbers. The representations are sometimes helpful in understanding aspects of OPERATIONS such as addition, subtraction, multiplication, and division, by seeing cases where they arise in representations of the numbers. But they are NOT the numbers themselves. **Achieve**- AZ made the CCSS example part of their requirement. AZ included part of the CCSS example in their requirement, possibly increasing the rigor for this standard. **Wurman**-It is incoherent to argue that "Notes are not included within the standard unless it [sic] would provide limits to the standard or clarification to the standard" and then delete a clearly limiting note ("Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade"). Similar incoherence in removing good illustrative examples, while claiming that clarifying examples will not be removed, has ben observed many times in this rewrite.	No revisions needed. Examples will be included in supporting document.	
Measurement and Data (MD)			**Abercrombie**-The standards are written with clarity, are measurable, and have sufficient breadth and depth. The addition of the standards around time and money are sound and add to the breadth of this domain; these standards are also appropriately placed in the grade progression		
5.MD.A	**Convert like measurement units within a given measurement system.**			No revisions needed.	
5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems.	No substantive change. This is common core rebranded.		No revisions needed.	
5.MD.B	**Represent and interpret data.**			No revisions needed.	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
5.MD.B.2.	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	Removing examples is not a "refinement." There is no substantive change and the standard remains consistent with 2010 common core.	**Wurman**-without the example this standard is opaque and likely to be interpreted identically as is 4.MD.B.4.	Example is included in the standards for clarity. To be consistent among grade levels format has been adjusted to (e.g.)	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots **(e.g.,** given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally**).**
5.MD.C	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**				
5.MD.C.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of *n* cubic units.	Identical to 2010 common core standard.	**Milgram**-.ONE SHOULD CLEARLY REALIZE THAT THERE IS A HUGE PROBLEM WITH PART (B) OF THE ORIGINAL STANDARD AND FIX IT. The issue is that, while (b) is exactly true as stated, there is a huge tendency to take it as a total definition of having a volume of n cubic units. This would make it impossible to assign any volume to figure such as prisms with triangular bases, since it is impossible to pack them without gaps or overlaps using unit cubes. You need to involve serious mathematicians in fixing these kinds of foul-ups.	In part (b) of this standards " A solid figure *which* can be…" refers to shapes that can be filled without gaps or overlaps. 5th grade is the first time students are beginning to explore concepts of volume and this is not a foul up on the workgroup's part. Examples will be in supporting documents. No revisions needed.	
5.MD.C.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	Identical standard remains consistent with common core 2010.	**Milgram**-As usual, there is a problem the committee doesn't seem to recognize. The standard only refers to special figures that can be decomposed without gaps or overlap into cubes, but even in fifth grade one wants to be able to determine the volumes of somewhat more general solids.	" A solid figure which can be…" refers to shapes that can be filled without gaps or overlaps. 5th grade is the first time students are beginning to explore concepts of volume. Examples will be in a supporting documents. No revisions needed.	

Coding	Draft Standard - as of 8/2016	Public Comment - Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard- 12/2016
5.MD.C.5	Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in a real-world context involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base (making the connection between additive and multiplicative b. Understand and use the formulas *V = w x l x h* and *V = B x h* , where in this case B is the area of the base (*B = l x w*), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving mathematical problems and problems in a real-world context	The language of subsection "a." directs instructional technique. In order to limit it to the "what," it should state, "Find the volume of a right rectangular prism with whole-number side lengths." Leave the "how" to the teacher or the school/school district. **How do you measure understanding? Recognition can be measured..Point to the triangle (they recognize it is a triangle) but understand what makes it a triangle is tough to measure. **Does a 5th grader understand "real-world problem solving and how to link that to everyday work and decision making?" Where is the research to back this up? **Language manipulated to some small degree, but meaning doesn't change. This standard remains unaltered from 2010 common core.	**Milner-**Why is "real world" kept in 5.MD.C.5 but changed to "in a real world context" in other standards? In part a. the removal of "Represent threefold whole-number products as volumes" detracts from the standard (same comment made above for 3.MD.C.7b). In part c. "applying this technique" is meaningless because no technique is mentioned. **Milgram-.**Be very careful here. Technically, the additivity principle is very delicate and only holds for volumes of special solid figures. **Achieve**-AZ removed the requirement to represent "three-fold whole-number products" as volumes. AZ changed "recognize" to "understand," making the AZ standard less easily measured. They also removed the explanation of how to find volumes of composed figures. **Wurman**- Why change the "x" to "●"? Other standards in this grade (e.g., 5.NF.B.4 and 5.NF.B.5) still use the "x"	Changes to the standard have been made to remove the "how". As per Milner's review, standards as changed to "in real world context" in addition, represent threefold whole-number products as volumes was restored from the original standard. As per Wurman review, "x" has been restored to be consistent with other 5th grade standards.	Relate volume to the operations of multiplication and addition and ~~solve real world and mathematical problems involving volume~~. **solve mathematical problems and problems in real-world contexts involving volume.** a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. **Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication).**
					(cont.) b. Understand and use the formulas V = *l x w x h* and V = *B* x *h* , where in this case B is the area of the base (*B* = *l x w*), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts.
Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.	No revisions needed.	
5.G.A	**Graph points on the coordinate plane to solve mathematical problems as well as problems in a real-world context.**				**Graph points on the coordinate plane to solve mathematical problems as well as problems in ~~a~~ real-world context.**

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5.G.A.1	Understand and describe a coordinate system as perpendicular number lines that intersect at the origin (0 , 0). Identify a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number (x) indicates the distance traveled on the horizontal axis, and the second number (y) indicates the distance traveled on the vertical axis.	"called axes" should remain...later refer to "axis" but don't relate them to the perpendicular number lines	**Milgram**-POSSIBLY HUGE CONFUSION BETWEEN RAYS AND LINES HERE. I WOULD HAVE TO LOOK UP ALL MENTIONS OF NUMBER LINES IN PREVIOUS GRADES TO SEE IF IT IS REALLY TRUE THAT THE DEFINITION OF NUMBER LINE IS REALLY A RAY WITH NON-NEGATIVE FRACTIONS OR WHOLE NUMBERS AS LABELS. **Achieve**-By including the example as part of this standard, AZ specifically identifies the variables as x and y, making it less likely that students would use other variables more appropriate to a real world context. The CCSS makes the effort to allow for any variable and uses x and y only in a parenthetical example. **Milner-**5.G.A.1 should include the names abscissa and ordinate. "Understand that the first number (x, called abscissa) indicates the distance traveled on the horizontal axis, and the second number (y, called ordinate) indicates the distance traveled on the vertical axis."	"In the first quadrant of the coordinate plane" as added to clarify this standards. "called axes" was restored to the standards as recommended by technical review.	Understand and describe a coordinate system as perpendicular number lines, **called axes**, that intersect at the origin (0 , 0). Identify a given point **in the first quadrant of the coordinate** plane ~~located~~ using an ordered pair of numbers, called ~~its~~ coordinates. Understand that the first number (x) indicates the distance traveled on the horizontal axis, and the second number (y) indicates the distance traveled on the vertical axis.
5.G.A.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.		**Milgram**-Referring to my comment on 5.G.A.1 above, here it seems clear that number lines do include negative numbers. Then we do have a serious problem with 5.G.A.1. Make this situation clear and coherent PLEASE.	No revisions needed.	
5.G.B	**Classify two-dimensional figures into categories based on their properties.**			No revisions needed.	
5.G.B.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.		**Milgram**-.Category has a very specific meaning in mathematics. It is not interchangeable with "set." If you mean to change the definition of category to "set," this should be explained in the glossary. In particular, "category" should be defined in the glossary as a set. **Wurman**-Without the example, parsing this standard will be a challenge to elementary teachers.	Examples will be included in a supporting documents. No revisions needed.	
5.G.B.4	Classify two-dimensional figures in a hierarchy based on properties.		**Milgram**-This standard needs to be limited by examples. **Wurman**-Actually, elementary teachers would be helped by some examples here.	Examples will be included in a supporting documents. No revisions needed.	
SMP	**Standards for Mathematical Practices**		**Achieve**-The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.		

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5.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				
5.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

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5.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.				
5.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				

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5.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
5.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
5.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				

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5.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				