6th Grade Arizona Mathematics Standards

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
6th Grade Mathematics Standards					
Ratio and Proportion (RP)			**Carlson** -These standards are coherent and logical. **Abercrombie**-These standards are clear, measurable, and developmentally appropriate. The inclusion of the limits in standard 7.RP.A.3 are appropriate and useful. No suggestions for refinements were identified. The standards are written so that they will be unambiguously interpreted across the state. **Milner-**The fundamental concepts are introduced in the wrong order. Rates need to be defined AFTER proportional relationships. This fact becomes crystal clear in 7.RP.A.2b.	Reason for no change (Milner comment) The 6-7 Ratios and Proportional Relationships progression (2011) document states, "Rates are at the heart of understanding the structure ... providing a foundation for learning about proportional relationships in seventh grade." (pg. 5). Support documents will assist in providing educators with guidance in relationship to the scope and sequence of proprotional realtionships and rate introduction.	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**				
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	Possible re-wording "Understand the concept of a ratio and use RELEVANT language to describe a ratio-relationship between two quantities." **I think the deletion of the example is appropriate and allows for flexibility in instruction.	**Carlson-**6.RP.A.1: "Understand the concept of a ratio…" What do you want them to understand? If you aren't explicit, then you fail your question G: there are no clear expectations that will be broadly interpreted in the same way across schools. There is a lot of research about productive meanings for ratio in the literature, so there is no reason not to be explicit here. **Milner-**6.RP.A.1 needs to be placed in a real-world context. Otherwise ANY two numbers a and b are "in a ratio relationship", namely a:b. **Milgram-**.I have no idea what "ratio language" might mean. Ratios are actually very subtle objects and were first developed and explained by Euclid himself in his famous book V. Sad to say, very, very few teachers (and possibly even fewer "math educators") have any idea of what goes on with them or how they are defined. In particular, they are NOT numbers, though in special cases, they can be represented by quotients.	Based on Carlson, Milner, Milgram's comments and cited research, edits were made. Additional examples for this standard will be included in a supporting document. Lamon's Teaching Fractions and Ratios for Understanding (3rd ed.) states, "Often ratio language is used as an alternate way of expressing a multiplicative relationship. For example, there were 2/3 as many men as women at the concert." NCTM's Developing Essential Understanding of Ratios, Proportionals and Proportional Reasoning states "A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit. There are two ways to form a ratio, both of which involve coordinating two quantities. One way is by comparing two quantities multiplicatively. The second way is by joining or composing the two quantities in a way that preserves a multiplicative relationship.	Understand the concept of a ratio **as comparing two quantities multiplicatively or joining/composing the two quantities in a way that preserves a multiplicative relationship** and use ratio language to describe a ratio relationship between two quantities. *For example, "There were 2/3 as many men as women at the concert.*
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)	The clarification of the expectations for complex vs. non-complex fractions is helpful. I think the deletion of the example is appropriate and allows for flexibility in instruction.	**Milner**-In 6.RP.A.2 the concept of unit rate is not introduced properly. The essential component, two co-varying quantities, is missing. The examples suggest this but were removed and, moreover, this needs to be explicitly included rather than suggested. In the first deleted example, the numbers of cups of sugar and cups of flour are related variables/quantities (linearly, in fact), and those numbers are related by the latter being equal to the former multiplied by the unit rate of flour-to-sugar. Rate cannot be defined for just two numbers; it requires two co-varying quantities (in a proportional relationship at this stage). **Milgram**-See my comments on standard 6.RP.A.1 above. Here, I also have no idea what "rate language" might mean. I would also strongly urge putting the example in the original 6.RP.A.2 back. Finally, it seems to me that the people modifying this standard didn't really understand what the original writers meant by "complex fractions." They are fractions that have the form (a/b)/(c/d), as distinguished from fractions of the form a/b (where a, b, c, and d are non-zero whole numbers). It might be worth noting that the reason I did not ask that the examples in 6.RP.A.1 be restored is that the second example is not correct.	Based on Carlson, Milner, Milgram's comments and cited research, edits were made. Examples for this standard will be included in a supporting document. The 6-7 Ratios and Proportional Relationships progression (2011) document states, "It is important for students to focus on each of the terms "for every," "for each," "for each 1," and "per" because these equivalent ways of stating ratios and rates are at the heart of understanding the structure ..."	Understand the concept of a unit rate a/b associated with a ratio a : b with b ≠ 0, and use rate language **(e.g. for every, for each, for each 1, per)** in the context of a ratio relationship. (Complex fraction notation is not an expectation for unit rates in this grade level.)

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6.RP.A.3	Use ratio and rate reasoning to solve mathematical problems and problems in a real-world context. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Wording change helps with clarification. **The rewording of "solve mathematical problems and problems in a real-world context" creates consistency across grade levels and allows for flexibility in instruction. **Clear, concise and to the point. I support this adoption.	**Achieve-**AZ removed the reference to data collected from measurements. **Milgram**-Let me repeat that the terms "ratio reasoning" and "rate reasoning" have no meaning what-so-ever in actual mathematics. I wish this document did not use them.	Based on Achieve's comment, edits were made to include reference to data collected from measurements.	Use ratio and rate reasoning to solve mathematical problems and problems in a real-world context **(e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).** a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
The Number System (NS)			**Carlson** -These standards are coherent and logical. **Abercrombie**-These standards are clear, measurable, and contain sufficient breadth and depth. The vertical alignment of these standards is excellent, and the refinements made to the standards are useful. The standards are written so that they will be unambiguously interpreted across the state.		
6.NS.A	**Apply and extend previous understanding of multiplication and division to divide fractions by fractions.**				**Apply and extend previous understanding of multiplication and division to divide fractions by fractions.**
6.NS.A.1	Interpret and compute quotients of fractions to solve mathematical problems and problems in a real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. (In general, (a/b) ÷ (c/d) = ad/bc.)	The change in language and deletion of examples is appropriate. **Take out visual models and just do the algorithm. **There are many different algorithms...but only one STANDARD algorithm. **"AZMerit shows what students don't know. It should also show what the less proficient student does know. For example, if a student has a hard time finding a model for 3/4 divided by 2/3, can he do the actual steps of dividing fractions? If so, he could be rated Proficient 2. If he can choose the model that goes with it, he can be ranked Prof 1. I strongly recommend you tag questions on all math standards using this two-step process." **This standard embodies on of the most famous mathematical "don't ask just do it" algorithms of elementary school. The inversion of the second fraction and multiplying continues to mystify most people, and it does not seem as if this standard addresses this situation at all. The original one's example at least highlighted the connection between division and multiplication, and the new one seems to place the focus more squarely on the standard procedure, which does little to instill understanding.	**Achieve**-Removing the "e.g." in this CCSS gives the impression that only visual models and equations are required. AZ removed the CCSS specific example but kept the general one.	Based on Achieve's comment, an edit was made to include a specific example.	Interpret and compute quotients of fractions to solve mathematical problems and problems in real-world context involving division of fractions by fractions using visual fraction models and equations to represent the problem. ***For example, create a story context for 2/3 ÷ 3/4 and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that 2/3 ÷ 3/4 = 8/9 because 3/4 of 8/9 is 2/3.*** *In general, a/b ÷ c/d = ad/bc.*
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**				**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.B.2	Fluently divide multi-digit numbers using a standard algorithm.	The language "a standard algorithm" allows for choice in instruction and is appropriate. **There is a disconnect between k-5 and 6-8 standards. In 5th grade, they don't have to use the standard algorithm and then in 6th grade they are expected to fluently apply the standard algorithm Where do we teach the standard algorithm? **There are many different algorithms...but only one STANDARD algorithm. **Clear, concise and to the point. I support this adoption.	**Achieve**-AZ changed "the" to "a," implying that there may be multiple standard algorithms. **Milgram**-what I would strongly suggest is that somebody talk to a younger person who was educated in China, and had their standard course in algorithms in lower high school. (It is a standard course and all Chinese students are expected to take it.)	An algorithm is defined as "a set of instructions/steps used to solve a problem or obtain a desired result in every case". No action required.	

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6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	The phrase "a standard algorithm" allows for flexibility and choice and is appropriate. **Clear, concise and to the point. I support this adoption.	**Achieve**-AZ changed "the" to "a," implying that there may be multiple standard algorithms. **Milgram**-Be careful here. It should be understood that there does not exist any algorithm, let alone a “standard one” for doing these operations with infinite decimals, except for very special cases, for example where the infinite decimal is ultimately repeating.	No action required.	
6.NS.B.4	Understand the greatest common factor, understand the least common multiple, and use the distributive property. a. Find the greatest common factor of two whole numbers less than or equal to 100. b. Find the least common multiple of two whole numbers less than or equal to 12. c. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	The changes to this standard create clarity and provide a clear outline of each piece of the standard.	**Achieve**-AZ split this CCSS into four parts, the stem and three sub-parts, and removed the example. The change from "find" to "understand" in the AZ stem for this standard represents an increase in rigor but is more difficult to measure. The sub-parts for this AZ standard return to the more easily measured performances of "find" and "use." **Milgram**-As above, I have no idea of what “Understand the greatest common factor” or “Understand the least common multiple” might mean. Both are definitions, and definitions just ARE. Perhaps, the authors want to say that students should understand why these definitions are important. If so, they should say so. **Milner**-6.NS.B.4 is not one standard because there is no unifying thread across its three parts. It is the ill-advised merging of three distinct standards into one. There should be one separate standard for each part, including a justification for why the lcm and gcd exist for any finite set of positive integers.	Based on comments from Achieve, Milgram and Milner, edits have been made. Support documents will address justification of why the LCM and GCF exist for any finite set of positive integers.	**Use previous understands of factors to find** the greatest common factor and the least common multiple. ~~and use the distributive property.~~ a. Find the greatest common factor of two whole numbers less than or equal to 100. b. Find the least common multiple of two whole numbers less than or equal to 12. c. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. **For example, express 36 + 8 as 4(9+2).**
6.NS.C	**Apply and extend previous understandings of numbers to the system of rational numbers.**		**Milner**-6.NS.C.9 should not be removed!!! Students have no clue about the meaning of the symbol “%” (i.e. “ % = 1/100 ”). It could be reworded “Understand that fractions, decimals, and percents are three different ways of representing numbers, and fluently convert from one way to another.” 7.NS.A.2 contains conversion of fraction to decimal.	No change needed. Percents are addressed in 6.RP.3c	**Apply and extend previous understanding of numbers to the system of rational numbers.**
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world context, explaining the meaning of 0 in each situation.	Deletion of the example is appropriate.	**Wurman**-The examples were helpful and there is no need to remove them. **Milner**-6.NS.C.5 should not contain the word “directions” because the standards do not ever mention or define the direction of a number.	Based on Wurman's comment, the examples will be included in a supporting document.	
6.NS.C.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	The deletion of the example is appropriate.	**Milgram**-A rational number IS NOT a point on the number line. It is a number. Probably what the author really meant to say is something like “represent a rational number as a point on the number line,” and give an example such as representing 6/11 **Milner**-6.NS.C.6c should not mention integers separately because at this grade level students already know that integers are rational numbers.	Based on Milgram and Milner's comments, edits were made.	Understand a rational number **can be represented** as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and **positive** rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

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6.NS.C.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world contexts. d. Distinguish comparisons of absolute value from statements about order, especially when considering values	The changes made to this standard are appropriate and provide clarification and flexibility.	**Achieve**-part d:The word “especially” in a standard is awkward. If the intent is that students be able to work with the comparisons both in and out of context, it should be clearly stated.	Based on Achieve's comments, edits were made.	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line. b. Write, interpret, and explain statements of order for rational numbers in real-world context. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in real-world context. d. Distinguish comparisons of absolute value from statements about order **in mathematical problems and problems in real-world context.**
6.NS.C.8	Solve mathematical problems and problems in real-world context by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	The change in wording is appropriate. **Real world application with 4 quadrants is impossible because no one really uses 4 quadrants.		No revision necessary	
Expressions and Equations (EE)			**Carlson**- In the high school standards (at least the current ones) it explicitly discusses evaluating as linked to using an input value to determine the corresponding output value, and solving an equation as using an output value to determine the corresponding input value. I argue that this way of thinking and terminology should be used in grades 6-8 as appropriate both because they are extremely powerful ways to think about the processes by also because it opens up multiple solution paths and methods for checking the reasonableness of solutions. **Abercrombie**-In terms of developmental appropriateness, the 6th grade standards in this domain are likely to be quite challenging for 6th graders since students at this age are just beginning to be able to think representationally and abstractly, requirements for understanding algebraic expression and equation. Providing some limit to the complexity of the algebraic expressions would make these standards more developmentally appropriate, such as limiting the number of variables in an expression, or the types of operations included in the expressions. That said, the standards are written clearly and are measurable, and interpretation of these standards		
6.EE.A	**Apply and extend previous understanding of arithmetic to algebraic expressions.**			**Apply and extend previous understanding of arithmetic to algebraic expressions.**	
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **No change is appropriate. **I support the adoption of this standard.	**Milgram**-As stated this standard is too general. Examples are needed to limit it.	This standard builds on and supports 5.OA.A.1 1- Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).as wells as 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product). This standard also builds a foundation for 7.EE.B.4 Use variables to represent quantities in mathematical problems and problems in a real-world context and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px+q>r or px+q<r, where p, q, and r are specific rational	

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6.EE.A.2	Write, read, and evaluate algebraic expressions. a. Write expressions that record operations with numbers and variables. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in a real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **The changes are appropriate. **I support the adoption of this standard.	**Wurman**-Actually, where part (a) says "letters standing for numbers" is not equivalent to "variables."	No revision necessary Wurman's feedback is based on the 2010 standard language and not the draft standard language as the draft standard states "...numbers and variables." This standard builds on and supports 5.OA.A.1 Use parentheses in numerical expressions, and evaluate expressions with this symbol as wells as 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. This standard also builds a foundation for 7.EE.B.4 Use variables to represent quantities in mathematical problems and problems in a real-world context and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px+q>r or px+q<r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	Write, read, and evaluate algebraic expressions. a. Write expressions that record operations with numbers and variables. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. c. Evaluate expressions **given** specific values of their variables. Include expressions that arise from formulas used to solve mathematical problems and problems in real-world context. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
6.EE.A.3	Apply the properties of operations to generate equivalent expressions.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. This is no change from the common core idea that Diane Douglas has been so strongly against. This is a ruse. **The deletion of the example is appropriate. **Which properites, please be specific **I am in support of this adoption.	**Wurman-**The examples are helpful to clarify the scope of the standard and should be left in. **Milgram**-What is the meaning here of "equivalent expressions?" There is no universally understood mathematical concept for equivalent expressions. Generally, in the context of the Common Core standards, what we meant was that two expressions are equivalent if they involve the same objects (or variables) and when evaluated on each of these objects give the same value. Since this is not "standard" it needs to be written down somewhere in this document.	This standard builds on and supports 5.OA.A.1 Use parentheses in numerical expressions, and evaluate expressions with this symbol as wells as 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. This standard also builds a foundation for 7.EE.B.4 Use variables to represent quantities in mathematical problems and problems in a real-world context and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px+q>r or px+q<r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	
6.EE.A.4	Identify when two expressions are equivalent.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **The deletion of the example is appropriate. **I support the adoption of this standard.	**Wurman**-"Algebraic expressions are equivalent"? **Milgram**-See the comments for 6.EE.A.3 above.	This standard builds on and supports 5.OA.A.1 Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations).as wells as 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product). This standard also builds a foundation for 7.EE.B.4 Use variables to represent quantities in mathematical problems and problems in a real-world context and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px+q>r or px+q<r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	
6.EE.B	**Reason about and solve one-variable equations and inequalities.**				

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6.EE.B.5	Understand solving an equation or inequality as a process of reasoning to find the value(s) which make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	The change in wording clarifies the standard, and the deletion of the example is appropriate. **I support the adoption of this standard.	**Achieve**-AZ uses a slightly different choice of wording but the meaning is the same and may improve clarity. **Wurman-**The original language made an effort to frame solving an equation in a manner accessible to a beginner. The rewrite uses mathematical jargon. **Milgram-**.I am concerned about the phrase "as a process of reasoning." An arbitrary reasoning process has no expectation of being successful in finding the values that make the expression true. Much better would be simply "Understand solving an equation or inequality as finding the values of the variables that make the equation or inequality true."	Based on technical review, edits were made.	Understand solving an equation or inequality as a process of reasoning to find the value(s) **of the variables that ~~which~~** make that equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.B.6	Use variables to represent numbers and write expressions to solve mathematical problems and problems in a real-world context; understand that a variable can represent an unknown number or any number in a specified set.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **Change in wording appropriate. **I support the adoption of this standard.	**Carlson**-6.EE.B.6: "Use variables to represent numbers…" Students should be expected to see variable as a way of representing all of the values of a varying quantity, and see evaluating a function for a value of an input variable or solving an equation relative to choosing from among all of these possible values some subset that produce a given outcome. Math education research findings have repeatedly documented that students emerge from grade school mathematics without a strong concept of variation and tend to see variables as just unknowns, the one value that when substituted for x makes a statement true. We need to specifically support students in initially seeing variables as a letter that stands for the varying values of a varying quantity (varying distance in feet of a car from a stop sign as it drives away from the stop sign). Formulas and functions should then be introduced as constructs that define how two varying quantities are changing together (how they covary). Again, numerous researchers have documented that seeing variables as varying and functions as defining how two quantities change together are essential ways of thinking for understanding fundamental ideas in calculus. Variation and covariational reasoning should be supported from the earliest possible moments in students' mathematical experiences. [This comment applies to the entirety of the EE strand]. After students have established a covariation view of functions the idea of a variable as an unknown can be logically introduced when "solving an equation for some value of the input quantity when a value of the output quantity is given" (e.g., give $f(x) = 5x – 9$, solve $17 = 5x – 9$ for x.) **Milgram-**In general, there is no reason that a variable needs to be a number at all. There are many cases where the variable are points in a certain set such as the surface of a sphere or a torus.	The workgroup determined no edits were needed.	Use variables to represent numbers and write expressions to solve mathematical problems and problems in ~~a~~ real-world context; understand that a variable can represent an unknown number or any number in a specified set.
6.EE.B.7	Solve mathematical problems and problems in a real-world context by writing and solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and $p/x = q$ for cases in which p, q and x are all non-negative rational numbers.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **The addition of wording provides clarification and is appropriate. **I support the adoption of this standard.	**Achieve**-AZ added two variations on the CCSS equations. However, $p/x = q$ would not be appropriate at this level since students have not been introduced to rational expressions. It is likely that this is a typo and it should be $x/p = q$. Indiana, as referenced in the technical notes, includes $x/p = q$ but not $p/x = q$ in their standard 6.AF.5. **Milgram**-Reasonable standard. **Milner**-In the proposed 6.EE.B.7, $p/x = q$ should rather be $x/p = q$ since this standard is about linear equations.	Based on the feedback from Achieve and Milner, this was changed to make it mathematically correct. Changed from p/x to x/p	Solve mathematical problems and problems in ~~a~~ real-world context by writing and solving equations of the form $x + p = q$, $x - p = q$, $px = q$, and ~~p/x~~ $x/p = q$ for cases in which p, q and x are all non-negative rational numbers.
6.EE.B.8	Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in a real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number line.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **The addition of greater than or equal to and less than or equal to is appropriate for the grade level. **I support the adoption of this standard.	**Milner**-In the proposed 6.EE.B.8, the last words "on number line" should either be "on number lines" or "on a number line."	Based on Milner's comment, edits were made.	Write an inequality of the form $x > c$, $x < c$, $x \geq c$, or $x \leq c$ to represent a constraint or condition to solve mathematical problems and problems in ~~a~~ real-world context. Recognize that inequalities have infinitely many solutions; represent solutions of such inequalities on number line**s**.
6.EE.C	**Represent and analyze quantitative relationships between dependent and independent variables.**				
6.EE.C.9	Use variables to represent two quantities to solve mathematical problems and problems in a real-world context that change in relationship to one another; write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	This standard is developmentally inappropriate for 6th graders as many do not have the basic math skills to go from the concrete to the abstract. We Need to eliminate this and all standards in expressions and equations. **the change in wording and deletion of the examples is appropriate. **I support the adoption of this standard.	**Wurman**-Again, the original language was careful to embed a clear definition of the terms in the standard, while the rewrite uses plain mathematical jargon without scaffolding proper definitions. The original seems better. **Milner**-In the proposed 6.EE.C.9, the wording is in an incorrect semantic order, it should read "Use variables to represent two quantities that change in relationship to one another to solve mathematical problems and problems in a real-world context."	Based on technical review, edits were made.	Use variables to represent two quantities ~~to solve mathematical problems and problems in a real-world context~~ that change in relationship to one another **to solve mathematical problems and problems in real-world context**; write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
<u>Geometry (G)</u>			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.		

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6.G.A	**Solve real-world and mathematical problems involving area, surface area, and volume.**				**Solve ~~real-world and~~ mathematical problems and problems in real-world context involving area, surface area, and volume.**
6.G.A.1	Find the area of polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in a real-world context.	What other shapes, be specific **I like this standard. However, its inclusion of decomposing a polygon into triangles and rectangles contradicts the manner in which the criteria was applied to earlier standards, specifically 3.MD.C.7. I like the inclusion of the decomposition here, and I think that it should be added back in 3.MD.C.7.d. **OK as is. **I support the adoption of this standard.	**Achieve**-The CCSS provides more detail about some of the specific polygons required. Unfortunately, the AZ modification, apparently meant to remove redundancy, loses the parallel language found in critical area 5 of the front matter in the Grade 6 standards. The CCSS provides more detail about some of the specific polygons required. Unfortunately, the AZ modification, apparently meant to remove redundancy, loses the parallel language found in critical area 5 of the front matter in the Grade 6 standards. Perhaps it would be clearer and more consistent to say, "Find the area of polygons by composing into rectangles or decomposing into triangles and other polygons." (This would match the sort of clarification made in 7.NS.A.1.) A teacher looking to see where areas of triangles are addressed in the progression may not see this modification as encompassing that notion. **Milgram-**Far too vague. There must be examples to clarify what is expected here. First, is it obvious that every polygon can be decomposed into rectangles? I don't believe this for a moment. (In fact it is very easy to construct counter-examples unless you allow infinite decompositions. But it would be idiotic to be discussing limits in sixth grade.)	Based on Achieve and Milgram's comments, edits were made. Edit made to remove "a" from "in a real world context".	Find the area **of right triangles, other triangles, special quadrilaterals,** and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques to solve mathematical problems and problems in ~~a~~ real-world context.
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formula V = B·h, where in this case, B is the area of the base (B = l x w) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in a real-world context.	To avoid directing instructional techniques, this should read, "Find the volume of a right rectangular prism with fractional side lengths." the "...by packing it with unit cubes.." gets into the "how" which should be left to the teacher/school/school district. **Where do we find fractional unit cubes to place in these boxes you are referring to	**Milgram-**Somewhat imprecise and disorganized, but this can be made into a solid standard.	The "...by packing it with unit cubes..." in the standard is not an instructional/curricular directive but rather indicates how to build the understanding conceptually to support the learning progression. Edit made to remove "a" from "in a real world context".	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formula *V = B·h* , where in this case, *B* is the area of the base (*B = l x w*) to find volumes of right rectangular prisms with fractional edge lengths in mathematical problems and problems in real-world context.
6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in a real-world context.	I support this adoption.		Edit made to remove "a" from "in a real world context".	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques to solve mathematical problems and problems in ~~a~~ real-world context.
6.G.A.4.	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in a real-world context.	Surface is an extremely hard concept for 6th graders. To have to apply it would send them over the edge or do algebra with would be tramatic **I support the adoption of this standard.		Edit made to remove "a" from "in a real world context".	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques to solve mathematical problems and problems in ~~a~~ real-world context.
Statistics and Probability (SP)			**Abercrombie-**The standards in this domain are very well written – they are clear, measurable, demonstrate a logical progression of knowledge in terms of breadth and depth, and are easily interpreted. Moving 8.SP.B.1 from 7th grade to 8th grade enhances the knowledge progression across grades. The standards are developmentally appropriate.		
6.SP.A	**Develop understanding of statistical variability.**				
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers.	Throughout this draft the term "recognized" has been changed to "understand" with the rationale that recognize cannot be measured but understand can...now recognize is used? Be consistent. I do not agree anyway as I think you can measure if someone can recognize something but not if they understand it ;) **I support the adoption of this standard.	**Wurman**-Examples are helpful and shouldn't have been removed. **Milgram**-This is horribly misstated and almost certainly represents a serious misunderstanding of the subject.	Based on technical review, example was restored.	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for variability in the answers. ***For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.***
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	I support the adoption of this standard.	**Milner**-In 6.SP.A.2 "which" should rather be "that" (better English usage). Moreover, "can be described by its center, spread, and overall shape" is very problematic, indeed false. The standard should read "Understand that a set of data collected to answer a statistical question has a distribution whose general (or overall) characteristics can be described by its center, spread, and overall shape."	Based on Milner's comment, edits were made.	Understand that a set of data collected to answer a statistical question has a distribution **whose general characteristics** ~~which~~ can be described by its center, spread, and overall shape.

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6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a variation measurement uses a single number to describe the spread of the data set.	Let's make sure we are using ratios that 6th graders understand. They do not understand gas mileage. **Which units of center and spread methods, be specific **I support the adoption of this standard. **Throughout this document the term "recognize" has been replaced with "understand" stating the rationale that understand is measurable and recognize is not. Be consistent. I disagree...you can tell if someone recognizes something but not if they understand it.	**Milner-**In 6.SP.A.3 the word “variation” is used as a synomym for “spread” in 6.SP.A.2. In 6.SP.B.5 yet a third word, “variability” is introduced. Why are three different terms being used for one concept? **Wurman**-"Variation measurement" is an odd way to replace "measure of variation." Measurement implies direct measurement, while measure can be a derived value. Variation is derived. **Milgram-**For a general distribution the usual situation would be that, to describe it will require at least an infinite number of invariants of the distribution.	Based on technical review, edits were made.	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a ~~**variation measurement**~~ **measure of variation** uses a single number to describe the spread of the data set.
6.SP.B	**Summarize and describe distributions.**				
6.SP.B.4	Display and interpret numerical data in plots on a number line including dot plots, histograms, and box plots.	IQR is tough. Easy to figure out, but has little meaning to 6th graders.	**Achieve**-AZ increased rigor by adding the requirement to interpret.		Display and interpret numerical data by creating ~~in~~ plots on a number line including histograms, dot plots, and box plots.
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	Measures of variability should not be taught at 6th grade. They struggle to make real world connections to the concept. I support the adoption of this standard without the measures of variability. **Mean absolute deviation is nothing that a 6th grader needs to know. They have no idea what it is and why it is useful. It is beyond them in SO many ways. Save it for older grades/classes that focus on stats. **Use the same terminology--is it spread or variability; is it center or shape; thanks for taking out range and mode; Median Absolute Deviation is what the real world uses.		Based on the GAISE report and the summary of the CBMS Survey on statistical education, in order for students to understand and use measures of center they need an underlying understanding of variability; therefore, the learning progression requires that we begin with variability to build the concept of measures of center.	Summarize numerical data sets in relation to their context~~, such as~~ by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
SMP	**Standards for Mathematical Practice**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade’s content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.	Grade level specific examples will be included in support documents for each standards for mathematical practice.	
6.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

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6.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				
6.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.				
6.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
6.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				

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6.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
6.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				
6.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

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7th grade Mathematics Standards					
Ratio and Proportion (RP)			**Carlson** -These standards are coherent and logical. **Abercrombie**-These standards are clear, measurable, and developmentally appropriate. The inclusion of the limits in standard 7.RP.A.3 are appropriate and useful. No suggestions for refinements were identified. The standards are written so that they will be unambiguously interpreted across the state. **Milner**-The fundamental concepts are introduced in the wrong order. Rates need to be defined AFTER proportional relationships. This fact becomes crystal clear in 7.RP.A.2b.	Reason for no change (Milner comment) The 6-7 Ratios and Proportional Relationships progression (2011) document states, "Rates are at the heart of understanding the structure ... providing a foundation for learning about proportional relationships in seventh grade." (pg. 5)	
7.RP.A	**Analyze proportional relationships and use them to solve mathematical problems and problems in a real-world context.**				**Analyze proportional relationships and use them to solve mathematical problems and problems in real-world context.**
7.RP.A.1	Compute unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like and different units.		**Milgram**-Actually, this version is basically incoherent. The original version of this standard is FAR BETTER. Leave it the way it was! **Achieve-**Complex fractions are used as examples in the CCSS rather than included in the wording of the standard. AZ specifically calls out complex fractions in this standard's requirements and removes the CCSS examples. This is a nice modification that makes the distinction from 6.RP more clear. **Wurman**-Yet again, the examples are clarifying and should be retained. The "or" in "like or different units" was incorrectly replaced by "and." Do we really expect only problems that have BOTH like AND different units?	Based on Wurman's comment an edit was made to change "and" to "or". Exmaples will be included in the suppport documents.	Compute unit rates associated with ratios involving both simple and complex fractions, including ratios of quantities measured in like **or** different units.

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.		**Milgram**-Again, the original version of this entire standard (including parts (a), (b), (c) and (d)) is far better than this one. [This is particularly the case for part (c).] **Wurman**-The examples in sub-standard (a) clarify the standard and should be restored.	Based on Milgram and Wurman's comments, the examples were restored.	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship **(e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).** b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. ***For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*** d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems. (Limited to: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.)		**Milner**-In 7.RP.A.3 the examples were meant to indicate desirable applications, not comprehensive limits. Why preclude the ratio of legs to people, for example? **Milgram**-Reasonable standard. **Achieve**-AZ changed the CCSS examples to limitations. By making this change, AZ appears to exclude other applications of the standard. For an example, see the activity provided by Illustrative Mathematics for 7.RP.3 (https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/102), which no longer matches the AZ standard. **Wurman**-Retain the examples as examples otherwise, for example, time & distance problems will be forbidden!	Based on Milner, Achieve, and Wurman's comments, edits were made.	Use proportional relationships to solve multi-step ratio and percent problems **(e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error**).

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
<u>**The Number System (NS)**</u>			**Carlson** -These standards are coherent and logical. **Abercrombie**-These standards are clear, measurable, and contain sufficient breadth and depth. The vertical alignment of these standards is excellent, and the refinements made to the standards are useful. The standards are written so that they will be unambiguously interpreted across the state.		
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**		Milner- should begin “Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide any rational numbers, except division by zero.” “Apply and extend previous understandings of addition and subtraction to add and subtract any rational numbers.”	Edits made based on Milners comment.	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide any rational numbers **except division by zero.**
7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.		**Milgram-**I'm afraid I have no idea of what a horizontal or vertical number line diagram might be. Please clarify, or use more standard terms d. Apply properties of operations as strategies to add and subtract rational numbers. This needs to be clarified. Perhaps an example or two might help. **Achieve**-AZ specifically calls out integers as part of the set of rational numbers. This clarification helps teachers spot the progression of integers in the standards. **Wurman**-- The addition of "integers" to "rational numbers" is wrong-headed, dumbing down the standard by focusing on integers. - Restore the clarify examples to (a). - (b) could use an example to clarify the "interpret sums of rational numbers by describing real-world context." Surely this clause can't simply mean to use world problem with addition of integers, decimals, and fractions: this was already previously addressed in grades 4,5, and 6. **Milner-**7.NS.A.1 should begin “Apply and extend previous understandings of addition and subtraction to add and subtract any rational numbers.”	Milner's comment is addressed in the standard cluster heading. Examples will be included in the supporting document. . Apply and extend previous understandings of addition and subtraction- deleted from stem of standard since this is a repeat of the cluster heading.	~~Apply and extend previous understandings of addition and subtraction to~~ Add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world context. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world context. d. Apply properties of operations as strategies to add and subtract rational numbers.

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7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide integers and other rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.		**Milgram**-Terminating in 0's is a SPECIAL CASE of "eventually repeats." I would strongly suggest rephrasing (d) the part of (d) that starts "know that the decimal form of a rational ..." as follows: "know that the decimal form of a rational number eventually repeats. For example, a rational number of the form m (a whole number) divided by a power of ten eventually has 0 as its repeating term. Similarly m/3 has either 3 or 6 as it's repeating term as long as m is not divisible by 3." **Wurman**-- The added integers in "integers and rational numbers" should be removed, unless the purpose is to dumb down the standard by focus on integers; - sub-standard (a) should be clarified. It is unclear how the multiplication and division operations can be "extended" from fractions to rational numbers when fractions ARE rational numbers; - sub-standards (d) should add "form" in "Convert a rational number to decimal form using ..." Decimals are not some new kind of numbers.	Based on Wurman's feedback an edit was made. Apply and extend previous understandings of addition and subtraction- deleted from stem of standard since this is a repeat of the cluster heading.	~~**Apply and extend previous understandings of multiplication and division and of fractions to**~~ Multiply and divide integers and other rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world context. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world context. c. Apply properties of operations as
7.NS.A.3	Solve mathematical problems and problems in a real-world context involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)		**Milgram**-Probably would be worthwhile to remind readers that complex fractions are fractions of the form (a/b)/(c/d), with a, b, c, and d all integers with b, c, and d, non-zero. The term "complex fraction" is not necessarily "standard." **Wurman**-A minor comment: carrying low-level explanations in the name of consistency to higher grades seems uncalled for. "Four operations with rationals" is perfectly clear at this grade. **Milner-**7.NS.A.2 should begin "Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide any rational numbers, except division by zero."	Based on Milgram's comment, edits were made. Milner's comment is addressed in the standard cluster heading.	Solve mathematical problems and problems in real-world context involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions where $a/b \div c/d$ when $a,b,c,$ and d are all integers and b,c, and $d \neq 0$.

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Expressions and Equations (EE)			**Carlson**- In the high school standards (at least the current ones) it explicitly discusses evaluating as linked to using an input value to determine the corresponding output value, and solving an equation as using an output value to determine the corresponding input value. I argue that this way of thinking and terminology should be used in grades 6-8 as appropriate both because they are extremely powerful ways to think about the processes by also because it opens up multiple solution paths and methods for checking the reasonableness of solutions. **Abercrombie**-In terms of developmental appropriateness, the 6th grade standards in this domain are likely to be quite challenging for 6th graders since students at this age are just beginning to be able to think representationally and abstractly, requirements for understanding algebraic expression and equation. Providing some limit to the complexity of the algebraic expressions would make these standards more developmentally appropriate, such as limiting the number of variables in an expression, or the types of operations included in the expressions. That said, the standards are written clearly and are measurable, and interpretation of these standards should be unambiguous across the state.		
7.EE.A	**Use properties of operations to generate equivalent expressions.**				
7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.				
7.EE.A.2	Rewrite an expression in different forms in a problem context and understand the connection between the structures of the different forms.	I like the like the greater DOK and clarification of expectation.	**Milgram**-My view is that the original version with it's example is much clearer than this revision. **Achieve-**The CCSS standard addresses how rewriting an expression can help make better sense of the relationships between quantities in a problem. AZ changed the meaning of this CCSS by requiring rewriting and leaving out how the new version of an expression can "shed light on the problem." They have made the rewriting more about the expressions themselves, rather than the context. This modification also lacks clarity. It is not clear what is meant by "the connection between the structures of different forms." **Wurman**-The removal of the example made this standard even more opaque than it originally was. Presumably that was not the purpose. **Milner**-In the proposed 7.EE.A.2, the wording should be changed to “Rewrite an expression in a problem context in different forms and understand the connection between the structures of the different forms and its meaning in the particular context.”	Based on technical review, edits were made and the example was restored for clarity.	Rewrite an expression **in different forms, and understand the relationship between the different forms and their meanings in a problem context.** ***For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."***

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.EE.B	**Solve mathematical problems and problems in a real-world context using numerical and algebraic expressions and equations.**				**Solve mathematical problems and problems in real-world context using numerical and algebraic expressions and equations.**
7.EE.B.3	Solve multi-step mathematical problems and problems in a real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate and assess the reasonableness of answers using mental computation and estimation strategies.		**Milgram-.**I would delete the last phrase in the second sentence "using mental computation and estimation stragegies." Also, I strongly suggest that you PUT BACK THE FIRST EXAMPLE, which for correctness and precision should be revised as follows: "For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50per hour." **Achieve**-AZ removed the CCSS example, the requirements for specific number sets required, the strategic use of tools (addressed in MP.5), and the application of the properties of operations (addressed in 7.NS). These two deletions have removed references to the Practices and to making connections across the domains. Both are important to making sure teachers do not lose sight of the importance of both. **Wurman**-The rewrite is reasonable, yet how will students learn conversion among different forms of rationals if standard AZ.6.NS.C.9 was suggested for removal in the previous grade??	Based on technical review, edits were made and the example restored for clarity.	Solve multi-step mathematical problems and problems in real-world context posed with positive and negative rational numbers in any form. Convert between forms as appropriate and assess the reasonableness of answers. ~~**using mental computation and estimation strategies.**~~***For example, If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 per hour.***

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.EE.B.4	Use variables to represent quantities in mathematical problems and problems in a real-world context, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px+q>r or px+q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.		**Milgram**-Use variables to represent quantities inmathematical problems and problems in a real-world context, and construct simple equations and inequalities to solve problems by reasoning about the quantities.I would strongly suggest removing the last phrase “by reasoning about the quantities.” This is both difficult to test and mostly instructions for pedagogy, cutting down on the teachers best judgment .I would strongly suggest putting back the example. **Wurman-**Rewrite is more or less fine language-wise, but I suggest to remove the "specific" from "specific rational numbers." This gives the impression that they must be some specific numbers, while the intent here is simply to say they can be any rational numbers. Or simply call them "rational constants." **Milner**-In the proposed 7.EE.B.4b, non-strict inequalities should be included for consistency with 6.EE.B.8.	Based on technical review, edits were made. Examples will be included in a support documents.	Use variables to represent quantities in mathematical problems and problems in real-world context, and construct simple equations and inequalities to solve problems ~~**by reasoning about the quantities**~~. a. Solve word problems leading to equations of the form *px+q=r* and *p(x+q)=r,* where *p, q,* and *r* are specific rational numbers. Solve equations of these forms **fluently.** Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. b. Solve word problems leading to inequalities of the form px+q>r or px+q < r, where p, q, and r are ~~**specific**~~ rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
Geometry (G)			**Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.		
7.G.A	**Draw, construct, and describe geometrical figures and describe the relationships between them.**				**Draw, construct, and describe geometrical figures, and describe the relationships between them.**
7.G.A.1	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.		**Milgram-**The major question I have here is HOW CAN YOU POSSIBLY TEST THIS STANDARD. But there are other issues as well. First, the condition for the existence of a triangle with given side lengths a, b, and c, is that the sum of any two of them is GREATER THAN OR EQUAL to the remaining length, and I do not recall that this condition has been mentioned in the standards list to this point. (I believe it first appears in the high school geometry standards.). And there are similar issues the other cases as well. In short there appears to be no background for this standard. In fact, I think the best thing you can do here is to delete the entire standard.		Draw ~~(freehand, with ruler and protractor, and with technology)~~ geometric shapes with given conditions **using a variety of methods.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.		**Milgram**-Again, there is a huge issue with testing this. And there are many possibilities for the resulting figures. If you insist on keeping this standard I strongly suggest that you limit it using examples of what is to be expected and tested. **Milner-**7.G.A.3 contains examples that should be removed for consistency with many other standards in which examples were removed. Examples are not included within a standard unless an example would provide limits to the standard or clarification to the standard, which here they do not.	Due to limited instructional resources, multiple examples will be included in support documents.	Describe the two-dimensional figures that result from slicing three-dimensional figures. ~~as in plane sections of right rectangular prisms and right rectangular pyramids.~~
7.G.B	**Solve mathematical problems and problems in a real-world context involving angle measure, area, surface area, and volume.**				**Solve mathematical problems and problems in real-world context involving angle measure, area, surface area, and volume.**
7.G.B.4	Understand and use the formulas for the area and circumference of a circle **to** solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Will they be given a formula sheet? They no longer need to "know" the formulas, just understand them...We think they actually need to both know and understand	**Milgram**-How are you going to test this "informal derivation?" I would strongly suggest that the second phrase starting "give an informal ..." be deleted, and that at least one example of the kind of problem that you expect to be solved be included. **Achieve-**AZ changed "know" to "understand and use." This wording has the same meaning and intent, but rigor is increased.	Based on Milgram's comment, examples will be included in the support documents. As stated in the introduction, the formula should be developed from a foundation of conceptual understanding and formula mastery should include this understanding as well as use of the formula in specified applied problems.	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.		**Milgram-**There are a vast number of problems that could be constructed to test this standard ranging from roughly fourth grade difficulties all the way up to advanced college geometry courses. I would strongly suggest examples to show what kinds of facts are expected and how difficult the problems are to be. **Milner-**7.G.B.5 should have "a multi-step problem" in plural. A good refinement to add here would be the non-commutativity of transformations. For example: "Understand that two plane transformations of a figure may produce different resuts when applied in different order."	Based on Milner's comment, edits were made.	Use facts about supplementary, complementary, vertical, and adjacent angles in ~~a~~ multi-step problem**s** to write and solve simple equations for an unknown angle in a figure.
7.G.B.6	Solve mathematical problems and problems in a real-world context involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.		**Milgram-**Same comments as for the above standard, 7.G.B.5.	Based on Milgram's comment, examples may be included in the supporting document.	Solve mathematical problems and problems in ~~a~~ real-world context involving area~~, volume and surface area~~ of two~~- and three-~~dimensional objects composed of triangles, quadrilaterals, **and other** polygons. ~~cubes, and right prisms~~. Solve mathematical problems and problems in real-world context involving volume and surface area of three-dimensional objects composed of cubes and right prisms.
<u>Statistics and Probability (SP)</u>			**Abercrombie-**The standards in this domain are very well written – they are clear, measurable, demonstrate a logical progression of knowledge in terms of breadth and depth, and are easily interpreted. Moving 8.SP.B.1 from 7th grade to 8th grade enhances the knowledge progression across grades. The standards are developmentally appropriate.		
7.SP.A	**Use random sampling to draw inferences about a population.**				
7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.		**Milgram**-Vague. As written too general to test.	The workgroup determined no edits are needed.	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.		**Milgram**-It would be extremely helpful to give examples to limit this standard and show what is expected. **Wurman**-Yet again, good clarifying examples are removed.	Based on technical review, example was restored.	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. ***For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.***
7.SP.B	**Draw informal comparative inferences about two populations.**				
7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (mean absolute deviation).	I like the clarification	**Milgram**-And how do you intend to test this standard? Give examples or delete. **Achieve**-In the CCSS, the mean absolute deviation is used in the example as a way of demonstrating the difference between centers and allows for other measures of variability. AZ's inclusion of "(mean absolute deviation)" at the end of the standard makes it seem that this defines MAD rather than serving to clarify MAD as the measure of variability that is expected here. **Wurman**-Yet again, good clarifying examples are removed.	Based on technical review, example was restored.	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. ~~**(mean absolute deviation).**~~***For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.***

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.		**Milner**-In 7.SP.B.4 the example is important as clarification for the standard; clearly the intention of the standard is not finding the means for each distribution and then mechanically saying that they are different and which is larger. The applications must be meaningful and avoid, for example comparing the mean height of students in a school with the mean annual rainfall in Seattle in the last 100 years. **Milgram**-PUT BACK THE EXAMPLE. **Wurman**-Yet again, good clarifying examples are removed. Teachers already have difficulty with statistics in the middle school. This will surely make them worse.	Based on technical review, example was restored.	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. ***For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book .***
7.SP.C	**Investigate chance processes and develop, use and evaluate probability models.**				
7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.	This standard should stay in the 8th grade with the other probability standards. This standard does not fit with the statistics and doesn't allow for in depth probability exploration when separated from the rest of the probability standards in the 8th grade.	**Carlson**-7.SP.C.5: "Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of an event occurring." Be more specific, since the wording of this standard allows for teachers to specifically teach, and students to develop, fuzzy and unproductive meanings for probability (for example a person saying "there's a 50-50 probability because I don't know what the outcome will be) that are not consistent with the mathematical definitions of probability. Replace this standard with the definition of probability as the long-term relative frequency of an event. **Milner-**In 7.SP.C.5 "Larger numbers indicate greater likelihood" is essential for the expected understanding. In the Notes the word "involved" is misspelled. **Wurman**-Yet again, good clarifying examples are removed. Teachers already have difficulty with statistics in the middle school. This will surely make them worse.	Probability and statistics are interrelated and probability is a natural outgrowth of statistical analysis which is why they occur within one domain. The 6th grade SP standards lay the foundation of statistical analyses which will lead to foundational probability concepts in 7th grade that culminate in more complex probability concepts in 8th grade and high school (a learning trajectory). Based on technical review, edits were made. Based on Wurman's comments,e xamples will be provided in the support documents.	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. **Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.**

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	This standard should stay in the 8th grade with the other probability standards. This standard does not fit with the statistics and doesn't allow for in depth probability exploration when separated from the rest of the probability standards in the 8th grade.	**Wurman**-Yet again, good clarifying examples are removed. Teachers already have difficulty with statistics in the middle school. This will surely make them worse.	Based on Wurman's comments example was restored.	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. ***For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.***
7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	This standard should stay in the 8th grade with the other probability standards. This standard does not fit with the statistics and doesn't allow for in depth probability exploration when separated from the rest of the probability standards in the 8th grade.	**Wurman**-Yet again, good clarifying examples are removed. Teachers already have difficulty with statistics in the middle school. This will surely make them worse.	In the 2010 standards, all middle grades probability standards were in 7th grade. Probability and statistics are interrelated and probability is a natural outgrowth of statistical analysis which is why they occur within one domain. The 6th grade SP standards lay the foundation of statistical analyses which will lead to foundational probability concepts in 7th grade that culminate in more complex probability concepts in 8th grade and high school (a learning progression). Based on technical feedback, examples were restored.	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. ***For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*** b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. ***For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?***

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SMP	**Standards for Mathematical Practices**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.	Grade level specific examples will be included in support documents for each standards for mathematical practice.	
7.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				
7.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
7.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
7.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
7.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				
7.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
8th grade Mathematics Standards					
Number Systems (NS)			**Carlson** -These standards are coherent and logical. **Abercrombie-**These standards are clear, measurable, and contain sufficient breadth and depth. The vertical alignment of these standards is excellent, and the refinements made to the standards are useful. The standards are written so that they will be unambiguously interpreted across the state. **Milner**-A good refinement of this domain would be the addition of a standard about the density of rational and of irrational numbers among the reals. For example, "8.NS.A.3: Understand that given any two distinct rational numbers, $a < b$ say, there exist a rational number c and and irrational number d such that $a < c < b$ and $a < d < b$. Similarly, given any two distinct irrational numbers, $a < b$ say, there exist a rational number c and and irrational number d such that $a < c < b$ and $a < d < b$."	Based on Milner's comment, this a new standard added to the progression. Examples will be included in support documents.- 8.NS.A.3	
8.NS.A	**Understand that there are numbers that are not rational, and approximate them by rational numbers.**				Understand that there are **irrational numbers**, and approximate them using rational numbers.
8.NS.A.1	Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.		**Milgram**-There are at least two very important topics and standards being confused here. First, "every number has a decimal expansion," second, "rational numbers are those with decimal expansions that terminate in zeros or eventually repeat." Also, the statements themselves are terribly imprecise. The correct second statement is "rational numbers are exactly those numbers with decimal expansions that EVENTUALLY repeat. Typically, one can have a number like 472.5791333333... that start repeating only after a certain point. These are the rationals. Also, one needs to understand that DECIMAL EXPANSION also needs to be expanded on. It is far from obvious or even easy. **Achieve**-Finally, there are occasional inconsistencies between the changes in the standards and the changes listed in the technical notes. It is unclear in these instances which information reflects the latest intention of the reviewers. Standard 8.NS.A.1, for example, removed converting decimal expansions, yet that change was not mentioned in the notes. It is not clear if the change is a mistake or intentional. **Milner**-8.NS.A.1 should rather end with "Know that numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called irrational."	Based on technical review comments, edits were made. Milner's comment, "numbers whose decimal expansions do not terminate in zeros or in a repeating sequence of fixed digits are called rational" can potentially be included in the glossary.	**Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.**
8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.		**Milgram**-This standard is not appropriate. "Rational approximation" alone is something that needs a great deal of independent discussion. It is much better simply to leave this to college courses. **Wurman-** Both the original and the rewrite should replace "value of expressions" simply by "their values." For example, it is unclear what is the "expression" to be evaluated in the case of pi by eight graders, or whether square root of an integer is an expression or just a notation. The suggested edit avoids this issue. **Milner**-8.NS.A.2 should end with "and estimate the value of expressions involving irrational numbers."	Based on Wurman's comment, an edit was made.	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate **their values**.
8.NS.A.3				Based on Milner's comment, this a new standard added to the progression. Examples will be included in support documents.	Understand that given any two distinct rational numbers, $a < b$, there exist a rational number c and an irrational number d such that $a < c < b$ and $a < d < b$. Given any two distinct irrational numbers, $a < b$, there exist a rational
Expressions and Equations (EE)			**Carlson**- In the high school standards (at least the current ones) it explicitly discusses evaluating as linked to using an input value to determine the corresponding output value, and solving an equation as using an output value to determine the corresponding input value. I argue that this way of thinking and terminology should be used in grades 6-8 as appropriate both because they are extremely powerful ways to think about the processes by also because it opens up multiple solution paths and methods for checking the reasonableness of solutions. **Abercrombie-**In terms of developmental appropriateness, the 6th grade standards in this domain are likely to be quite challenging for 6th graders since students at this age are just beginning to be able to think representationally and abstractly, requirements for understanding algebraic expression and equation. Providing some limit to the complexity of the algebraic expressions would make these standards more developmentally appropriate, such as limiting the number of variables in an expression, or the types of operations included in the expressions. That said, the standards are written clearly and are measurable, and interpretation of these standards should be unambiguous across the state.	Based on Carlson's comments, examples will be included in support documents that utilize the high school terminology.	
8.EE.A	**Work with radicals and integer exponents.**				
8.EE.A.1	Understand and apply the properties of integer exponents to generate equivalent expressions.	Where is the progressions document please?	**Milner**-In 8.EE.A.1, the adjective numerical is an essential qualifier of expressions because this standard is not intended for variables.	Based on Milner's feedback, the term numerical was added as an adjective.	Understand and apply the properties of integer exponents to generate equivalent **numerical** expressions.

8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. a. Evaluate square roots of perfect squares less than or equal to 225, and rewrite in equivalent form non-perfect squares. b. Evaluate cube roots of perfect cubes less than or equal to 625, and rewrite in equivalent form non-perfect cubes.	Adding rewriting non-perfect squares and cubes will add at least a week of teaching time we do not have since nothing was removed from the standards. **I like the limits on root values. **I like the limits on root values. **I would just like some clarity. Does "rewrite non-perfect squares and cubes" mean to simplify radicals? **This is an important numeracy skill that needs to extend to the high school standards or it needs to be explicitly stated that working with square root and cube root values is securely held knowledge. Something to consider, at what level are students expected to combine irrational numbers (add, subtract, multiply divide). Should this be written explicitly as a standard? **Does "equivalent form" mean simplified or an	**Milgram-**Notation is not MAGIC. It is simply a convenience to avoid continuously using the same words over and over. It is better to state a relevant standard in the form "Demonstrate familiarity with the use of square root and cube root symbols in mathematical expressions." Evaluate square roots of perfect squares less than or equal to 225, and rewrite non-perfect squares in equivalent form.This is something that fourth grade students can easily learn. It is really far too elementary to delay it till grade 8 Evaluate cube roots of perfect cubes less than or equal to 625, and rewrite non-perfect cubes in equivalent form.This is a more appropriate example for this standard at the eighth grade level. **Achieve-**AZ split this CCSS into parts and added limitations on the size of the perfect square and cube roots and removed knowing that sqrt(2) is irrational. The notes claim this is now addressed in 8.NS.1, but that connection is unclear. In addition to other changes, AZ includes "rewrite non-perfect squares in equivalent form" and "rewrite non-perfect cubes in equivalent form." The intention here is not mathematically clear. For example, what are students expected to do with √7? This appears to overlap with A2.N-RN.A.2 and will be a time consuming addition to the standard. The technical notes indicate this is "a foundational concept that is part of the progression to Algebra" but such a claim is far from apparent. **Milner-**In 8.EE.A.2a, the end should read "rewrite square roots of non-perfect squares in equivalent form." The same applied to part b for cube roots.	Based on public comment the original 8.EE.A.2 in the 2010 standards, was unclear. Therefore, limits were included along with an elaboration on the type of roots to both improve clarity and maintain the learning progression. Based on Milner's comments, edits were made.	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. **Know that √2 is irrational.** a. Evaluate square roots of perfect squares less than or equal to 225. ~~**and rewrite in equivalent form non-perfect squares.**~~ b. Evaluate cube roots of perfect cubes less than or equal to **1000.** ~~**625 and rewrite in equivalent form non-perfect cubes.**~~

8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times larger or smaller one is than the other.		**Milgram**-The only issue I have here is the final phrase "express how many times larger or smaller one is than the other." There are many ways of measuring how much bigger one number is than another, even when they are written in scientific notation. One should not imply that the only way of doing this is with the exponent, even though it is the most important. **Wurman**-This standard is problematic and rather meaningless in both the original and the rewritten form. There are very few numbers that can be correctly represented by this standard, as it applies ONLY to single digit integers multiplied by a power of 10. All other numbers are *approximations*, but the word "approximate" is absent here. The best solution seems to fold this standard into the next one (8.EE.A.4) about scientific notation, which should address the issue of approximation and the ease of comparison between numbers in scientific notation.	The workgroup determined that no changes were necessary.	
8.EE.A.4	Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.	This is another skill that should extend into high school. I don't think it is mentioned int eh high school standards, but it comes up in upper level math classes. Students have issues with interpreting scientific numbers reported to them using technical devices. Might be something to mention in the high school document and to cross reference with the science standards.	**Milgram**-Good standard, except, probably for the last sentence, which might be interpreted as requiring that the standard only should be tested with numbers "generated by technology." I would strongly suggest that this last sentence be deleted. **Wurman**-See comment on the previous standard.	Based on Milgram's comments, edits were made.	Perform operations with numbers expressed in scientific notation including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. ~~Interpret scientific notation that has been generated by technology.~~
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**				
8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.		**Milgram**-I strongly disagree with the removal of the example. I would agree that, as written, the example is confusing, but it represents an absolutely key aspect of the standard. I would only suggest that it be made clear that in the example the intent is to use LINEAR equations and understand that the larger the SLOPE, the faster a particle (represented by its' x-coordinate) will be traveling along the line. **Wurman-**The standard speaks only of a single way to represent proportional relationships, so the example is important to clarify what is meant by "different ways."	Based on Milgram and Wurman's comments, the examples were restored.	Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. ***For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed*** .
8.EE.B.6	Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at (0, *b*).		**Milgram-**See my comments for the standard directly above. In many respects, this standard is too subtle for grade 8 unless the grade 8 course is a real course in algebra rather than, as is the case here, a course in pre-algebra. It is meant to tie the GEOMETRIC definition of a straight line to the graph of a linear equation, and is supposed to be a basic application of the standard above. **Achieve-**AZ version includes both coordinates for the intercept. This is an improvement on the CCSS version. **Wurman**-Actually specifying the intercept of "vertical axis at (0,b)" is a tautology and certainly not any more accurate than just "vertical axis at b."	The workgroup determined that no change were necessary.	
8.EE.C	**Analyze and solve linear equations, inequalities, and pairs of simultaneous linear equations.**		**Achieve**-AZ added inequalities to the 8.EE.C cluster. This is a time-consuming change and it is not clear why this needs to happen here rather than in Algebra 1.		
8.EE.C.7	Solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.	Will student's be required to graph linear inequalities as well? Adding inequalities to our standards will require more teaching time we simply do not have! If you are going to add concepts some need to be removed! Accept		In response to public comment: This standard does not require the graphing of inequalities. The inclusion of inequalities strengthens the learning progression to ensure no gap between 6th grade and high school and supports students' understanding of multiple solution situations. Adding "fluently" supports the learning progression.	**Fluently** solve linear equations and inequalities in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solution. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x = a* , *a = a* , or *a = b* results (where *a* and *b* are different numbers). b. Solve linear equations and inequalities with rational number coefficients, including solutions that require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.	https://twitter.com/azedschools/status/764269072376819712 On Aug 12, ADE Tweeted and example where systems of 3 linear equations are part of this standard. Looking at Algebra 1, Algebra 2, and 8th grade, I am not sure where systems of three equations belong. 8th grade is specifically stating pairs of equations. Are systems of three equations disappearing from the standards required for all students in the state of AZ?	**Milgram-**I again disagree with the removal of the examples. This is a very difficult standard for students at the pre-algebra level, and without the examples to limit the standards, the standard will either be interpreted too simplistically or will be too difficult for the typical student to handle. **Wurman-** - Consider adding inequalities as in the previous standard - Consider explicitly mentioning the cases of no solution and infinite number of solutions in (b), as an example of no solution is used in the original. **Milner-**In 8.EE.C.8c, the wording should be "Solve mathematical problems and problems in a real-world context", for consistency.	Based on Milgram's, Milner's, and Wurman's comments, examples will be included in support documents. Based on Wurman's comments, edits were made to the standard.	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations **including cases of no solution and infinite number of solutions.** Solve simple cases by inspection**.** c. Solve **mathematical problems and problems in real-world context** ~~**and mathematical problems**~~ leading to two linear equations in two variables.
Functions (F)			**Carlson**-I am concerned about the absence of covariational reasoning in the standards as a way of thinking about functions (representing the coordination of the values of two co-varying quantities such that a graph emerges as a trace showing constraints in how the quantities change in tandem, formulas as a representation of the restriction on how the values of varying quantities change together). Again, there is a broad body of research demonstrating the importance of covariational reasoning in developing a robust and powerful meaning for functions and representations of function relationships. For example, seeing graphs as emergent while coordinating the values of covarying quantities helps students avoid the common "trap" of seeing graphs as pictures of an event or physical objects (such as a wire). Having students work with dynamic visualizations of events and construct emergent graphical representations by tracking how the values of two quantities change together should be included and emphasized in the standards, not just because it helps students understand graphs in Algebra courses, but because it develops key insights that support a conceptual development of the ideas of Calculus. **Abercrombie**-The standards in this domain are measurable, interpretable, have good vertical and horizontal alignment, and are easily interpreted. I have no suggestions for refinement for the standards in this domain.	Based on Carlson's comments, edits to 8.F.B.4 were made.	
8.F.A	**Define, evaluate, and compare functions.**			s	
8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)		**Milgram**-It should be understood that this IS NOT so much a standard as a DEFINITION. (But, frankly, we were concerned that typical eighth grade teachers have never learned how to deal with definitions.) What we hoped would happen is that the standard would be tested by having students determine whether a graph is the graph of a function or not. (It is not the graph of a function if there is more than one y-value for any particular x-value.)	The workgroup determined no edits were needed. Based on Milgram's comments, specific examples will be included support documents.	
8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).		**Milner**-8.F.A.2 needs clarification by example. Clearly the intention of the standard is not listing for the first function some of its properties and for the second function some of its properties and then mechanically saying which ones are properties of both functions and **which are properties of one but not of the other.** **Milgram**-Again, it is crucial that a limiting example be part of this standard. I would agree that the included example could easily be improved on, but there really should be an **example.** **Wurman**-The example was helpful to clarify that the purpose is to compare similar functions (only linear?) rather than different ones such as linear and exponential. The proposed revision. certainly is wide open to comparing any function to any other function, whatever such comparison may mean (e.g., discrete with continuous, quadratic with exponential). Should be clarified!	Based on technical review, example was restored.	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). ***For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.***
8.F.A.3	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.			No revision necessary	
8.F.B	**Use functions to model relationships between quantities.**				
8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.		**Milgram**-Bad standard. What does it mean to "construct a function" here? Does it mean that the student is given a verbal description of a situation and is asked to model it with an appropriate function? If it means something else, then this should be clarified. And what does the construction of this function have to do with determining the rate of change and or the initial value?	Based on Milgram's feedback, edits were made.	~~**Construct a function**~~ **Given a description of a situation, generate a function to** model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. **Track how the values of the two quantities change together.** Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.

8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing linear and nonlinear graphs. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.		**Wurman-**The example clarified that the standard is after basic relationships such as increase, decrease, or remain flat (constant). It also indicated that relationships such as oscillating, converging, or asymptotic are not expected here. Without an example, anything goes.	Based on Wurman's feedback, the example was restored.	Describe qualitatively the functional relationship between two quantities by analyzing a graph **(e.g., where the function is increasing or decreasing, linear or nonlinear).** Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Geometry (G)			**Carlson**-In the 8th grade standards you should expand the understanding of what is preserved under rigid transformations. Tt should also be noted that rigid transformations map points to points, lines to lines, line segments to line segments, rays to rays, angles to angles, (etc.).” Understanding this property is key to leveraging transformations rigorously to prove theorems (and not just to pay lip-service to the notion that transformations form the foundation of congruence and similarity). For example, justifying the vertical angle theorems using transformations requires that one knows that a 180 degree rotation through any point on the line carries the line back onto itself. By giving a more complete definition of rigid motion it makes it more clear about what transformations would count as a rigid motions (and why others would not) as well as provides a more rigorous and mathematically sound foundation for using transformations in meaningful ways to prove theorems. Otherwise these standards are coherent and follow both a logical progression as well as being placed at grade levels to support understanding in other strands at those levels. **Abercrombie-**In general, the standards are measurable, clear, contain breadth and depth, and are developmentally appropriate. The vertical and horizontal alignment is clear. The focus on real-world application is a strength.		
8.G.A	**Understand congruence and similarity.**				
8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations. Properties include: line segments taken to line segments of the same length, angles taken to angles of the same measure, parallel lines taken to parallel lines.		**Milgram-**.It should be clarified whether there are further properties that might be asked about. My own view is that the three properties (a), (b), and (c) in the original standard are all that should be asked about in questions constructed to represent this standard. **Achieve**-AZ uses a different format for this standard. This wording has the same meaning, intent, and rigor as the CCSS. **Wurman**-The reason for removal of the "lines are taken to lines" is mathematically incoherent and the phrase must be restored. Also restore "taken" in "line segments are taken to line segments ..." either explicitly or by implicit transfer from the previous clause.	In response to technical review, edits were made.	Verify experimentally the properties of rotations, reflections, and translations. Properties include: **lines are taken to lines**, line segments **are taken** to line segments of the same length, angles **are** taken to angles of the same measure, parallel lines **are** taken to parallel lines.
8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.		**Wurman**-- a cleaner way is to speak about "one can be obtained from the other" rather than in terms of "first" and "second," as congruence is reflexive. - should speak about the sequence that demonstrates congruence rather than exhibits congruence.	Based on Wurman's comments, his suggested wording was integrated into the standard.	Understand that a two-dimensional figure is congruent to another **if one can be obtained from the other** ~~the second can be obtained from the first~~ by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that **demonstrates congruence.** ~~exhibits the congruence between them.~~
8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates (rotations and dilations about the origin).		**Achieve**-AZ added the limitation for rotations and dilations. There should be some justification for this limitation, as there doesn't seem to be any profound mathematical or pedagogical reasons for this restriction. (See the Illustrative Mathematics activities related to this standard for possibilities.) **Wurman**-Actually it is not a reasonable limitation. It will force translation to always be the last, implying the order is important. No need to dumb down eight grade content.	In response to technical review, edits were made to exclude the limit.	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates **~~(rotations and dilations about the origin)~~**.
8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.		**Milgram**-It should probably be understood that, in actuality, the material before the semi-colon is A DEFINITION. (Two two-dimensional figures are similar IF AND ONLY IF the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.) Then the rest of the “standard” represents what kind of question will be appropriate to test student understanding. It should be delimited, though, since, as phrased, it would be too easy to construct questions that are far more complex than the standard actually expects in eighth grade. **Wurman-** Same comment as for 8.G.A.2.	In response to technical review, edits were made.	Understand that a two-dimensional figure is similar to another **~~if the second~~ if, and only if, one** can be obtained from the **~~first~~ other** by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that **~~exhibits the similarity between them~~ demonstrates similarity.**
8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*			Based on Milgram's comment, examples may be included in the supporting document.	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.B	**Understand and apply the Pythagorean Theorem.**				

8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.		**Milgram**-Totally ambiguous. This is not a standard in any way, shape or form. There are a number of so called proofs out there that are actually incorrect (for example, the well known tangram proof, which rests on unproved assumptions about area). This so called standard requires, at a minimum examples of the kinds of proofs that are to be explained, and examples of what "satisfactory" explanations of the proofs consist of. **Wurman**-Actually, replacing "explain the proof" by a simple "prove" would go a long way towards clarity. As it is, people incessantly argue whether proof is expected or not.	Based on technical review, edits were made.	~~**Explain a proof of**~~ **Understand** the Pythagorean Theorem and its converse.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two dimensions and in three dimensions in regards to slant height.		**Milgram**-Reasonable standard **Achieve**-The inclusion of the parenthetical statement, "(in regard[s] to slant height)," lacks specificity. Are the three-dimensional applications limited to slant height? **Wurman**-The limit to slant height in three dimensions may be reasonable, but it is a mystery why must it be only in mathematical problems.	Based on technical review, edits were made.	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world **context** and mathematical problems in two **and three** dimensions. ~~**and in three dimensions in regards to slant height**~~.
8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		**Milgram**-Reasonable standard	No revision necessary.	
8.G.C	**Solve real-world and mathematical problems involving volume(s) of cylinders, cones, and spheres.**				
8.G.C.9	8.G.C.9 - Understand and use given formulas for the volumes of cones, cylinders and spheres to solve real-world and mathematical problems.		**Milgram**-Examples are critically needed to limit and illustrate the intent of this standard. **Achieve-**AZ changed "know" to "understand," appearing to increase the rigor for this standard. While "understand" is typically considered a higher demand than "know," this phrasing states that the formulas will be "given," rather than "known" by the student. **Wurman**-Actually "know" is the correct word here, as students will rarely "understand" the volumes of spheres and cones in this grade.	Based on comments from technical review, examples may be included in the supporting document. As stated in the introduction, the formula should be developed from a foundation of conceptual understanding and formula mastery should include this understanding as well as use of the formula in specified applied problems.	Understand ~~**and use given the**~~ formulas for ~~**the**~~ volumes of cones, cylinders and spheres **and use them** to solve real-world **context** and mathematical problems.
Statistics and Probability (SP)			**Abercrombie-**The standards in this domain are very well written – they are clear, measurable, demonstrate a logical progression of knowledge in terms of breadth and depth, and are easily interpreted. Moving 8.SP.B.1 from 7th grade to 8th grade enhances the knowledge progression across grades. The standards are developmentally appropriate.		
8.SP.A	**Investigate patterns of association in bivariate data.**				
8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns of association between two quantities.		**Wurman**-Examples are extremely helpful to clarify this standard and should be restored.	Based on Wurman's feedback, edits were made.	Construct and interpret scatter plots for bivariate measurement data to investigate and describe patterns **such as clustering, outliers, positive or negative association, linear association, and nonlinear association**. ~~of association between two quantities.~~
8.SP.A.2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.			No revision necessary.	
8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.			No revision necessary.	
8.SP.A.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies to describe a possible association between the two variables.		**Wurman**-Relative frequencies of WHAT??	Based on Wurman's feedback, edits were made.	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies **calculated for rows or columns** to describe ~~**a**~~ possible association between the two variables.
8.SP.B	**Investigate chance processes and develop, use, and evaluate probability models.**				

8.SP.B.5	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. Identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.	Why is this being added? We don't have time to teach the standards we have currently and this doesn't even fit in with anything that we already teach. This should remain in 7th grade.	**Achieve**-See the Grade 7 alignment with CCSS 7.SP.8. Note: The coding of the AZ standards differs from that of the CCSS. This may cause problems for teachers who search nationally for materials aligned to 8.SP.1, which does not exist in the CCSS. **Wurman**-The use of "and" in the list of methods in (b) implies an exhaustive list but the clause speaks of "such as." Are no other representation methods allowed?	This strengthens the learning progression to ensure no gap between 7th grade and high school. Based on Wurman's review, edits were made. A crosswalk including coding will be available that links the 2010 coding/standards to the final standards. This will allow teachers to continue to search for aligned resources. 7.SP.C.8 = 8.SP.B.5	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using ~~**methods such as**~~ organized lists, tables, ~~**and**~~ tree diagrams **and other methods**. Identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events.
SMP	**Standards for Mathematical Practices**		**Achieve-**The ADSM revised the language for each of the eight Standards for Mathematical Practice and have helpfully included the practices at each grade level. Positioning the Practices with each grade's content standards shows a commitment to their emphasis and serves as a reminder for teachers to attend to them. Achieve recommends adding grade-specific descriptors for each grade level to tailor the message for different grade levels or bands to make them clearer and more actionable for educators.	Grade level specific examples will be included in support documents for each standards for mathematical practice.	
8.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				
8.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

8.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify				
8.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and				
8.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen				
8.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand				
8.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				
8.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
	High School Algebra 1		**Milner**-When separating the A1 standards from the corresponding A2 standards for exponential functions, in A1 the wording "exponential function with integer exponents" is used (and then reminded in A2). This restricts the functions to be sequences, which is not the intent of this standard. This needs to be fixed in many A1 and A2 standards (every time the issue appears). **Carlson-**I really appreciate the fact that the standards have been broken down by course (A1, G, A2). This is by far the best change in the standards. I am not as convinced about the benefits of stripping out the examples. In fact, I think the standards would benefit from multiple examples for EVERY standard. It seemed as if you found the inclusion of examples restricted the interpretation of the standard to only problem types like the given examples. I can understand that, but removing the examples creates a problem relative to your question G: "Are the standards written with clear student expectations that would be interpreted and implemented consistently across the state?" I am sure that the original purpose of including examples was to help ensure that the standards were interpreted in similar ways by all schools and by those creating the tests. Removing the examples makes it more likely that a variety of interpretations will exist (including those inconsistent with the intentions of the standards authors). Therefore, I recommend including several examples of questions where each standard would apply (at least 3) so that everyone reading the standard understands its purpose in similar ways but also sees the variety of ways in which the standard can be applied so that the examples do not create an overly narrow interpretation. I am concerned about the absence of covariational reasoning in the standards as a way of thinking about graphs (representing the coordination of the values of two co-varying quantities such that a graph emerges as a trace showing constraints in how the quantities change in tandem), as well as functions in general.	Based on Milner's comment, the phrase 'with integer exponents' will be removed from all relevant standards. Based on Carlson's comments, multiple examples will be found in supporting documents. Carlson's recommendations regarding covariation are pedagogical in nature and are best supported through professional development. Conditional Probability is an Algebra II standard (A2.S-CP.B.6) – only independent probability was moved down to Algebra I. Vertical Alignment is maintained by addressing probability standards in 7th grade (7.SP.C.5,6,7), 8th grade (8.SP.B.1), and Algebra I (A1.S-CP.A.2). These standards are assumed to be securely held knowledge by Algebra II.	
			Carlson : There is a wide body of research demonstrating the importance of covariational reasoning in developing a robust and powerful meaning for graphical representations and their connections to other representations (tables and formulas) and in avoiding the common "trap" where students see graphs as pictures of an event or physical objects (such as a wire). Having students work with dynamic visualizations of events and construct		
			Abercrombie-The changes made to the standards in this domain help specify differences between expectations for Algebra 1 and 2. However, moving the conditional probability and rules of probability standards to A1 does not contribute to improvements in horizontal and veritical alignment; in my opinion these standards taught in conjunction with with the conditional probability standards located in A2 would lead to more conceptual coherence, in terms of breadth and depth of content knowledge.		
Number and Quantity (N)					
The Real Number System (N-RN)					
A1.N-RN.B	**Use properties of rational and irrational numbers.**				
A1.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**We touch on this at the beginning of the year then the students never see it again until Algebra 2 when they cover irrational numbers. Should be removed from Algebra 1. **There is a need to state when students should add, subtract, multiply, divide irrational numbers in exact form. I think this standard implies that I need to sum and multiply irrational numbers, but the measurable concept is whether the results are real or irrational. A student can understand this concept and have no skill abilities when it comes to actually manipulating irrational numbers. Is this standard comprehensive enough? **The way the standards for high school were mapped to a course such as Algebra I, Geometry, etc. is a significant improvement in providing clarity for which standards are expected to be mastered by students in these courses. Thank you for providing this level of	**Milgram-**As a test, I'd like to hear the author of this standard show how he expects a student to respond, given the total lack of background surrounding the standard. All the arguments that I know showing these facts involve subtle points that it is very unlikely students would know without more background than is indicated in these standards.	Background knowledge for this standard is included in standards 8.NS.A.1, 8.NS.8.2, and 8.EE.A.2(a) and 8.EE.A.2(b). In these standards, students familiarize themselves with decimal expansions of rational and irrational numbers, allowing them to use this as a basis to address this standard. Examples and progressions will be included in supporting documents.	

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A1.N-Q.A	**Reason quantitatively and use units to solve problems.**				
A1.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Common Core "Use units as a way to understand problems and to guide the solution of multi-step problems" what is meant by "guide" solution?	**Milgram-**Examples are crucial for a standard like this. There are none either in the standard or near it. **Achieve-**While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	Per Milgram's suggestion, examples will be included in the supporting material Per Achieve's suggestion, see the revised standard.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, **include utilizing real-world context**.
A1.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem solving opportunities utilizing a real-world context.	Common Core	**Wurman-**Inclusion or real-world context makes sense here, but why problem solving opportunities? I thought the standards should not be about pedagogy. **Achieve**-The limitations and/or differences for the three required course are not clear in these AZ standards. In this case the "include" statement is redundant with the notion of descriptive modeling. Also, if the "utilizing real-world context" statement is important in AZ, why does Alg 2 not have the same additional statement, identifying it as a modeling standard.Additional note: There appears to be a typo in all of these additional statements to indicate modeling. In the Introduction (p 18) it says, "utilizing a real-world context." In every instance in the standards the "a" is left off.	The standard includes the statement 'utilizing real world context' so it is clear that this is a modeling standard per Page 18 of the introduction. Per Wurman's comment ~ 'problem-solving opportunities' is an open term giving teachers freedom over the types of problems they may use.	Define appropriate quantities for the purpose of descriptive modeling. Include problem-solving opportunities utilizing real-world context.
A1.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Common Core **How is this different from the A2?	**Wurman-** Actually, should be "appropriate to limitation on measurement or need for precision when reporting quantities." There is no limitation on "measuring" pi or significant limitation on measuring distances among celestial bodies, yet nobody would expect 20 digits for pi or more than 2-4 digits of precision for astronomical distances. **Achieve**-The limitations for the three required courses are not clear in these AZ standards.While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context,"	This standard is limited to measurement. Precision is part of the mathematical practices and inherent in the standards. Per Achieve's suggestion, see the revised standard	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities **utilizing real-world context**.
Algebra - (A)			**Milgram**-these Algebra I standards are as procedural as they come. There is virtually no development of knowledge and skills here.		
Seeing Structure in Expressions (A-SSE)					
A1.A-SSE	**Interpret the structure of expressions.**				
A1.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.	**I am a little unclear as to what this standard is referring to. **What do you mean by complicated? If this word is used here, it needs to be defined. Do you mean to say expressions with more than 3 parts? **Common Core	**Milgram**-Way too much material on "expressions" what ever they are supposed to be. We really have no idea since there are way too few examples. **Achieve-**While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	Per Milgram's suggestion, examples will be included in the support material According to the definition of mathematical modeling on page 18 of the introduction, modeling is not necessary for this standard but could be included at a teacher's discretion. Our revision intentionally removed the modeling statement as it is not always applicable. Per the public comments, the word 'complicated' was removed and examples will be included in the support material.	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret ~~**complicated**~~ expressions by viewing one or more of their parts as a single entity. ~~**For example, interpret P(1+r)n as the product of P and a factor not depending on P.**~~

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A1.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. Focus on numerical expressions, such as recognizing 53^2 - 47^2 as a difference of squares and see an opportunity to rewrite it in the form (53+47)(53-47). Focus on polynomial expressions in one variable, such as seeing an opportunity to rewrite a^2 + 9a + 14 as (a+7)(a+2).	You are mixing numeric expressions and algebraic expressions in this standard. Maybe the numeric expressions should be in the domain of Number and then include the structure of irrational numbers and then keep the algebraic expressions here. Common Core	**Achieve-**The two courses demonstrate their differences through the examples.It is not clear in Alg 1 whether "polynomial" expressions would be limited to degree 2. Typically we assume "polynomial" expressions include higher order. This is especially evident in the call to "extend polynomial expressions" in Alg 2.It is not clear if there is a specific reason for using "focus" in the additional notes in Alg 1 and "extend" as a verb in the AZ. Both appear to be instructions for the teacher as opposed to requirements for the student. "Extend" can read as a connector for someone reading from A1 to A2, but it can also mean that students are able to "extend" polynomial expressions. This should be made more clear.	Per Achieve's comment, see the revised standard. Additional examples will be provided in the support materials.	**Use structure to identify ways to rewrite numerical and polynomial expressions. Focus on polynomial multiplication and factoring patterns.**
A1.A-SSE.B	**Write expressions in equivalent forms to solve problems.**		**Milgram-**All these standards focus on very formal properties and there is virtually no discussion of specific student skills at, for example, skill in factoring quadratics with integer roots or where, e.g. the factors have forms like (x – 4)(2x + 3) = 2x^2 -5x – 12 where one root is a whole number but the other is a negative fraction.		
A1.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. Focus on expressions with integer exponents.	Common Core **Remove part b. They can use the formula to find the min/max. They do not have to complete the square. Spend more time on part a. using the quadratic equation to find zeros and then use the zeros to set graph and up factors, Use the zeros to answers real world problems and why one of the answers is not reasonable. **Suggested Rewording: Use the properties of exponents to write equivalent exponential functions expressions with integer exponents.	**Milgram**-This standard talks about one relatively special method for explaining or proving things. It is one among many methods that are sometimes helpful, and far, far from the most important. Yet it appears to be virtually the only method that is developed in these Algebra I standards. For what it is worth, it is probably worth pointing out that virtually all the better methods involve detailed analysis and some actual calculations. But, as I've already pointed out, this discussion of Algebra I is almost entirely procedural, and minimizes skill developments. **Wurman**-The exhortation to focus on integer exponents is uncalled for. Algebra 1 should deal in real exponents rather than repeat itself in future grade without any significant increase in depth. In fact, the corresponding standards A2.N-RN.1-2 should be moved to Algebra 1. **Achieve-**AZ includes more detail about the expectations in Alg 2.While this is an identified modeling standard in the CCSS, the AZ version only includes the phrase, "utilizing a real-world context," in Alg 2. This would be needed in Alg 1, as well, per the ADSM Introduction (see page 18).	According to the definition of mathematical modeling on page 18 of the introduction, modeling is not necessary for this standard but could be included at a teacher's discretion. Our revision intentionally removed the modeling statement as it is not always applicable. The phrase 'Focus on expressions with integer exponents' was removed.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. ~~**Focus on expressions with integer exponents.**~~
Arithmetic with Polynomials and Rational Expressions (A-APR)					
A1.A-APR.A	**Perform arithmetic operations on polynomials.**				
A1.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Common Core		General Comment. No action necessary.	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.A-APR.B	**Understand the relationship between zeros and factors of polynomials.**				
A1.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic and cubic polynomials in which linear and quadratic factors are available.	** like that the standard is clear as to what level the students need to know **Add? Know the fundamental theorem of algebra, namely that If is a polynomial of degree n then will have exactly n zeroes, some of which may repeat. I think that the fundamental theorem of algebra should be stated either in alg 1 or alg 2. Waiting till the plus standards is too late. However, I don't believe that the wording in the plus standards is appropriate here. Common Core	**Milgram**-As was the case with the previous standards I've discussed, this standard talks about the most elementary aspects of the techniques being talked about. The key phrase is "when suitable factorizations are available." There is no indication that students should learn how to FIND factorizations themselves, which is one of the major things standard Algebra I courses usually focus on, but not this one! **Achieve-**AZ provides limitations for Alg 2 that make it appear to be at a lower level than the unlimited Alg 1 requirement. An explanation may be needed to clarify the specific requirements for Alg 1.		
<u>Creating Equations (A-CED)</u>					
A1.A-CED.A	**Create equations that describe numbers or relationships.**				
A1.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing a real-world context. Focus on equations and inequalities that are linear, quadratic, or exponential with integer exponents.	Common Core	**Milgram-**Again, there have to be LIMITING examples to show what kinds of things are expected. But more important, there is no indication that students should develop skill in working with linear and quadratic equations and inequalities, let alone rational and exponential functions (the term "simple" does not begin to really limit this standard). **Wurman**-Real exponents should be allowed! **Achieve-**Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Wurman & Achieve suggestions: see Revised standard	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities that are linear, quadratic, or exponential. ~~**with integer exponents.**~~
A1.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Common Core		General Comment. No action necessary.	
A1.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	**There needs to be a lot more time allocated to solving and graphing linear inequalities before this can be successful for students. **It is not clear how many equations or inequalities should be in the system. In grade 8 pairs are stated and here is is open to interpretation. Please be clear. Provide some content limits here.	**Milgram**-Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. EXAMPLES to limit the standard, and again, I'd want to know what skills students are expected to learn for this.	Per Milgram's suggestion, examples will be included in the support material. The standard intentionally does not limit the number of equations or variables that teachers may choose to use.	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	**Remove this standard. No one rearranges a formula before using one in the real world. People plug in the information they have and solve for what they do not have. Student's time would be better spent solving equations with various missing variables, actually solve of something. **Remove the comma? Reword? Rearrange formulas to highlight a variable of interest. What is the purpose of saying that students are using the same reasoning as in solving equations? Do you mean that students are to explain their reasoning? Isn't that just a mathematical practice?	**Milgram-**Way overblown. Sure, such rearrangements are a helpful technique (among many, many others) but in practical terms and taking account of the usual skills exhibited by 9thgraders, it is far from the most important. So why is it given such a prominent place in these standards?	No action necessary	
<u>**Reasoning with Equations and Inequalities (A-REI)**</u>					
A1.A-REI.A	**Understand solving equations as a process of reasoning and explain the reasoning.**				
A1.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from linear to quadratic equations.		**Milgram**-This is an almost classic example of replacing development of basic skills in solving equations with, as a number of mathematicians would say "pretend algebra." **Wurman-**The term "extend" in "Extend from linear to quadratic equation" is unclear. Instead the standard should simply say "Explain each step in solving simple linear and quadratic equation as following from ..." and remove the last sentence in the proposed language. **Achieve-**It is not clear whether by "extend" AZ intends that both linear and quadratic are required in Alg 1. Then, in Alg 2 it is not clear whether quadratic equations are required or only rational and radical. If the former, how would rational expressions be an "extension" of quadratics? If so, how would that be explained mathematically?	Edits made per Wurman's comments. See the revised standard	Explain each step in solving **linear and quadratic equations** as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. ~~**Extend from linear to quadratic equations.**~~
A1.REI.B	**Solve equations and inequalities in one variable.**				
A1.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.		**Milgram-**This should have been a major standard with lots of sub-standards to enumerate various key cases that students need to learn and develop skills to work with such examples. Then the next standard, A1.A-REI.B.4 should be PART OF the list of substandards. Also, it should be understood that the examples in (a) and (b) below are among the most trivial possible. One should go considerably further than this in any reasonable Algebra I course.	Examples will be included in support materials.	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.A-REI.B.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – k)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Focus on solutions for quadratic equations that have real roots. Include cases that recognize when a quadratic equation has no real solutions.	**Does this mean Algebra I is only responsible for completing the square when a is 1? **Do not use the method of completing the square! Give then the quadratic formula after they used -b/2x. Once the students have mastered the quadratic formula then show them how it is linked to to the line of symmetry formula. Students need to comfortable with a formula before they can explore the whys and whats behind the formula.	**Achieve**-AZ includes limitations for Alg 1 but is consistent with the requirements of the CCSS. Does "focus on solutions..." really mean "limit to solutions..." here?	Examples will be included in the support materials.	
A1.A-REI.C	**Solve systems of equations.**				
A1.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		**Milgram**-Another example of the same sort of thing, replacing the development of basic skills with generalities that aren't nearly as important.	No action necessary	
A1.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing a real-world context.	What just with graphs? Can't students solve approximately using tables? When I look at this standard I want to limit algebra 1 to just systems of two equations but is is still not clear and then I wonder where systems of three equations go. This standard needs to be compared to the grade 8 standard.	**Milgram**-Needs examples to clarify the level of the systems that are to be solved and the methods that might be used. (How many linear equations in how many variables, the types of reduction methods for solving these systems and the discussion of the kinds of things that happen when there are dependency relations among the equation, and so on.) **Achieve-**Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	The standard intentionally does not limit the number of equations or variables that teachers may choose to use.	Solve systems of linear equations exactly and approximately ~~(e.g., with graphs)~~, focusing on pairs of linear equations in two variables. Include problem solving opportunities utilizing real-world context.
A1.A-REI.D	**Represent and solve equations and inequalities graphically.**				
A1.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve, which could be a line.	Still Common Core		General Comment. No action necessary.	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.A-REI.D.11	Explain why the *x* -coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Focus on cases where f(x) and/or g(x) are linear, absolute value, quadratic and, exponential functions with integer exponents.	Still Common Core	**Milgram-**.In all the cases but the linear one with independent equations, there can be multiple solutions. Consequently, there should be detailed discussions of the kinds of examples that would be appropriate. As it stands, there are far too many possible difficulties to simply state the standard without this information. **Wurman-**It is nice that here, at least, absolute functions were finally mentioned. Yet this does not replace the need to mention them also in other standards such as A1.A=CED.1 or A1.A-REI.3 to be taught in context of both equations and inequalities. The "with integer exponents" clause should be eliminated. **Achieve**-AZ added detail to define differences between the two algebra courses. It is not clear whether "focus on" means "limit to" or "include." See earlier comments on using "Extend" in a standard.It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.While this is an identified modeling standard in the CCSS, the AZ Alg 1 version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Wurman & Achieve suggestions: see Revised standard According to the definition of mathematical modeling on page 18 of the introduction, modeling is not necessary for this standard but could be included at a teacher's discretion. Our revision intentionally removed the modeling statement as it is not always applicable. Per Milgram's suggestions, examples will be included in the supporting materials.	Explain why the *x* -coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Focus on cases where f(x) and/or g(x) are linear, absolute value, quadratic, and exponential functions. ~~**with integer exponents.**~~
A1.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary in the case of a strict inequality, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	Not clear to the number of equations in the system Still Common core	**Milgram-**As happens far too often in these standards, this one cuts the situation down to the most elementary cases possible. At least one case where one has three inequalities in two variables with a non-trivial solution region should be included.	The standard intentionally does not limit the number of equations or variables that teachers may choose to use.	
Functions (F)					
Interpreting Functions (F-IF)					
A1.F-IF.A	**Understand the concept of a function and use function notation.**				
A1.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	I think this is going to promote only calculations. This should talk more about a relationship in which two quantities that vary simultaneously such that one quantity uniquely determines another quantity. I think that this will allow students to be able to conceptualize the idea of a function being an infinite collection of points that follow a specific rule. Still Common core	**Milgram-**This is not a standard as it stands. It is a definition. What is needed are examples of the kinds of problems that would be appropriate to test this "standard."	Per Milgram's suggestion, examples will be included in the support material	
A1.F-IF.A.2	Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	Common Core	**Milgram-**Again, there must be examples for this "standard." Virtually all the obvious ones that I am aware of are entirely trivial or far too difficult for ninth grade students. **Achieve-**AZ changed function notation to "function notations." Is the plural intentional? If so, which notations beyond that in A1.F-IF.A.1 are expected?	Per Milgram's suggestion, examples will be included in the support material Per Achieve's suggestion: thanks for the edit!	~~Use function notation, E~~Evaluate a function for inputs in the domain, and interpret statements that use function notation in terms of a context.

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A1.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	**I'd recommend clarification as exponential and logarithmic functions can be defined recursively while linear and polynomial functions being defined recursively is not always useful as those functions are additive. **Sequences need more context. Please explain how recursive sequences would fit into function interpretations. Is there a standard in Algebra 1 on sequences and series? **There needs to be a little more here. This is the first formal introduction to sequences. Either more support needs to be given as to what to teach students here or more scaffolding needs to be given to teachers in the standards. This standard is almost too general. Maybe sequences only belongs in algebra 2?	**Milgram**-Without the example, this becomes essentially meaningless, entirely trivial, or far too difficult for Algebra I. Also, it is worth noting that Algebra I students generally find recursion very, very difficult. I would probably put this standard into Algebra II, which would almost surely be a better course to have recursion in it. **Wurman-**The example definitely promoted clarity and should be restored.	Per Milgram & Wurman's suggestion, examples will be included in the support material Per the public comments, examples/supports will be provided in the support materials.	
A1.F-IF.B	**Interpret functions that arise in applications in terms of the context.**		**Milner-**In A1.F-IF.B there is a contradiction of sense between "intervals where the function is increasing" and "exponential (functions) with integer exponents" since the former require the function to be defined over some interval while the latter precludes that		
A1.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing a real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, absolute value, quadratic, and exponential with integer exponents and piecewise-defined functions. (limited to the aforementioned functions).	Why include problem solving opportunities in this standard. There are other standards that will use this skill that are "real world". Reading this standard is awkward. Still Common Core	**Milgram**-As with other standards here, this standard focuses on the most trivial aspects on one of the deepest areas in most Algebra I courses. Rather than developing any real skills, this standard is satisfied by elementary hand-waving. **Achieve**-AZ adds the requirement to apply functions to real-world contexts and limitations for Alg 1. By adding "include problem solving," AZ makes measurability more difficult.It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.AZ's inclusion of graphs of rational functions seems to require the expectation in F-IF.7d (+). Are graphs of rational functions a part of Alg 2 in AZ? They are not included in the AzMERIT specifications.Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context." See earlier comments on using "focus" and "extend" in a standard.	Per technical review, please see the revised standard.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums. Focus on linear, absolute value, quadratic, ~~and~~ exponential ~~with integer exponents~~ and piecewise-defined functions (limited to the aforementioned functions).
A1.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	Common core	**Wurman**-The example definitely promoted clarity and should be restored. **Achieve**-It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.It seems like there should be an Alg 2 version of this CCSS, since it represents an important skill related to all function types.While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	Per Wurman's comment, examples will be included in support mateirals Per Achieve's comment, this appears to be a duplicate comment from the previous standard	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing a real-world context. Focus on linear, absolute value, quadratic, and exponential with integer exponents.	Common core	**Milner**-Concerning average rate of change, A1.F-IF.B.6 and A2.F-IF.B.6, most teachers and students do not even understand the concept of "change" for a function. It would be a monumental step forward if the concept were specifically mentioned as a standard ("Change of a quantity is a difference between two values of the quantity"). Similarly, introduce the concept of rate of change of two variables that are related to each other and co-vary (vary together) from the words in the name: rate is a ratio (or quotient); if u and v are co-varying variables, the rate of change of u with respect to v as u varies from u1 to u2 and v varies from v1 to v2 is the ratio of their changes, that is (u2 - u1) / (v2 - v1). **Achieve**-AZ includes detail to define differences in Alg 1 and Alg 2.It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Milner's comment, examples will be in the support materirals. Per Achieve's comment, see the revised standard	Calculate and interpret the average rate of change of a **continuous** function (presented symbolically or as a table) ~~over a specified~~ on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Focus on linear, absolute value, quadratic, and exponential functions. ~~**with integer exponents.**~~
A1.F-IF.C	**Analyze functions using different representations.**				
A1.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, exponential with integer exponents, quadratic, and piecewise-defined functions.	I think square root and cube root still need to be included here. Important foundational knowledge for geometry. **is end behavior a key feature? This comment is also for A1.F-IF.B.4	**Milgram**-This is another difficult situation. I agree that examples (c), (d), (e) are inappropriate given the quality of the preceding standards, but (a) and (b) are essential to the standard. Without these limiting examples, the standard could mean anything. **Achieve**-It is not clear why work with exponential functions would be limited to functions with integer exponents in Alg 1. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.	This standard is extended in Algebra II to include square root and cube root functions (A2.F-IF.C.7) and the graphing of these functions is purposefully left out of Algebra I. However, evaluating square roots and cube roots is an 8th grade standard (8.EE.A.2) which is a more direct support for the Geometry standards Per Milgram's comment, examples will be included in the support materials. Per Achieve's comment, see the revised standard Per the public comment, end behavior is an Algebra II concept.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include linear, exponential ~~**with integer exponents**~~, quadratic, and piecewise-defined functions (limited to the aforementioned functions).
A1.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square of a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	Once again, get rid of completing the square. Students do not have to use completing the square to show zeros, symmetry, max/min,or interpret a graph. Students are better off spending more time with less formulas so they really understand and use those formulas with ease. That way when they get to Algebra 2 they remember the formulas from Algebra 1 and are ready for the next formulas (ie. completing the square)	**Wurman-**As already mentioned, allowing exponential functions but limiting the exponents to integers is wrong headed. Sub-standard (b) should be restored.	Sub-standard (b) was moved to Algebra II	

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).Focus on linear, absolute value, quadratic, exponential with integer exponents and piecewise-defined functions (limited to the aforementioned functions).	I think square root and cube root still need to be included here. Important foundational knowledge for geometry. **Get rid of this standard all together or completely re-work how it is taught and tested. The only reason it is a standard is because Arizona was going to give the PARCC test and this was on it. This is a standard, that when teachers run out of time they do not teach. Teachers figure the students can figure it out from what they know already about the different functions. It goes horribly wrong most of the time.	**Achieve-**As mentioned earlier, AZ needs to clarify what "exponential [functions] with integer exponents" means.	Per Achieve's comment, see the revised standard	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).Focus on linear, absolute value, quadratic, exponential ~~**with integer exponents**~~ and piecewise-defined functions (limited to the aforementioned functions).
Building Functions (F-BF)					
A1.F-BF.A	**Build a function that models a relationship between two quantities.**				
A1.F-BF.A.1	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. Focus on linear, absolute value, quadratic, exponential with integer exponents, and piecewise-defined functions (limited to the aforementioned functions)	**The relationship between the quantities is not clear. **Detailed explanation on what the difference is between a explicit expression and a recursive process.	**Milgram**-The "examples" are far too general. Better examples are essential. **Achieve-**Clarification is needed regarding the intent of "exponential [functions] with integer exponents." AZ added detail to define differences between the two algebra courses. This is particularly true for exponential functions.Again, AZ uses "focus" and "extend" as the verbs for specifics in Alg 1 and Alg 2, respectively. These appear to be messages to the teacher as opposed to requirements for the students.While this is a modeling standard in the CCSS, it does not have the AZ connection to modeling in Alg 1. Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Milgram's comment, examples will be given in a support materials Per the public comment, examples will be given in a support materials Per Achieve's comment, see the revised standard	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from **real-world** context. Focus on linear, absolute value, quadratic, exponential ~~**with integer exponents**~~, and piecewise-defined functions (limited to the aforementioned functions).
A1.F-BF.B	**Build new functions from existing functions.**				
A1.F-BF.B.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Focus on linear, absolute value, quadratic, exponential with integer exponents, and piecewise-defined functions (limited to the aforementioned functions).	f(k*x) is a very subtle transformation and difficult to see for even functions. Maybe this transformation should be moved to algebra 2. Students might better understand it after working with function compositions and having good procedural fluency with polynomial operations.	**Milgram-**This standard is again one that focuses on very formal aspects of functions. But unlike most of the previous topics this on is legitimately difficult, and, while far from basic, is very helpful in more advanced areas. In any case, it requires carefully thought out examples to clarify the kinds of questions **that would be appropriate for it in Algebra I.** **Achieve-**AZ provides more detail for the two algebra courses. It is not clear how exponential functions are being handled in the two courses. This needs clarity. See previous comments for more detail.	Per Milgram's comment, examples will be given in a supporting document Per the public comment, see the revised standard, which moves f(k*x) to an Algebra II concept.	Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)* , ~~**f(kx)**~~, and *f(x+k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph ~~using technology~~. Focus on linear, absolute value, quadratic, exponential ~~**with integer exponents**~~, and piecewise-defined functions (limited to the aforementioned functions).
Linear, Quadratic, and Exponential Models (F-LE)					
A1.F-LE.A	**Construct and compare linear, quadratic, and exponential models and solve problems.**				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to	I would say that linear equations have a constant rate of change of y with respect to x. This means that dy/dx=k where k is constant. This definition can be more useful. Likewise, exponential functions are growing by a constant factor such that f(x+1)/f(x)=k where k is constant.	**Milgram**-This standards is all over the place. Some of it is entirely trivial, but much of it is far too advanced for Algebra I. I recommend that it be deleted.		
A1.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input/output pairs.				
A1.F-LE.A.3	Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.		**Milgram**-Without careful preparation, Algebra I students will never be able to handle the mathematics involved. I recommend that the standard be deleted.		
A1.F-LE.B	**Interpret expressions for functions in terms of the situation they model.**				
A1.F-LE.B.5	Interpret the parameters in a linear or exponential function with integer exponents in terms of a context.				Interpret the parameters in a linear or exponential function with integer exponents **utilizing real world context.** ~~in terms of a context~~.
Conceptual Category - Statistics and Probability (S)					
<u>Summarize, represent, and interpret data on a single count or measurement variable. (S-ID)</u>					
A1.S-ID.A	**Summarize, represent, and interpret data on a single count or measurement variable.**		**Milgram**-The remaining standards should all be deleted.		
A1.S-ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).	Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know.	**Achieve**-AZ attempts to add more detail about the intended purpose of data representation in an effort to lift this standard past the middle school version in 6.SP.4. However, clarity is needed in the additional statement. Comparison would be between two sets of data rather than comparing statistics. It would make more sense to say, **"… for the purpose of comparing two or more data sets."**	Per Achieve's comment, see the revised standard	Represent **real-value** data with plots ~~on the real number line (dot plots, histograms, and box plots)~~ **for the purpose of comparing two or more data sets**.

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know. **Does this include students being able to calculate the standard deviation from data sets?		Per public comment, examples will be given in support materials.	
A1.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know. **Are students expected to calculate statistical measures? Which ones?	**Achieve**-The purpose of the AZ addition is not clear here. The placement of the parentheses makes it appear in the AZ version that "data sets" are equivalent to the three types of plots. Also, it is not clear why they have included these three types of representations in this interpretation requirement. Are these the only displays included in the requirement? Why would a visual display be required at all?	Achieve's comment doesn't align to the draft standard - no action necessary	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of ~~extreme data points~~ outliers **if present.**
A1.S-ID.B	**Summarize, represent, and interpret data on two categorical and quantitative variables.**				
A1.S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know.			Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data, including joint, marginal, and conditional relative frequencies. Recognize possible associations and trends in the data.
A1.S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Focus on linear models. b. Informally assess the fit of a function by plotting and analyzing residuals.	**Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know. **Define how much residual knowledge should be known. **Part B: Is this going to be done with statistical software? Also I feel that analyzing residuals should be moved out of Algebra 1 standards. **How appropriate is analyzing and plotting residuals for freshmen? This seems like a higher math level standard.		Per public comment, examples will be given in support materials.	
A1.S-ID.C	**Interpret linear models.**				
A1.S-ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.				Interpret the slope **as a** rate of change and the ~~intercept~~ constant term of a linear model in the context of the data.

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.S-ID.C.8	Compute and interpret the correlation coefficient of a linear fit.	Is the computation done with the use of calculators or statistical software?	**Achieve**-AZ removed "using technology," which indicated the skill is to use the calculator, and that such computations, by hand, are not the intention. The emphasis in the CCSS is to teach the ability to use the calculator as a tool.	Per the Achieve and public comment, the use of technology as a tool is inherent in the standard via Mathematical Practice Standard 5	Compute and interpret the correlation coefficient of a linear ~~fit.~~ relationship.
A1.S-ID.C.9	Distinguish between correlation and causation.	Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know.			
Conditional Probability and the rules of Probability (S-CP)					
A1.S-CP.A	**Understand independence and conditional probability and use them to interpret data.**				
A1.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	This standard should stay in Algebra 2 with the other probability standards. This standard does not allow for in depth probability exploration when separated from the rest of the probability standards in Algebra 2. **Unnecessary in algebra I **teaching subsets and sample space is not a small concept, given the number of standards in Algebra I, I believe these standards should remain in Algebra II **Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know.		Students first see compound probability in 8th grade (8.SP.B.1). Keeping A1.S-CP.A.1 I in Algebra I maintains a progression that builds to the conditional probability concepts in Algebra II	Describe events as subsets of a sample space ~~(the set of outcomes)~~ using characteristics ~~(or categories~~) of the outcomes, or as unions, intersections, or complements of other events. ~~(“or,” “and,” “not”).~~
A1.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	This standard should stay in Algebra 2 with the other probability standards. This standard does not allow for in depth probability exploration when separated from the rest of the probability standards in Algebra 2. **Unnecessary in algebra I **Teaching conditional probability and rules for independent events is a large concept, given the number of standards in Algebra I, I believe these standards should remain in Algebra II. **Only here because it is on some state test. Does not fit with any of the other Algebra standards. We should not be teaching to some test the state decides students should be able to pass. We should be teaching what the need to know.		Students first see compound probability in 8th grade (8.SP.B.1). Keeping A1.S-CP.A.1 I in Algebra I maintains a progression that builds to the conditional probability concepts in Algebra II	**Use the Multiplication Rule for independent events to** understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
SMP	**Standards for Mathematical Practice**				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				
A1.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				
A1.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
A1.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
A1.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
A1.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				
A1.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard 12/2016
High School Geometry			**Carlson-**I applaud the fact that you have retained a focus on transformations as the foundation of congruence and similarity in Geometry. Similar to my comments for Algebra I and Algebra II, I believe that removing the examples was a mistake. I think the standards would benefit from multiple examples for EVERY standard. It seemed as if you found the inclusion of examples restricted the interpretation of the standard to only problem types like the given examples. I can understand that, but removing the examples creates a problem relative to your question G: “Are the standards written with clear student expectations that would be interpreted and implemented consistently across the state?” I am sure that the original purpose of including examples was to help ensure that the standards were interpreted in similar ways by all schools and by those creating the tests. Removing the examples makes it more likely that a variety of interpretations will exist (including those inconsistent with the intentions of the standards authors). Therefore, I recommend including several examples of questions where each standard would apply (at least 3) so that everyone reading the standard understands its purpose in similar ways but also sees the variety of ways in which the standard can be applied so that the examples do not create an overly narrow interpretation. Moving the equations of conic sections to the plus standards was an excellent choice as it does not belong in Geometry, and is also not a necessary learning goal for standard Algebra 2 students.		
			Carlson con't- G.G-CO.B.6: I made a comment about the definition of rigid motion for the glossary section, but I will repeat it here (because it needs to be beefed up), as well as make the case that the definition of rigid motion should be written out in the standard specifying exactly what students should learn about it. A rigid motion is a transformation that maps points to points, lines to lines, line segments to line segments with the same length (and thus preserves the distances between two points and their image points), rays to rays, and angles to angles of the same measure. The definition in the glossary only talks about preserving lengths and angle measures, but without the full definition you lose a lot of the rigor of proofs based on transformation arguments. For example, you can rigorously justify the vertical angle theorem using transformations only if you establish that a 180 degree rotation of a line using any point on the line as the center of rotation maps the line onto itself. Doing this for the two intersecting lines (using the intersection point as the center of rotation) ensures that the vertical angles map onto one another, which means that they have the same measure (since angle measure is preserved). It isn’t quite enough to only say that lengths and angle measures are preserved for a rigorous proof. **Abercrombie-**The instances of rewording in these standards (e.g. G.G.-SRT.B.5) place emphasis on conceptual mathematical thinking required, which is an improvement in the standards. Moving G.G-GPE.A.2&.3 to the plus standards is a sound decision, as this material seems beyond the depth and breadth of the rest of the Geometry standards. All standards are measurable, describe breadth and depth of content, demonstrate horizontal and vertical alignment, and are eaily interpretable.		
Number and Quantity (N)					
Quantities (N-Q)					
G.N-Q.A	**Reason quantitatively and use units to solve problems.**		**Wurman**-These standards don't really belong in Geometry		

Geometry

G.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	How about 1. Use units as a guide to understanding problems, choose and interpret units in formulas, choose and interpret the scale in graphs. **All of this can be incorporated into other standards. These are not stand alone topics. As students learn different formulas, graphs, ect they can be taught how units will help then understand the problem better.	**Milgram**-Examples needed	See revised standard	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, **include utilizing real-world context**.
G.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. Include problem solving opportunities utilizing real-world context.	2. Define appropriate quantities when modeling and include "real world" problems. **This seems very low for high school students. They should already know appropriate quantities. This should be removed. **This standard needs more clarification. **Need a better description of this standard. **Not sure what this means for geometry> From PARCC "For example, in a situation involving periodic phenomena, the student might autonomously that amplitude is a key variable in a situation, and then choose to work with peak amplitude" Need some support for this standard in a geometry setting.	**Milgram-**Examples needed	See revised standard	Define appropriate quantities for the purpose of descriptive modeling. **Include problem-solving opportunities utilizing real-world context.**
G.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**Again, too low for high school students. They should already know reasonableness and limitations when working with quantities. **Is this only for real world situations? For mathematical situations we want exact values.	**Milgram**-Examples needed **Achieve-**The limitations for the three required courses are not clear in these AZ standards.		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities **utilizing real-world context**.
Geometry (G)					
<u>Congruence (G-CO)</u>					
G.G-CO.A	**Experiment with transformations in the plane.**				

Geometry

G.G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	How about 1. Know precise definitions of segment, ray, angle, polygon (triangle, quadrilateral, etc.), circle, perpendicular, and parallel based on the undefined terms of point, line, and plane. **Is distance around a circular arc referring to circumference? **Geometry emphasizes an understanding of the attributes and relationships of geometric objects which can be applied in diverse contexts – interpreting a technical drawing, estimating the amount of wood needed to frame a house, or drawing computer graphics. There are many types of geometry but school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates).		The suggested rewording limits the scope and intent of the standard.	
G.G-CO.A.2	Represent and describe transformations in the plane as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.	How about 2. Describe transformations as functions that take points in the coordinate plane as inputs and give other points in the coordinate plane as outputs. Recognize translations, reflections, and rotations as rigid motions and dilations as non-rigid motions. **Although transformations are important they are not the main focus of Euclidean geometry, but instead are a visual and special method of understanding the theorems and postulates of geometry. I believe Arizona's standards are over emphasizing transformations to the exclusion of virtually other approach.	**Milgram**-As stated, this is far too general. It also, actually, consists of multiple standards. First, there are "geometric transformations" in geometry. These are transformations that preserve length (though defined in this way they are hard to work with, so they are usually defined at the high school level as transformations that preserve length and angles). These transformations preserve congruence. Second, there are transformations that preserve similarity. These are the transformations that preserve area, though, again, this is an advanced definition. In high school they are usually defined as transformations that multiply length by a fixed positive real number and preserve angles. Third, there are transformations that do not preserve either length or area. They are NOT GEOMETRIC, so should not really be discussed much, if at all, in a geometry course. **Wurman**-Clarity? The original attempted (a) instill the sense of representation of transformations using aids such as transparencies or software, and then (b) treat them as functions. The supposedly clearer results simply dropped (a) and just focused on (b). Myopic.	The suggested rewording limits the scope and intent of the standard. Per Milgram and Wurman's comment, examples and guidance will be included in the support material.	
G.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	How about 3. Use reflection and rotation symmetry, to describe the transformations necessary to map a figure onto itself.	**Milgram-**Be clearer about whether trapezoids are general, since the general trapezoid has NO such geometric transformations except the identity, but some non-rectangular trapezoids do have a non-identity symmetry.	The suggested rewording expands the scope beyond the intent of the standard and complicates the measurability of this standard Per Milgram's comment, terms in this standard (especially Trapezoid) will be rigorously defined in the glossary.	
G.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	How about 4. Determine definitions of reflections, rotations, and translations. **Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes, as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.	**Milgram**-"Develop" is not testable without a huge amount of effort.	The suggested rewording is addressed in the standards under 8.GA Per Milgram's comment, assessment takes many forms and we believe this can be assessed in the classroom	

G.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.	How about 5. Given a pair of congruent figures, specify a series of transformations that will map the one onto the other. **Similarity transformations (rigid motions followed by dilations) define similarity, formalizing it as "same shape" and "scale factor". These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent	**Milgram**-Be more precise. There are an uncountable number of "transformations" that will do this for ANY geometric figure. I think it is necessary to replace "transformations" by "geometric transformations." **Wurman-**Like in A.2 above, a truncated mis-interpretation.	The suggested rewording limits the scope and intent of the standard.	Given a geometric figure and a rotation, reflection, or translation draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.
G.G-CO.B	**Understand congruence in terms of rigid motions.**				
G.G-CO.B.6	Use geometric definitions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	How about 6. Given two figures, show that they are congruent by finding a series of rigid motions (reflections, rotations, and/or translations) that will map one onto the other. **Split this into 2 parts; Part 1:Use geometric definitions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; Part 2:given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental to this study are the rigid motions: translations, rotations, reflections, and combinations thereof. All are assumed to preserve distance and angle measure (and therefore shapes).	**Milgram-**.Limit by specifying the types of figures under consideration. **Achieve**-AZ changed "descriptions" to "definitions." This overlaps with G-CO.A.4.	This is an almost identical re-wording of standard 8.GA.2 Per Achieve's comment, this change was intentional to indicate that G-CO.A.4 has students develop their definitions of rigid motions, where this standard has them formalize and apply these definitions. Per Milgram's comment, the standard was purposefully drafted without specific figure limitations	
G.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	**How about 7. Use the definition of congruence in terms of rigid motion to show that two triangles are congruent. Two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. Once the triangle congruence criteria (SSS, SAS, ASA, and AAS) are established, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. In advanced classes the SSA congruence criterion can be studied. Although it	**Migram**-As stated, this is extremely difficult to do correctly. It should be limited by indicating the types of information that students should assume known.	Per Milgram's comment, examples and guidance will be provided in support material	

G.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	8. Use the triangle congruence criteria of SSS, SAS, ASA, and AAS to prove triangles congruent. Limiting 7 and 8 to just transformations in not formalizing geometry. Transformations as a formal method of proof is limited to SSS. To use it for the others requires some method of assuring that the angles have the same measure which cannot be guaranteed by measuring with a protractor. **Need to add all the other ways triangles are similar, for example HL (hypotenuse leg), and AA should be here as well. Do all triangle similarities at the same time, one right after another. ****Where will AAS and HL fit into the standards?**	**Milgram**-As stated, this is extremely difficult to do correctly. It should be limited by indicating the types of information that students should assume known.	Per Milgram's comment, examples and guidance will be provided in support material. Per the public comments, see the revised standard.	Explain how the criteria for triangle congruence (ASA, **AAS,** SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G.G-CO.C	**Prove geometric theorems.**				
G.G-CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Why not list all the appropriate theorems, I don't want to guess which are to be stressed and which are not important. **CCSS high school math standards remove the teaching of geometry proofs. Teaching geometry without proofs is impossible!	**Milgram**-This is a good, well stated standard but it would benefit from a description of the things that students should assume.	The standard explicitly includes the theorems that are to be emphasized - any additional theorems are a curricular decision. Per Milgram's comment, examples and guidance will be provided in support material	
G.G-CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	Same here, list all that are important, don't make people guess. **Break into 2 parts Part 1: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent--that is easy and quick Part 2: the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.--this is more complicated and takes longer for the students to grasp.	**Milgram-**This is a good, well stated standard. But see the comment on the standard directly above.	The standard explicitly includes the theorems that are to be emphasized - any additional theorems are a curricular decision. Per Milgram's comment, examples and guidance will be provided in support material	
G.G-CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and rectangles are parallelograms with congruent diagonals.	Again, list all appropriate theorems.	**Milgram**-see comments directly above. **Achieve**-By removing "conversely," AZ has made the standard more clear.	The standard explicitely includes the theorems that are to be emphasized - any additional theorems are a curricular decision. Per Milgram's comment, examples and guidance will be provided in support material	

G.G-CO.D	**Make geometric constructions.**		**Milner**-In G.G-CO.D it is imperative that geometric constructions are presented also as theorems that need to be proved. For example, the method for bisecting an angle needs a proof before it can be accepted as actually bisecting it.		
G.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods. Constructions include: Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	Make formal geometric constructions with a variety of tools and methods.--This should be done with patty paper or with a computer program. No one in the real world does constructions by hand. The textbook I use has a real world video for constructions and the man in that even says everyone uses a computer program. Only those few that want to write those programs need to know how to do this.	**Milgram**-see comments directly above.	Per Milgram's comment, examples and guidance will be provided in support material	Make formal geometric constructions with a variety of tools and methods. Constructions include: copy~~ing~~ segments; copy~~ing~~ angles; bisect~~ing~~ segments; bisect~~ing~~ angles; construct~~ing~~ perpendicular lines, including the perpendicular bisector of a line segment; and construct~~ing~~ a line parallel to a given line through a point not on the line.
G.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**Spend what too much time teaching how to do this with no why to back it up. Again, only do this if there is a computer program for it. Or better yet remove this standard and let the engineering and drafting (on computer) classes teach this. **Constructing a hexagon inscribed in a circle serves no practical purpose for a sophomore level student. **I feel that constructing a hexagon goes above and beyond what students need to accomplish at the sophomore level. **Need to add "with a variety of tools and methods" to match G.G-CO.D	**Milgram**-see comments above.	Per Milgram's comment, examples and guidance will be provided in support material Per the public comments, see the revised standard	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle; **with a variety of tools and methods**.
Similarity, Right Triangles, and Trigonometry (G-SRT)					
G.G-SRT.A	**Understand similarity in terms of similarity transformations.**				

G.G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	How about 1. Verify experimentally the properties of dilations. A dilated segment is parallel to the original segment. The dilation of a segment is shorter or longer than the segment in the ratio given by the scale factor. Apply center and scale factor to a given geometric figure to create a similar figure.	**Milgram**-What does one mean by "verify experimentally?" Are students expected to use technology? If not, what are they expected to use?	The suggested rewording alters the intent of the standard. The original standard focuses on verifying properties of dilations through experimentation leading to proofs of similar figures, while the suggested rewording narrows the focus to the skill of applying dilations to create similar figures. Per Milgram's comment, 'verify experimentally' is purposefully included throughout the geometry progression. Additional guidance and examples will be provided in the support materials.	
G.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	How about 2. Given a pair of geometric figures in the coordinate plane use similarity transformations to determine whether the figures are or are not similar. Define similar triangles as having corresponding angles congruent and corresponding sides proportional.	**Milgram**-Seems to be a repeat of standards above.	The suggested rewording changes the intent of the standard by narrowing its scope to the coordinate plane and reducing the level of rigor from 'explain' to 'define'.	
G.G-SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	Does this mean that SAS and SSS are not to be taught? **This should be with the other triangle similarity theorems (SAS, HL, SSS ect). Teach them all at once together. Make them all one standard and each similarity statement can be a different sub standard. **This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra.	**Milgram**-Seems to be a repeat of standards above.	Standard was revised to include SAS and SSS similarity as a bridge to trigonometry and to make the expectation explicit.	Use the properties of similarity transformations to establish the AA, **SAS, and SSS** criterion for two triangles to be similar.
G.G-SRT.B	**Prove theorems involving similarity.**				

G.G-SRT.B.4	Prove theorems about triangles. Theorems include: an interior line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	What about 4. An altitude drawn to the hypotenuse of a right triangle divides the triangle into two right triangles which are similar to the original. Use this theorem to prove the Pythagorean Theorem. A line intersecting two sides of a triangle divides the sides proportionally if and only if it is parallel to the third side. An angle bisector of a triangle divides the opposite side into two segments whose lengths are proportional to the lengths of the other two sides. **This standard can not be taught without first teaching geometric proofs. Students need to be taught geometric proofs before they can prove theorems. Without learning geometric proofs, teaching geometry is pointless. The 2010 CCSS eliminated proofs. Failing to put them back into the standards is educational malpractice. **The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions	**Milner**-In G.G-SRT.B.4 the "conversely" should be explained rather than removed: it is meant to require that students learn the proof that if the other two sides are divided proportionally, then the lines are parallel. **Achieve**-AZ's addition makes the standard clearer.	The suggested rewording is describing performance objectives rather than a standard. Per Milner's suggestion, 'conversely' was left intact in the standard.	
G.G-SRT.B.5	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing real-world context.		**Milgram**-Too general as stated. Must have examples.	Per Milgram's comment, examples and guidance will be provided in support material	
G.G-SRT.C	**Define trigonometric ratios and solve problems involving right triangles.**				
G.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		**Milgram**-"Understand" is not something that is easily tested, so this is, at best, a questionable standard. Leaving that aside, though, what properties of similarity are being used? It would seem to me that the SINGLE question here is very algebraic in nature: If A/B = C/D and all four numbers are non-zero, then A/C = B/D, (multiply both sides by B/C). Consequently, I wonder if this is not more appropriate for eighth grade, Algebra I, or Algebra II.	Per Milgram's comment, examples and guidance will be provided in support material	
G.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.	Good standard.	**Milgram**-see comments above.	Per Milgram's comment, examples and guidance will be provided in support material	
G.G-SRT.C.8	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles in applied problems.	**I need some clarity. Does "including inverse trigonometric ratios" mean using the reciprocal trig ratios (sec, csc, and cot) or does it mean using inverse trig functions? It seems you have mixed the two vocabulary terms. **Very appropriate in Geometry. **Thank you for including inverse trig ratios.	**Milgram**-This seems ok, given that you want to include aspects of trigonometry in the first year geometry course. On the other hand, I'm not so sure this is a good idea. **Achieve**-Inverse functions are an Alg 2 topic in AZ, possibly putting this requirement out of order. It is not clear whether the Geometry course typically comes before or after Alg 2 in AZ. While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context" per the ADSM Introduction (see page 18).	Per the public comments, the standards specify the inverse trigonometric ratios specifically, which are separate from the reciprocal trigonometric ratios. Additional examples and guidance will be provided in the support materials. Per Achieve's comment, see the revised standard.	Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles **utilizing real-world context** ~~in applied problems~~.
Circles (G-C)					
G.G-C.A	**Understand and apply theorems about circles.**				

G.G-C.A.1	Prove that all circles are similar.	Not prove--Understand that all circles are similar.	**Milgram**-What assumptions are being made in order to prove this result? One way of doing this is to first CAREFULLY define circles of positive radius r as ALL POINTS IN THE PLANE HAVING A FIXED DISTANCE r FROM A GIVEN POINT c. Then, it can be argued that a translations followed or preceded by a dilation takes any circle to any other. But without such a definition of a circle, this standard is likely to be impossibly difficult for your normal student.	Per Milgram's comment, terms in this standard (especially Circle) will be rigorously defined in the glossary. Examples will be included in support material.	
G.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	Please be specific. The number of relationships here is quite vast. What is emphasized and what is not. **Break into 2 parts Part 1: Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; Part 2: the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	**Milgram**-Needs examples.	The standard explicitly includes the relationships that are to be emphasized - any additional relationships are a curricular decision. Per Milgram's comment, examples and guidance will be provided in the support material.	
G.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Constructing the these circles, is based on the concurrency of bisectors, which are not in the standards.	**Milgram-**Again, what properties are students expected to assume known here?	Constructing bisectors is included in the standards under G-CO.D.12. Per Milgram's comment, examples and guidance will be provided in the support material.	
G.G-C.B	**Find arc lengths and areas of sectors of circles.**				
G.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.	Converting between degrees and radians is an additional concept which is not mandatory. The width of the standards is already significantly broad. This can easily be moved to Algebra II where a much more in depth discussion already occurs. **This standard would be easier to read if you had parts a. b. c. etc There is a lot of stuff here. Stuff that should be here.	**Achieve**-The AZ Geometry standard added conversion between degrees and radians. This makes the standard less focused and also makes it unrelated to the cluster.	Per Achieve's and the public comments, converting radians to degrees is introduced as a geometry standard (as opposed to an Algebra II standard) because the concept of a radian is rooted in the calculation for arc lengths of circles.	
Expressing Geometric Properties with Equations (G-GPE)					
G.G-GPE.A	**Translate between the geometric description and the equation for a conic section.**		**Milgram**-Very difficult standard for students in geometry. Usually done in Algebra II or a more advanced course.		
G.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Completing the square to find the center and radius of a circle given by an equation in standard form is a complex algebra. This along with the study of the equations of parabolas, ellipses and hyperbolas are extremely complex and requires algebra skills above the level of geometry students. Conics have traditionally been a 2nd semester standard in algebra II or pre-calculus.		Completing the square is firmly held knowledge from Algebra I, specifically standard A1.A-REI.B.4.	

G.G-GPE.B	**Use coordinates to prove simple geometric theorems algebraically.**				**Use coordinates to prove ~~simple~~ geometric theorems algebraically.**
G.G-GPE.B.4	Use coordinates to prove or disprove simple geometric theorems algebraically. Theorems Include: proving or disproving geometric figures given specific points in the coordinate plane; proving or disproving if a specific point lies on a given circle.	Wording here should be changed to "prove" instead of "prove and disprove". By definition, a theorem is a theorem because it is proved. It is impossible to disprove a theorem. **Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena	**Milner**-In G.G-GPE.B.4 "proving or disproving geometric figures" is meaningless. Also, "proving or disproving if a specific point lies on a given circle" should read "proving or disproving that a specific point lies on a given circle". **Milgram**-What is the world to the authors mean by "proving or disproving geometric figures?" Can't they manage to be at least reasonably precise? **Achieve**-This standard is mathematically problematic. This standard adds, "disprove simple geometric theorems" and "disproving geometric figures." By the definition a "theorem" cannot be disproved and disproving a figure makes no sense. In the CCSS, the examples ask that a theorem be used to disprove a condition or attribute. This is different from "disproving" the theorem.Also, the AZ version of the CCSS example may be construed to mean that only those two theorems are included. In the AZ final example, "disproving if..." should probably be "disproving that..."	The words 'prove' and 'disprove' are used to define the level of rigor involved in this standard. However, the phrase 'theorem' was changed to 'relationship' to better represent the intent of the standard. Per Technical Review comments, see the revised standard	Use coordinates to **algebraically** prove or disprove ~~simple~~ geometric **relationships** ~~theorems algebraically~~. ~~Theorems~~ **Relationships** include: proving or disproving geometric figures given specific points in the coordinate plane; and proving or disproving if a specific point lies on a given circle.
G.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems, including finding the equation of a line parallel or perpendicular to a given line that passes through a given point		**Milgram-**This is a more reasonable standard for this course and 10th grade geometry students than those above relating to conic sections.		
G.G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		**Milgram**-see comment above		
G.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.		**Milgram**-.No! put back the "e.g."!!!	Per Milgram's comment, examples and guidance will be included in the support materials	
Geometric Measurement and Dimension (G-GMD)					
G.G-GMD.A	**Explain volume formulas and use them to solve problems.**				

G.G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	**How about 1. Give an informal argument for the formulas for the circumference of a circle and area of a circle. Use Cavalieri's Principle to give an informal argument for the volume formulas of a cylinder and prism. Use experimental methods go give an informal argument for the volume formulas of a cone and pyramid. (+) Give an informal argument for the volume of a sphere should be optional. **circumference of a circle, area of a circle--Those are not volume! They should not be under the volume standard. **Use dissection arguments, --what does this even mean? What do we want the student to do with this? **Calalieri's principle is unnecessary. It is easy logic. There is no teaching or learning behind it. It just makes sense. It should not be a standard. Maybe an embedded one at most, and a topic of discussion at least. **Needs more clarification. **I feel that dissection arguments and informal limit arguments go beyond what is necessary at the sophomore level.		The standard has been revised to promote clarity and measurability. References to the area and circumference of a circle were already addressed in 7th grade, specifically 7.G.B.4, and are now securely held knowledge	~~Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.~~ **Analyze and verify the formulas for the volume of a cylinder, pyramid, and cone.**
G.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing a real-world context.	Prisms should be included. Why is surface area not a part of the standards? Great standard.	**Milner-**In G.G-GMD.A.3 the word "context" should be plural. **Achieve-**AZ added their modeling requirement to apply to a real-world context as suggested in the ASDM Introduction for standards that require modeling with mathematics.Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Standards addressing the volume of prisms begin in 6th grade, specifically 6.G.A.2 and continues through the middle school grades. Surface area is addressed in standard 7.G.B.6. Both of these are considered firmly held knowledge in high school	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems utilizing real-world context.
G.G-GMD.B	**Visualize relationships between two-dimensional and three-dimensional objects.**				
G.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Great applicable standard.			
Modeling with Geometry (G-MG)					
G.G-MG.A	**Apply geometric concepts in modeling situations.**				

G.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects	**Get rid of this and add Geometric probability. It goes along with standard G.G-MG.A.3. Applying geometric models to solve design problems. For example: how much space the cardboard tube in a roll of TP takes up? Or playing darts--what is the probability of hitting a certain area? **This standard is too vague. Specific examples would be appreciated. **This standard is too vague. Please define.	**Milgram-**Please, please, please, go back to the original formulation and put back the "e.g." **Achieve**-While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context" per the ADSM Introduction (see page 18).		Use geometric shapes, their measures, and their properties to describe objects **utilizing real-world context.**
G.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations	Density does not go or belong in Geometry. It is here because of a question or two on a state test. Leave it in Science. Add Geometric probability or scale drawing instead.	**Milgram**-see comments above. **Achieve-**While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context" per the ADSM Introduction (see page 18).		Apply concepts of density based on area and volume in modeling situations **utilizing real-world context.**
G.G-MG.A.3	Apply geometric methods to solve design problems		**Milgram**-see comments above. **Achieve-**While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context" per the ADSM Introduction (see page 18).		Apply geometric methods to solve design problems **utilizing real-world context.**
G.SMP	**Standards for Mathematical Practice**				
G.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

G.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				
G.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the				

G.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
G.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
G.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				

G.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				
G.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

Algebra 2

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review - Fall 2016	Workgroup Notes	Redline/Final Mathematics Standard - 12/2016
	Algebra 2 Standards		**Carlson**-I really appreciate the fact that the standards have been broken down by course (A1, G, A2). This is a nice improvement in the standards. I encourage the writers to reconsider using examples to make more the intent of each standard more clear. In fact, I think the standards would benefit from multiple examples for EVERY standard. It seemed as if you found the inclusion of examples restricted the interpretation of the standard to only problem types like the given examples. I can understand that, but removing the examples creates a problem relative to your question G: "Are the standards written with clear student expectations that would be interpreted and implemented consistently across the state?" I am sure that the original purpose of including examples was to help ensure that the standards were interpreted in similar ways by all schools and by those creating the tests. Removing the examples makes it more likely that a variety of interpretations will exist (including those inconsistent with the intentions of the standards authors). Therefore, I recommend including several examples of questions where each standard would apply (at least 3) so that everyone reading the standard understands its purpose in similar ways but also sees the variety of ways in which the standard can be applied so that the examples do not create an overly narrow interpretation.		

			Carlson -In the introduction to the A2 standards you discuss the seeming importance of transformations and want students to draw generalizations about the graphs of all functions affected by the same kind of transformation [related to standard A2.F-BF.B.3]. I think we really miss the boat when we restrict our focus of transformations to graphical representations (and there is no indication in the standards that you intend the exploration to extend beyond graphical representations). Transformations of functions can be a rich area of exploration where a focus on the relative inputs and outputs of functions with a relationship like g(x) = f(x -2) can help students focus on the meaning of arguments, function outputs, domains and ranges, relationships of function values represented in tables, using a transformation to modify a formula if, say, you want to change the units of the input or output quantity, etc. This supports function reasoning, multiple representations, etc., connecting to countless other standards in the course, but almost none of this gets leveraged when the focus is only on graphical representations. In addition, a lot of the research into covariational reasoning demonstrates that students tend to think of graphs like pictures or static wires, and transformations as manipulations of some physical objects as opposed to an emergent model of how two quantities change together, and the research is pretty clear about the relatively dire implications for students with the former view. I highly recommend expanding and revising the transformations standards to explicitly go beyond graphical representations and to make connections to other standards that can be leveraged and supported with this broader scope. I also think that students' common misunderstandings about graphs, including ways of thinking about transformations, can be addressed by supporting covariational reasoning and including its development as a goal within the standards [I am out of space here – see my A1 comments.] I applaud the move to space out the statistics standards. As it was, Algebra 2 was very bloated with standards and the set of statistics standards expected to be taught at that level was just too overwhelming. Moving some to Algebra I and some to plus standards and fourth year courses was a good move. Ideally, I would have liked Arizona to follow CCSS initial recommendations to include most of probability in Geometry instead of the algebra courses (and that is still my first choice and something I think you should consider)		
	Algebra 2 Standards		**Milner**-Algebra 2 (and the glossary) needs the concepts of abscissa and ordinate as first and second coordinates of an ordered pair of numbers. It is disgraceful that high school graduates can only refer to them (wrongly, of course) as "x-coordinates" and "y-coordinates" for points that are frequently plotted on coordinate axes that are not labeled x and y. The A2 standards would improve if the concept of periodic function were introduced before trigonometric functions, since it is in fact unrelated to those. The way it is done students get the very wrong idea that all periodic functions are trigonometric. When separating the A1 standards from the corresponding A2 standards for exponential functions, in A1 the wording "exponential function with integer exponents" is used (and then reminded in A2). This restricts the functions to be sequences, which is not the intent of this standard. This needs to be fixed in many A1 and A2 standards (every time the issue appears). **Abercrombie**-The changes made to these standards help define the differences between A1 and A2. The deletion of the Quantities (N-Q) standards makes sense, as these standards are indeed integrated throughout A1, A2 and Geometry. The standards are measurable, clear, describe breadth and depth of content, and are interpretable.		

Algebra 2

Number and Quantity (N)					
The Real Number System (N-RN)					
A2.N-RN.A	**Extend the properties of exponents to rational exponents.**				
A2.N-RN.A.1	Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	As long as in A1.N students have already simplified radicals.	**Milner**-A2.N-RN.A.1 needs the example that was removed. **Milgram**-As usual, put in at least one example. Far too general as it stands. **Achieve-**The slight change in wording in the AZ causes no significant change in meaning or rigor.	Per Milner and Milgram's comment, examples will be included in supporting documents	
A2.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Good! This is still common core standard.			
A2-N-Q.A	**Reason quantitatively and use units to solve problems.**				
A2-N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**Need to determine the content limits from A1 to A2. Currently it is exactly the same. **Great standard but a bit complex to understand. Rewrite in a more comprehensible way. This is still common core standard.	**Milgram**-NEEDS EXAMPLES to bound the types of questions that will appear on tests of this standard. **Achieve**-While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	See revised standard. Examples will be included in support documents.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays, **include utilizing real-world context**.
A2.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.	Need to include real-world context as well in A2, it is already included in A1.	**Milgram**-The example in the comments column (D) at least should be made part of the "standard." Without bounds the range of questions will be too broad. **Achieve**-The limitations and/or differences for the three required course are not clear in these AZ standards. In this case the "include" statement is redundant with the notion of descriptive modeling. Also, if the "utilizing real-world context" statement is important in AZ, why does Alg 2 not have the same additional statement, identifying it as a modeling standard.	See revised standard	Define appropriate quantities for the purpose of descriptive modeling. **Include problem-solving opportunities utilizing real-world context.**
A2.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**Define limitations and forms of choosing level of accuracy. This is still common core standard. **ok.	**Achieve-**The limitations for the three required courses are not clear in these AZ standards.		Choose a level of accuracy appropriate to limitations on measurement when reporting quantities **utilizing real-world context**.
A2.N-CN.A	**Perform arithmetic operations with complex numbers.**				

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A2.N-CN.A.1	Apply the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Write complex numbers in the form (*a+bi*) with *a* and *b* real.	This is still common core standard.	**Milgram**-Much better as a standard than the original. (This may well be the first time I've seen an actual improvement in this column.)	No action needed	
A2.N-CN.C	**Use complex numbers in polynomial identities and equations.**		**Milner-**With the changes made, N-CN never defined complex numbers. In particular, the relation i2 = –1 makes no sense for students who "know" that squares are never negative.	Definition of complex numbers included in the glossary.	
A2.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.	This is still common core standard.	**Milgram**-There should still be limits. What kinds of coefficients, integer, rational, or real? Use quadratic formula or not?	No action needed	

Algebra 2

Algebra (A)					
Seeing Structure in Expressions (A-SSE)					
A2.A-SSE.A	**Interpret the structure of expressions.**				
A2.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. Extend polynomial expressions to multivariable expressions. Focus on rational or exponential expressions seeing that $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.	This is still common core standard. **I agree with this standard. Fits algebra 2 curriculum and helps students expand their knowledge of expressions. **Please use correct mathematical notation when providing examples of division and/or fractions. There are many software or typesetting options in which to include proper notation such as MathType. It is important to correct mathematical notation.	**Achieve**-The two courses demonstrate their differences through the examples. It is not clear in Alg 1 whether "polynomial" expressions would be limited to degree 2. Typically we assume "polynomial" expressions include higher order. This is especially evident in the call to "extend polynomial expressions" in Alg 2.It is not clear if there is a specific reason for using "focus" in the additional notes in Alg 1 and "extend" as a verb in the AZ. Both appear to be instructions for the teacher as opposed to requirements for the student. "Extend" can read as a connector for someone reading from A1 to A2, but it can also mean that students are able to "extend" polynomial expressions. This should be made more clear	Per Achieve's comment, see the revised standard. Additional examples will be provided in the supporting document.	**Use structure to identify ways to rewrite polynomial and rational expressions. Focus on polynomial operations and factoring patterns.**
A2-A-SSE.B	**Write expressions in equivalent forms to solve problems.**				
A2.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem solving opportunities utilizing a real-world context and focus on expressions with rational exponents. c. Use the properties of exponents to transform expressions for exponential functions.	This is still common core standard.	**Milner**-A good example for A2.A-SSE.B.3 may be e2t -2et + 1 = (et - 1)2, to underscore that previously learned structures and concepts need now to be combined with newly learned ones. **Milgram**-There should be an example here. **Achieve**-AZ includes more detail about the expectations in Alg 2.While this is an identified modeling standard in the CCSS, the AZ version only includes the phrase, "utilizing a real-world context," in Alg 2. This would be needed in Alg 1, as well, per the ADSM Introduction (see page 18).	Examples will be included in support documents	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Include problem-solving opportunities utilizing real-world context and focus on expressions with rational exponents. ~~c.~~Use the properties of exponents to transform expressions for exponential functions.
A2.A-SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.	This is still common core standard. What about sums of arithmetic sequences? Are students expected to use sigma notation? This standard might need some more pieces to it.	**Milgram**-As stated, probably a very poor standard. How do you test in a useful manner for "derive the formula?" But deleting the example was a poor idea. I would make the standard read "Use the quadratic formula to solve problems such as calculating mortgage payments on a fixed rate mortgage."	Examples will be included in support documents Per the public comment: sums of arithmetic sequences are not in the draft standards. The use of sigma notation is a curricular decision, but is not required by the standards.	
Arithmetic with Polynomials and Rational Expressions (A-APR)					
A2.A-APR.B	**Understand the relationship between zeros and factors of polynomials.**				

A2.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a* , the remainder on division by *(x – a)* is *p(a)* , so *p(a) = 0* if and only if *(x – a)* is a factor of *p(x)* .	**This is very unclear. It seems to be very strangely worded. It is talking about the Remainder Theorem and talking about factors of the polynomial, or zeroes of the function. I think this should be clarified. **I find this standard to be appropriate for algebra 2 students. Division with polynomials should be owned at this level.	**Milgram**-It should be understood that this formula is very elementrary.	Per Public Comment, see the revised standard	Know and apply the Remainder **and Factor** Theorem: For a polynomial *p(x)* and a number *a* , the remainder on division by *(x – a)* is *p(a)* , so *p(a) = 0* if and only if *(x – a)* is a factor of *p(x)* .
A2.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Focus on quadratic, cubic, and quartic polynomials including polynomials for which factors are not provided.	Need to clarify if we are looking only at real zeros or complex zeros. Can we name the Fundamental theorem of algebra here? This is still common core standard.	**Achieve**-AZ provides limitations for Alg 2 that make it appear to be at a lower level than the unlimited Alg 1 requirement. An explanation may be needed to clarify the specific requirements for Alg 1.	No action needed	

A2.A-APR.C	**Use polynomial identities to solve problems.**				
A2.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.		**Milner**-In A2.A-APR.C.4 it will be very unclear to teachers and students what is meant by "and use them to describe numerical relationships". **Milgram**-Must have examples to limit it. For example, what about polynomials in more than one variable?	Per Milner and Milgram's comments, examples will be provided in supporting documents	
A2.A-APR.D	**Rewrite rational expressions.**				
A2.A-APR.D.6	Rewrite rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)* , where *a(x)* , *b(x)* , *q(x)* , and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)* , using inspection, long division, or for the more complicated examples, a computer algebra system.	This is still common core standard.	**Milgram-**.The last phrase "using inspection, long … " is pure pedagogy. DELETE IT. Also, put back the word simple. Without it, the standard is way too general.	No action needed	
Creating Equations (CED)					
A2.A-CED.A	**Create equations that describe numbers or relationships.**				
A2.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing a real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions with real exponents.	This is still common core standard.	**Milgram-**Again, way too general. Must have examples. **Achieve-**Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Milgram's comment, examples will be included in supporting documents. Per Achieve, see revised standard	Create equations and inequalities in one variable and use them to solve problems. Include problem-solving opportunities utilizing real-world context. Focus on equations and inequalities arising from linear, quadratic, rational, and exponential functions ~~with real exponents.~~
Reasoning with Equations and Inequalities (REI)					
A2.A-REI.A	**Understand solving equations as a process of reasoning and explain the reasoning.**				

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A2.A-REI.A.1	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Extend from quadratic equations to rational and radical equations.	This is still common core standard.	**Milgram**-Probably the standard as written is far too difficult to test. Needs examples. Or even better, get rid of it. Of special concern is the second part: "extend from quadratic ..." which simply seems incomprehensible to me. **Achieve-**It is not clear whether by "extend" AZ intends that both linear and quadratic are required in Alg 1. Then, in Alg 2 it is not clear whether quadratic equations are required or only rational and radical. If the former, how would rational expressions be an "extension" of quadratics? If so, how would that be explained mathematically?	Per Milgram's comment, examples will be included in supporting documents. Per Achieve, the phrase 'extend' is intentional for vertical alignment to indicate this standard directly connects to an Algebra I standard.	
A2.A-REI.A.2	Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	This is still common core standard.	**Milgram**-You can't be serious! PUT "simple" back, and give a limiting example. For example how large can the degree of the denominator be? **Achieve-**AZ removed "simple" as a limitation on rational and radical equations. They will need to indicate expected limits as these types of equations can be very demanding.	Examples will be included in supporting documents.	
A2-A-REI.B	**Solve equations and inequalities in one variable.**				
A2.A-REI.B.4	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	This standard seems to best fit in algebra 1 where there is more emphasis on one-variable equations. This is still common core standard.	**Milgram**-Again there must be ways to limit this. See the many comments above.	Examples will be included in supporting documents.	**Fluently** solve quadratic equations in one variable. ~~b.~~ Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
A2-A-REI.C	**Solve systems of equations.**				
A2.A-REI.C.7	Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	This is still common core standard. Comparing algebra 1 to algebra 2, systems of three equations seems to be missing and yet ADE tweeted and example showing a system of three equations as an 8th grade example. https://twitter.com/azedschools/status/764269072376819712	**Milgram**-The example is absolutely essential. Put it back!!	Examples will be included in supporting documents.	

A2-A-REI.D	**Represent and solve equations and inequalities graphically.**				
A2.A-REI.D.11	Explain why the *x* -coordinates of the points where the graphs of the equations *y=f(x)* and *y=g(x)* intersect are the solutions of the equation *f(x) =g(x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include problems in a real-world context. Extend from linear, quadratic, exponential with integer exponents to cases where *f(x)* and/or *g(x)* are polynomial, rational, exponential with real exponent, and logarithmic functions.	This is still common core standard.	**Achieve-**AZ added detail to define differences between the two algebra courses. It is not clear whether "focus on" means "limit to" or "include." See earlier comments on using "Extend" in a standard.It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.While this is an identified modeling standard in the CCSS, the AZ Alg 1 version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Achieve's comment, see revised standard	Explain why the *x* -coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include problems in real-world context. Extend from linear, quadratic, **and** exponential **functions** ~~**with integer exponents**~~ to cases where f(x) and/or g(x) are polynomial, rational, exponential ~~**with real exponent**~~, and logarithmic functions.
Functions (F)					
Interpreting Functions (F-IF)					
A2.F-IF.B	**Interpret functions that arise in applications in terms of the context.**				
A2.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing a real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, simple rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.	This is still common core standard.	**Achieve**-AZ adds the requirement to apply functions to real-world contexts and limitations for Alg 1. By adding "include problem solving," AZ makes measurability more difficult.It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.AZ's inclusion of graphs of rational functions seems to require the expectation in F-IF.7d (+). Are graphs of rational functions a part of Alg 2 in AZ? They are not included in the AzMERIT specifications.Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context." See earlier comments on using "focus" and "extend" in a standard	Per Achieve's comment, see revised standard.	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include problem-solving opportunities utilizing real-world context. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Extend from linear, quadratic and exponential ~~**with integer exponents**~~ **functions** to include polynomial, radical, logarithmic, ~~**simple**~~ rational, piecewise-defined, sine, cosine, tangent, and exponential functions ~~**with real exponents**~~.

A2.F-IF.B.6	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing a real-world context. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, tangent, and exponential functions with real exponents.	instead of continuous function, maybe consider a continuous interval. This is still common core standard.	**Milner-**Concerning average rate of change, A1.F-IF.B.6 and A2.F-IF.B.6, most teachers and students do not even understand the concept of "change" for a function. It would be a monumental step forward if the concept were specifically mentioned as a standard ("Change of a quantity is a difference between two values of the quantity"). Similarly, introduce the concept of rate of change of two variables that are related to each other and co-vary (vary together) from the words in the name: rate is a ratio (or quotient); if u and v are co-varying variables, the rate of change of u with respect to v as u varies from u1 to u2 and v varies from v1 to v2 is the ratio of their changes, that is (u2 - u1) / (v2 - v1). **Achieve**-AZ includes detail to define differences in Alg 1 and Alg 2.It is not clear why work with exponential functions in Alg 1 would be limited to functions with integer exponents. Since exponential functions have variable exponents, is it the intent that the computations should only include integer exponents? Or that functions are discretely defined only at integer inputs? This may be an error to be corrected or clarified.Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Milner's comment, additional examples and guidance will be provided in supplemental resources Per Achieve's comment, see the revised standard	Calculate and interpret the average rate of change of a continuous function (presented symbolically or as a table) on a closed interval. Estimate the rate of change from a graph. Include problem-solving opportunities utilizing real-world context. Extend from linear, quadratic and exponential **functions** ~~with integer exponents~~ to include polynomial, radical, logarithmic, rational, sine, cosine, tangent, exponential, and piecewise-defined functions.~~ with real exponents~~.
A2.F-IF.C	**Analyze functions using different representations.**				
A2.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include square root, cube root, polynomial, exponential with real exponents, logarithmic, sine, cosine, tangent and piecewise-defined functions.	This is still common core standard.	**Milgram**-Put back the limitations as described in (a), (b), and (c). **Achieve**-The requirements of AZ Plus overlaps with Alg 2, F-IF.B.4.While this is an identified modeling standard in the CCSS, the AZ version does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	Per Milgram's comment, the standard condensed (b) and (c) into a single standard while removing (a) because it is securely held knowledge from Algebra I. Per Achieve's comment: according to the definition of mathematical modeling on page 18 of the introduction, modeling is not necessary for this standard but could be included at a teacher's discretion. Our revision intentionally removed the modeling statement as it is not always applicable.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Functions include square root, cube root, polynomial, exponential ~~with real exponents~~, logarithmic, sine, cosine, tangent and piecewise-defined functions.
A2.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions and classify those functions as exponential growth or decay.		**Milgram**-PUT BACK EXAMPLES.	Per Milgram's suggestion, examples will be included in the supplemental material.	

A2.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, trigonometric, and exponential functions with real exponents.		Milner-A2.F-IF.C.9 needs clarification by example. Clearly the intention of the standard is not listing for the first function some of its properties and for the second function some of its properties and then mechanically say which ones are properties of both functions and which are properties of one but not of the other. Achieve-As mentioned earlier, AZ needs to clarify what "exponential [functions] with integer exponents" means.	Per Milner's comment, additional examples and guidance will be provided in supplemental resources Per Achieve's comment, see the revised standard	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.). Extend from linear, quadratic and exponential **functions** ~~with integer exponents~~ to include polynomial, radical, logarithmic, rational, piecewise-defined, trigonometric, and exponential functions~~ with real exponents~~.

Building Functions (F-BF)					
A2.F-BF.A	**Build a function that models a relationship between two quantities.**				
A2.F-BF.A.1	Write a function that describes a relationship between two quantities. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and all exponential functions. Include problem-solving opportunities utilizing a real-world context. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations and function composition.	This is still common core standard.	**Milner**-In A2.F-BF.A.1 the expression “standard function types” is used but never defined. It is imperative that the limits of this standard be explicit. Also, it is imperative that A2.F-BF.A.1a and A2.F-BF.A.1b specifically direct to be applied both to mathematical and real-life situations. **Milgram**-.Far too general. Put in examples to limit it and show the kinds of questions that are meant to put into a test of this countent. **Achieve-**Clarification is needed regarding the intent of "exponential [functions] with integer exponents." F-BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.* AZ added detail to define differences between the two algebra courses. This is particularly true for exponential functions.Again, AZ uses "focus" and "extend" as the verbs for specifics in Alg 1 and Alg 2, respectively. These appear to be messages to the teacher as opposed to requirements for the students.While this is a modeling standard in the CCSS, it does not have the AZ connection to modeling in Alg 1. Typo: According to the ADSM Introduction the phrase should be, "utilizing a real-world context."	Per Milner and Achieve's comment, see the revised standard Per Milgram's comment, additional examples and guidance will be provided in supplemental resources	Write a function that describes a relationship between two quantities. Extend from linear, quadratic and exponential **functions** ~~with integer exponents~~ to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and ~~all~~ exponential functions. Include problem-solving opportunities utilizing real-world context. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine ~~standard~~ function types using arithmetic operations and function composition.
A2.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	What about sequences that are not arithmetic or geometric? Are those sequences only dealt with in the plus standards? This is still common core standard.	**Achieve**-AZ changes the wording slightly but the meaning is essentially the same. Sequences are introduced in Alg 1 in F-IF.3. Will AZ students make the connection between recognition of sequences and applying them with this distance between them? Would it make more sense to address F-BF.2 in Alg 1? Or to include an Alg 2 version of F-IF.3 in AZ? While this is an identified modeling standard in the CCSS, the AZ version removes the modeling indicator but does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	Per Achieve's comment, additional examples and guidance will be provided in supplemental materials. According to the definition of mathematical modeling on page 18 of the introduction, modeling is not necessary for this standard but could be included at a teacher's discretion. Our revision intentionally removed the modeling statement as it is not always applicable.	
A2.F-BF.B	**Build new functions from existing functions.**				

A2.F-BF.B.3	Identify the effect on the graph of replacing *f(x)* by *f(x) + k* , *k f(x)* , *f(kx)* , and *f(x+k)* for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Extend from linear, quadratic and exponential with integer exponents to include polynomial, radical, logarithmic, rational, piecewise-defined, sine, cosine, and exponential functions with real exponents.	This is still common core standard.	**Milgram**-Algebra 1 and 2 and the need to address a limit between the two courses. TO PUT IT MILDLY, YOU HAVE NOT SUCCEEDED. THIS NEEDS TO HAVE MAJOR CHANGES MADE TO IT. IT IS VALUABLE , BUT FAR FROM ESSENTIAL, FOR STUDENTS TO SEE HOW THE KINDS OF TRANSFORMATIONS IN THE ORIGINAL STANDARD AFFECT FUNCTIONS. BUT IN ALGEBRA II STUDENTS CAN BE ASKED TO USE THESE TRANSFORMATIONS TO UNDERSTAND SOME FUNCTIONS, PARTICULARLY QUADRATIC FUNCTIONS. **Achieve**-AZ provides more detail for the two algebra courses. It is not clear how exponential functions are being handled in the two courses. This needs clarity. See previous comments for more detail.	Per Milgram's and Achieve's comment, Algebra II extends the standard from Algebra I to include even and odd functions as well as horizontal dilations (after adjusting the Algebra I standard). See the revised standard.	Identify the effect on the graph of replacing *f(x) by f(x) + k* , *kf(x*), *f(kx)* , and f(x+k) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Extend from linear, quadratic and exponential **functions** ~~with integer exponents~~ to include polynomial, radical, logarithmic, rational, sine, cosine, ~~and~~ exponential **functions,** ~~with real exponents~~. and piecewise-defined functions.
A2.F-BF.B.4	Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions *f* and *g* are inverse functions if and only if *f(x)=y* and *g(y)=x* for all values of *x* in the domain of *f* and all values of *y* in the domain of *g* . b. Understand that if a function contains a point *(a,b)* , then the graph of the inverse relation of the function contains the point *(b,a)* ; the inverse is a reflection over the line *y = x* .	Appreciate the additional language and clarification Appreciate the additional language and clarification Still common core	**Milner**-In A2.F-BF.B.4 the inverse function is confused with its graph. Moreover, the deletion of the composition of a function with its inverse completely obscures the essential defining condition of inverse. **Achieve-**There is a problematic mathematical issue in part b. The statement, "the inverse is a reflection over the line y=x" will only be true if the x-axis and y-axis quantities mean the same thing simultaneously - which would never happen in context.See the article "Inverse Functions: What Our Teachers Didn't Tell Us" written by Arizona educators (Mathematics Teacher, March 2011). There is also a need to improve precision in part b in that a GRAPH of the function, and not the function itself, contains the point (a, b)..."	Per Achieve's comment, see the revised standard	Find inverse functions. a. Understand that an inverse function can be obtained by expressing the dependent variable of one function as the independent variable of another, recognizing that functions *f* and *g* are inverse functions if and only if *f(x)=y* and *g(y)=x* for all values of *x* in the domain of *f* and all values of *y* in the domain of *g* . b. Understand that if a function contains a point *(a,b)* , then the graph of the inverse relation of the function contains the point *(b,a).* ~~the inverse is a reflection over the line *y = x* .~~ **c. Interpret the meaning of and relationship between a function and its inverse utilizing real-world context.**
Linear, Quadratic, and Exponential Models (F-LE)					
A2.F-LE.A	**Construct and compare linear, quadratic, and exponential models and solve problems.**				

A2.F-LE.A.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a* , *c* , and *d* are numbers and the base *b* is 2, 10, or *e* ; evaluate the logarithm using technology.	This is still common core standard.	**Milner**-In A2.F-LE.A.4 it should be specified that technology should be used to evaluate logarithms that are not readily found by hand or observation. **Achieve-**While this is an identified modeling standard in the CCSS, the AZ version removes the modeling indicator but does not include the phrase, "utilizing a real-world context," per the ADSM Introduction (see page 18).	Per Milner's comment, see the revised standard Per Achieve's comment: According to the definition of mathematical modeling on page 18 of the introduction, modeling is not necessary for this standard but could be included at a teacher's discretion. Our revision intentionally removed the modeling statement as it is not always applicable.	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a* , *c* , and *d* are numbers and the base *b* is 2, 10, or *e* ; evaluate ~~the~~ **logarithms that are not readily found by hand or observation** using technology.
A2.F-LE.B	**Interpret expressions for functions in terms of the situation they model.**				
A2.F-LE.B.5	Interpret the parameters in an exponential function with real exponents in terms of a context.	Is this skill also stated in A2.F-IF.C.8 ?		Per the public comment, see the revised standard which emphasizes the real-world context	Interpret the parameters in an exponential function ~~with real exponents~~ **utilizing real-world context** ~~in terms of a context~~.

Trigonometric Functions (F-TF)						
A2.F-TF.A	**Extend the domain of trigonometric functions using the unit circle.**					
A2.F-TF.A.1	Understand radian measure of an angle as the length of the arc on any circle subtended by the angle, measured in units of the circle's radius.	This is still common core standard. **Geometry standard? **How do you test this understanding? What are students suppose to be able to do with their understanding. This statement needs a little clarification so teachers know what students should be able to do with the understanding.				
A2.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of sine and cosine functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**Why not have the students show that they can visually identify the location of both sine and cosine. This is more general and produces more meaning. **Measurability if this standard is initially unclear. Students are to explain. Once they can explain, what are they supposed to do with their understanding that they explained? This is still common core standard.				
A2.F-TF.B	**Model periodic phenomena with trigonometric functions.**					
A2.F-TF.B.5	Create and interpret trigonometric functions that model periodic phenomena with specified amplitude, frequency, and midline.	These Items will be covered in Trigonometry, and should not be covered in Algebra 2				
A2.F-TF.C	**Prove and apply trigonometric identities.**					**~~Prove and~~ Apply trigonometric identities.**

A2.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	This is still common core standard. **This should be moved to a fourth year mathematics course. It is my opinion that not every student in Arizona can be successful with this concept. **This should be moved to the plus standards. Not all students in the state of AZ will understand this type of proof at the level it deserves. Finding the sine, cosine, tangent, etc of angles in unit circle doesn't require this identity. Keep the skill but move the proof.		Per the public comment, the standard has been revised to emphasize using the Pythagorean identity to find other trigonometric values for a common angle.	~~Prove~~ **Use** the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ ~~and use it~~ to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
Statistics and Probabilty (S)					

Algebra 2

Interpreting Categorical and Quantitative Data (S-ID)					
A2.S-ID.A	**Summarize, represent, and interpret data on a single count or measurement variable.**				
A2.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	Appropriate standard for this level. This is still common core standard.	**Milner-**A2.S-ID.A.4 needs to be rephrased from "…properties of a normal distribution to approximate a normal curve…" to "…properties of a normal distribution to approximate the given data by a normal curve…" **Achieve**-There are several differences in these two standards. AZ elected not to offer suggestions for tools to use in estimating the area under the normal curve. They added the requirement of non-symmetric data and consideration of outliers. This appears to be more demanding than the CCSS counterpart.	The comment does not seem to match the standard - unsure how to address these comments	Use the mean and standard deviation of a data set to fit it to a normal curve, and use properties of the normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, or tables to estimate areas under the normal curve.
A2.S-ID.B	**Summarize, represent, and interpret data on two categorical and quantitative variables.**				
A2.S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or chooses a function suggested by the context.	This is still common core standard.	**Milner**-In A2.S-ID.B.6 "chooses" needs to be "choose".	See revised standard	Represent data **of** two quantitative variables on a scatter plot, and describe how the quantities are related. Extend to polynomial and exponential models. ~~a.~~ Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.
A2.S-ID.C	**Interpret models.**				

A2.S-ID.C.10	Interpret parameters of exponential models.	This is still common core standard. **Is this with regression? What parameters? **I don't know what this would look like in my classroom. This item needs more support material, such as a supplemental document with examples. I did a quick google search and could not find adequate information for this. We do not have textbooks at our school and there is no reference for this topic. **is this the same thing as A2.F-LE.B.5? Interpret the parameters in an exponential function with real exponents in terms of a context.		Yes - this standard is referring to exponential regressions. This is a progression from the Algebra I standards A1.S-ID.C.7 through A1.S-ID.C.9, which have students interpret linear models. In Algebra II, students are expected to extend this to exponential models.	

Making Inferences and Justifying Conclusions (S-IC)					
A2.S-IC.A	**Understand and evaluate random processes underlying statistical experiments.**				
A2.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	This is another example of a cluster in isolation. If statistical experiments are going to be included, the other standards should have been left in here. HS.S-IC.B.3 through HS.S-IC.B.6 Having just tow standards for a topic goes against the spirit of the standards as currently written. I think you should either remove A2.S-IC.A.1 and A.2 or insert the other statistics standards that were removed in this draft. This is still common core standard. **Understand is a very generic term and hard to measure. This statement can be interpreted to mean a lot or very little. Please break apart this standard a little bit to make it more clear as to what students need to know at this level.	**Milner**-The language chosen for A2.S-IC.A.1 is very poor: "making inferences to be made". CHANGE IT to good English usage.	Standards A2.S-IC.A.1 and A2.S-IC.A.2 provide the foundation for 4th year mathematics courses. The comment does not seem to match the standard - unsure how to address this comment	
A2.S-IC.A.2	Explain if a specified model is consistent with results from a given data-generating process	This is another example of a cluster in isolation. If statistical experiments are going to be included, the other standards should have been left in here. HS.S-IC.B.3 through HS.S-IC.B.6 Having just tow standards for a topic goes against the spirit of the standards as currently written. I think you should either remove A2.S-IC.A.1 and A.2 or insert the other statistics standards that were removed in this draft. This is still common core standard. **Need more support material.	**Achieve-**AZ changed "decide" to "explain," increasing the rigor. Typo: The "if" in the AZ version should be "how," "whether," or "why."	Standards A2.S-IC.A.1 and A2.S-IC.A.2 provide the foundation for 4th year mathematics courses. Per Achieve's comment, see the revised standard	Explain ~~if~~ **whether** a specified model is consistent with results from a given data-generating process
A2.S-IC.B	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**			Included in Algebra 2 based on Workgroup and higher education input.	**Make inferences and justify conclusions from ~~sample surveys,~~ experiments, and observational studies.**

A2.S-IC.B.3	**NEW**			Included in Algebra 2 based on Workgroup and higher education input.	Recognize the purposes of and differences **between designed experiments,** sample surveys, ~~experiments,~~ and observational studies. ~~explain how randomization relates to each.~~
A2.S-IC.B.4	**NEW**	Students should understand the notion of sampling variability--that different samples from the same population can give different estimates. Consider adding a reduced version of this standard to Algebra 2: "Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes."		NEW - added based on public comment	Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes.

Conditional Probability and the Rules of Probability (S-CP)					
A2.S-CP.A	**Understand independence and conditional probability and use them to interpret data.**				
A2.S-CP.A.3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	This is still common core standard.	**Achieve-**The time gap between S-CP.2 and S-CP.3 seems large.	Per Achieve's comment, S-CP.2 is securely held knowledge from Algebra I	
A2.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	This is still common core standard.		General Comment. No action necessary.	
A2.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.	This is still common core standard.			Recognize and explain the concepts of conditional probability and independence ~~in everyday language and everyday situations.~~ utilizing real-world context.
A2.S-CP.B	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**				
A2.S-CP.B.6	Find the conditional probability of *A* given *B* as the fraction of *B's* outcomes that also belong to *A* , and interpret the answer in terms of the model.	This is still common core standard.			**Use Bayes Rule to** find the conditional probability of *A* given *B* as the fraction of *B's* outcomes that also belong to *A* , and interpret the answer in terms of the model.

A2.S-CP.B.7	Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)* , and interpret the answer in terms of the model.	This is still common core standard.		General Comment. No action necessary.	
A2.S-CP.B.8		This is the last piece of conditional probability. Could this be taught in algebra 2?		Moved back from Plus Standards based on higher education input and public comment.	Apply the general Multiplication Rule in a uniform probability model, *P* (*A* and *B*) = *P(A)P(B*|*A*) = *P(B)P(A*|*B)* , and interpret the answer in terms of the model.

Algebra 2

SMP	Standards for Mathematical Practice				
A2.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, "Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				
A2.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

A2.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or				
A2.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				

A2.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
A2.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				
A2.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				

A2.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				

Coding	Draft Plus Standards - as of 8/2016	Public Comment- Fall 2016	Technical Review	Workgroup Notes	Redline/Final Mathematics Standard - 12/2016
High School - Plus Standards					
Number and Quantity -N					
The Complex Number System (N–CN)					
P.N-CN.A	**Perform arithmetic operations with complex numbers.**				
P.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Division is not mentioned here or in Algebra II. Should this be mentioned in both places for coherency and the ability to build on knowledge. Since dividing with complex numbers is not in algebra 2, it should be listed here.		Per the public comment, division is intentionally included as a plus standard and not as an Algebra II standard	
P.N-CN.B	**Represent complex numbers and their operations on the complex plane.**				
P.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form, including real and imaginary numbers, and explain why the rectangular and polar forms of a given complex number represent the same number.				
P.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.				Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.
P.N-CN.B.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				
P.N-CN.C	**Use complex numbers in polynomial identities and equations.**				**Use complex numbers in polynomial ~~identities and~~ equations.**
P.N-CN.C.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.				Extend polynomial identities to the complex numbers. ~~For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.~~

P.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	This should be a regular standard. This is probably the most important concept associated with polynomial functions and students should have an understanding of its relationship to the solutions to functions.			Per the public comment, this standard was intentionally left as a plus standard.
<u>**Vector and Matrix Quantities (N–VM)**</u>					
P.N-VM.A	**Represent and model with vector quantities.**				
P.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, **v**).				Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. ~~(e.g., v, |v|, ||v||, v).~~
P.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				
P.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.				
P.N-VN.B	**Perform operations on vectors.**				
P.N-VM.B.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v** – **w** as **v** + (–**w**), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				

P.N-VM.B.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(**vx, vy**) = (**cvx**, **cvy**). b. Compute the magnitude of a scalar multiple cv using ||**cv**|| = |**c**|**v**. Compute the direction of **cv** knowing that when |**c**|**v** ≠ 0, the direction of **cv** is either along **v** (for **c** > 0) or against **v** (for **c** < 0).				Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. e.g., as **c**(**vx, vy**) = (**cvx, cvy**). b. Compute the magnitude of a scalar multiple **cv** using ||**cv**|| = |**c**|**v**. Compute the direction of **cv** knowing that when |**c**|**v** ≠ 0, the direction of **cv** is either along **v** (for **c** > 0) or against **v** (for **c** < 0).

P.N-VM.C	**Perform operations on matrices and use matrices in applications.**				
P.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.				Use matrices to represent and manipulate data. ~~e.g., to represent payoffs or incidence relationships in a network.~~
P.N-VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.				Multiply matrices by scalars to produce new matrices. ~~e.g., as when all of the payoffs in a game are doubled.~~
P.N-VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.				
P.N-VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				
P.N-VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				
P.N-VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				
P.N-VM.C.12	Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.				
Algebra - A					
Arithmetic with Polynomials and Rational Expressions					
P.A-APR.C	**Use polynomial identities to solve problems.**				
P.A-APR.C.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.				
P.A-APR.D	**Rewrite rational expressions.**				

P.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.				

Reasoning with Equations and Inequalities (A-REI)					
P.A-REI.C	**Solve systems of equations.**				
P.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.				
P.A-REI.C.9	Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).				
Functions - F					
Interpreting Functions (F–IF)					
P.F-IF.C	**Analyze functions using different representations.**				
P.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing				
Building Functions (F-BF)					
P.F-BF.A	**Build a function that models a relationship between two quantities.**				
P.F-BF.A.1	Write a function that describes a relationship between two quantities. c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*				
P.F-BF.B	**Build new functions from existing functions.**				
P.F-BF.B.4	Find inverse functions. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.				
P.F-BF.B.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.				

Trigonometric Functions (F–TF)					
P.F-TF.A	**Extend the domain of trigonometric functions using the unit circle.**				
P.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.				
P.F-TF.A.4	Use the units circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				
P.F-TF.B	**Model periodic phenomena with trigonometric functions.**				
P.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	This has poor wording. It should deal with what values will the inverse function accept before deciding how to restrict the domain of the original function.			The public comment does not address the intent of the standard.
P.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.				Use inverse functions to solve trigonometric equations utlizing real world context,~~that arise in modeling contexts~~; evaluate the solution~~s using technology~~, and interpret. ~~them in terms of the context~~.
P.F-TF.C	**Apply trigonometric identities.**				
P.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				
Geometry - G					
Similarity, Right Triangles, and Trigonometry (G-SRT)					
P.G-SRT.D	**Apply trigonometry to general triangles.**				
P.G-SRT.D.9	Derive the formula $A = ½ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the				
P.G-SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.				

P.G-SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).				Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. ~~(e.g., surveying problems, resultant forces).~~

Circles G-C					
P.G-C.A	**Understand and apply theorems about circles.**				
P.G-C.A.4	Construct a tangent line from a point outside a given circle to the circle.				
Expressing Geometric Properties with Equations (G-GPE)					
P.G-GPE.A	**Translate between the geometric description and the equation for a conic section.**				
P.G-GPE.A.2	Derive the equation of a parabola givena focus and directrix.	happy to see this as a plus standard.			
P.G-GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.				
Geometric Measurement and Dimension (G-GMD)					
P.G-GMD.A	**Explain volume formulas and use them to solve problems.**				
P.G-GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.				

Statistics and Probability - S					
Making Inferences and Justifying Conclusions (S-IC)					
P.S-IC.B	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**				
P.S-IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	This is an extremely important standard and one every students in AZ should be expected to master. Understanding reports in the media require critical thinking--something that this standard helps encourage. It is also being assessed on the SAT and PSAT.		Based on Higher Education input and public comment, a similar standard was included in Algebra 2.	
P.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	Students should understand the notion of sampling variability--that different samples from the same population can give different estimates. Consider adding a reduced version of this standard to Algebra 2: "Use data from a sample survey to estimate a population mean or proportion; recognize that estimates are unlikely to be correct and the estimates will be more precise with larger sample sizes."		Based on Higher Education input and public comment, a similar standard was included in Algebra 2.	Use data from a **random** sample ~~survey~~ to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
P.S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	This standard is challenging, but worthwhile for students as it connects the analysis of an experiment with the design. It will help reinforce what students are learning in science as well.		This standard was purposefully moved to the plus standards based on a committee of experts.	
P.S-IC.B.6	Evaluate reports based on data.	There is no standard more necessary for survival in the 21st century that this one. The amount of data--and reports from data--is growing exponentially and students need to be equipped to understand what they read or hear.			
Conditional Probability and the Rules of Probability (S-CP)					
P.S-CP.B	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**				
P.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	This is the last piece of conditional probability. Could this be taught in algebra 2?		Based on Higher Education input and public comment, standard **moved back to Algebra 2**	~~Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.~~

P.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.				
Using Probability to Make Decisions (S-MD)					
P.S-MD.A	**Calculate expected values and use them to solve problems.**				
P.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.				
P.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.				
P.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.				Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated. find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
P.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?				Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically. find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
P.S-MD.B	**Use probability to evaluate outcomes of decisions.**				

P.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.				Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
P.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).				Use **randomization** ~~probabilities~~ to make fair decisions **based on probabilities**. ~~(e.g., drawing by lots, using a random number generator).~~
P.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).				Analyze decisions and strategies using probability concepts. ~~(e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).~~

Contemporary Mathematics - CM					
Discrete Mathematics - (CM-DM)					
P.CM-DM.A	**Understand and apply vertex-edge graph topics**				
P.CM-DM.A.1	Study the following topics related to vertex-edge graph: Euler circuits, Hamilton circuits, the Travelling Salesperson Problem (TSP), minimum weight spanning trees, shortest path, vertex coloring, and adjacency matrices.				
P.CM-DM.A.2	Understand, analyze, and apply vertex-edge graph to model and solve problems related to path, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.				
P.CM-DM.A.3	Devise, analyze, and apply algorithms for solving vertex-edge graph problems.				
P.CM-DM.A.4	Extend work with adjacency matrices for graph, such as interpreting row sums and using the nth power of the adjacency matrix to count path of length n in a graph.				

P.MP	Standards for Mathematical Practice				
P.MP.1	Make sense of problems and persevere in solving them. Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.				

P.MP.2	Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.				

P.MP.3	Construct viable arguments and critique the reasoning of others. Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming, questioning, or debating the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.				
P.MP.4	Model with mathematics. Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				

P.MP.5	Use appropriate tools strategically. Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful; recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.				
P.MP.6	Attend to precision. Mathematically proficient students clearly communicate to others and craft careful explanations to convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.				

P.MP.7	Look for and make use of structure. Mathematically proficient students use structure and patterns to provide form and stability when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.				
P.MP.8	Look for and express regularity in repeated reasoning. Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.				