Arizona's Draft Standards
Mathematics

Glossary

ARIZONA DEPARTMENT OF EDUCATION

HIGH ACADEMIC STANDARDS FOR STUDENTS
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Arizona Mathematics Standards – Glossary

The purpose of this glossary is to help the user better understand and implement the Arizona Mathematics Standards. The glossary is **not** a comprehensive list of all mathematics terms and properties experienced in the K-12 classroom. It is possible that two different definitions exist for a mathematical term or action. Please work with your educational site to decide on the common definition understood by learners. For assessment purposes, **a definition that is in bold in the glossary will be the accepted assessment definition of the term or property (i.e., trapezoid).** The definitions in the glossary are general in nature.

Addition	The operation of joining 2 or more quantities.
Algorithm	A set of instructions/steps used to solve a problem or obtain a desired result in every case.
Amount	Quantity, number of, total, sum, size or extent.
Array	A rectangular arrangement of objects or elements organized into rows and columns, or a set of objects or elements organized into a specific pattern.
Associative property of addition	See Table 4 in this Glossary.
Associative property of multiplication	See Table 4 in this Glossary.
Auxiliary line	A line or line segment added to a diagram to help in solving a problem or proving a concept. Usually found in geometry and is indicated by a dashed or dotted line to indicate they were not part of the original diagram.
Bivariate data	Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.
Box and whisker plot	A method of visually displaying distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.
Categorical variable	Non-numerical categories used to describe data (e.g., breed, color, cities).
Circle	Circles of positive radius r as all points in the plane having a fixed distance *r* from a given point *c*.
Clustering	Collections of data elements in a data display that are in close proximity relative to the context.
Commutative property	See Table 4 in this Glossary.
Complex fraction	A fraction that has a fractional numerator, denominator, or both (e.g., $\frac{\frac{7}{11x^2}}{31y^5x^6s}$).
Complex number	A number that can be written in the form *a + bi* where *a* and *b* are real numbers *a*nd *i* is an imaginary number.

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Composite figure	A geometric figure that is composed of two or more simple polygons.
Congruent	Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).
Conic section	The four curves - circles, ellipses, parabolas, and hyperbolas. They are called conic sections because they can be formed by intersecting a right circular cone with a plane. When the plane is perpendicular to the axis of the cone, the resulting intersection is a circle. When the plane is slightly tilted, the result is an ellipse. When the plane is parallel to the side (one element) of the cone, it produces a parabola. When the plane cuts both extensions of the cone, it yields a hyperbola.
Contextualize	To place (as a word or activity) in a context.
Decimal fraction	A fraction whose denominator is some power of 10, usually indicated by a decimal point written before the numerator (e.g., 0.4 = 4/10).
Decontextualize	To remove from a context.
Descriptive modeling	Describes the phenomena or summarizes it in a compact form (e.g., graphs of observations).
Dilation	A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.
Digit	Any of the numerals 1 to 9 and usually the symbol 0.
Dilation	A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. The resulting image is the same shape as the original, but is a different size.
Distributive property	See Table 4 in this Glossary.
Division	The operation of determining how many times one quantity is contained in another.
Equation	A mathematical sentence that uses the equal sign (=) to show that two expressions are equal.
Expression	A mathematical phrase that combines numbers and/or variables using mathematical operations.
Factor theorem	When x-c is a factor of the polynomial then $f(c)$=0
Fluency	Fluently means *efficiently, accurately, flexibly, and appropriately*. Students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently.[1] • Efficiency—carries out easily, keeps track of sub-problems, and makes use of intermediate results to solve the problem.

[1] National Council of Teachers of Mathematics, Inc. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA.

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	• Accuracy—reliably produces the correct answer. • Flexibility—knows more than one approach, chooses a viable strategy, and uses one method to solve and another method to double-check. • Appropriately—knows when to apply a particular procedure. Please see standards 2.OA.B.2 and 3.OA.C.7 for standards related to addition and subtraction of within 20 and multiplication within 100. Both of these standards should mastery involves "from memory" as an outcome. By the end of 2nd and 3rd grade, these procedural fluency standards should be automatic recall by students in their respective grades.
Focus	A fixed point from which all other points are equidistant.
Fraction	The quantity a/b formed by a parts of size $1/b$.
Function (algebraic)	A mathematical relation for which each element of the domain corresponds to exactly one element of the range.
Identity property of addition	See Table 4 in this Glossary.
Identity property of multiplication	See Table 4 in this Glossary.
Interquartile range	The difference between the lower 25% and the lower 75%, the range the middle 50% lies. Also called the midspread or middle fifty, is a measure of statistical dispersion, being equal to the difference between the upper and lower quartiles.
Irrational numbers	A set of real numbers that cannot be expressed as a ratio of two integers (e.g., $\pi, \sqrt{2}$).
Iteration	The repetition of a pattern or sequence.
Iterative pattern/ sequence	A pattern/sequence generated by using an initial value and repeatedly applying the same rule.
Mathematical argument	The justification of a particular solution, algorithm, or method using logic, evidence, and mathematically sound reasoning.
Mean absolute deviation	The average distance between each data value and the mean.
Negative association	A relationship in paired data in which one variable's values tend to increase when the other decreases, and vice-versa. In a scatterplot, data tend to follow a pattern from the upper left to the lower right.
Numeral	A symbol or mark used to represent a number.
Outlier	An observation that lies outside the overall pattern of a distribution.
Pattern	A set or sequence of figures or numbers that are repeated in a predictable manner.
Patterns of association	The association between two quantities described as clustering, outliers, positive or negative, linear and nonlinear.

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Problem solving	Mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development.
Properties of equality	See Table 5 in this Glossary.
Properties of inequality	See Table 6 in this Glossary.
Properties of operations	See Table 4 in this Glossary.
Quantity/quantities	A specified or indefinite number or amount.
Quantitative reasoning	The ability to apply mathematical concepts to the interpretation and analysis of quantifiable information, expressed numerically or graphically, in order to solve a wide range of problems.
Quantitative variable	Represents a measurable quantity and is numerical (e.g., population, height, weight).
Ratio	A comparison of two numbers or quantities (e.g., 4 to 7 or 4 : 7 or $4/7$).
Rational expression	The quotient of two polynomials in the form $\frac{A}{B}$, where *A* and *B* are polynomials.
Rational number	A number that can be written as a fraction or as the quotient of two numbers a/b where $b \neq 0$.
Reasoning (mathematical)	The justification of a particular solution, algorithm, or solution method using logical and mathematically sound arguments.
Recursion	An inherently repetitive process by which the terms of a sequence can be computed from some or all of the preceding terms by an algorithmic procedure.
Reflection	A type of transformation that flips points about a line, called the *line of reflection.* Taken together, the image and the pre-image have the line of reflection as a line of symmetry.
Relative frequency	Conditional relative frequency: the ratio of a joint relative frequency and related marginal relative frequency. Joint relative frequency: the ratio f the frequency in a particular category and the total number of data values. Marginal relative frequency: the ratio of the sum of the joint relative frequency in a row or column and the total number of data values.
Remainder Theorem	If $f(x)$ is a polynomial in x then the remainder on dividing $f(x)$ by $x - a$ is $f(a)$.
Representation	Verb: the act of capturing a mathematical concept in some form. Noun: the form expressing a mathematical concept (e.g., equation, graph, written description, sketch, table, construction, manipulative).

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Rigid motion	A transformation that maps points to points, lines to lines, line segments to line segments with the same length (and thus preserves the distances between two points and their image points), rays to rays, and angles to angles of the same measure.
Rotation	A transformation where a figure is turned about a fixed point.
Scale factor	A number that represents the ratio of any two corresponding lengths in two similar geometric figures. *Note: the ratio of areas of two similar figures is the square of the scale factor and the ratio of the volumes of two similar figures is the cube of the scale factor.*
Similarity	Two figures are similar if one can be mapped onto the other using a series of rigid motions and at least one dilation.
Size	How big or small something is, by dimension, value or magnitude.
Standard deviation	A measure used to quantify the amount of variation or dispersion of a set of data values.
Subtraction	The operation of taking the distance between two quantities.
System of equations	A set of two or more equations that must all be true for the same value(s) (note: also referred to as simultaneous equations).
Transformation	A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the *pre-image*) and the points in another geometric object (the *image*). Reflections, rotations, translations, and dilations are particular examples of transformations.
Translation	A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size.
Trapezoid[2]	**A quadrilateral with at least one pair of parallel sides.**
Unit fraction	A fraction $1/b$ formed by 1 part when a whole is partitioned into b equal parts, a fraction with a numerator of 1 and a denominator is a positive integer.
Value	The numerical worth or amount.
Zeros of a function	In mathematics, a **zero**, also sometimes called a root, of a real-, complex- or generally vector-valued **function** f is a member x of the domain of f such that $f(x)$ vanishes at x; that is, x is a solution of the equation $f(x) = 0$. In other words, a "**zero" of a function** is an input value that produces an output of **zero** (0).

[2] For assessment purposes, **a definition that is in bold in the glossary will be the accepted assessment definition of the term or property.**

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Table 1. Common Addition and Subtraction Problem Types/Situations.[1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put Together / Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean ***makes*** or ***results*** in but always does mean ***is the same quantity as***.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

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Table 2. Common Multiplication and Division Problem Types/Situations.[1]

	Unknown Product	**Group Size Unknown** ("How many in each group?" Division)	**Number of Groups Unknown** ("How many groups?" Division)
	3 x 6 = ?	**3 x ? = 18 and 18 ÷ 3 = ?**	**? x 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example*.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example*.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example*.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? ***Area example*.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? ***Area example*.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? ***Area example*.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A straw hat costs $6. A baseball hat costs 3 times as much as the straw hat. How much does the baseball hat cost? ***Measurement example***. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A baseball hat costs $18 and that is 3 times as much as a straw hat costs. How much does a straw hat cost? ***Measurement example*.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A baseball hat costs $18 and a straw hat costs $6. How many times as much does the baseball hat cost as the straw hat? ***Measurement example*.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

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Table 3. Fluency Expectations across All Grade Levels and Courses.

Grade	Coding	Fluency Expectations
K	**K.OA.A.5**	Fluently add and subtract through 5.
1	**1.OA.C.5**	Fluently add and subtract through 10.
2	**2.OA.B.2**	Fluently add and subtract through 20. By the end of 2nd grade, know from memory all sums of two one-digit numbers.
3	**3.NBT.A.3** **3.OA.C.7**	Fluently add and subtract through 100. Fluently multiply and divide through 100. By the end of 3rd grade, know from memory all multiplication products through 10 x 10 and division quotients when both the quotient and divisor are less than or equal to 10.
4	**4.NBT.B.4**	Fluently add and subtract multi-digit whole numbers using a standard algorithm.
5	**5.NBT.B.5**	Fluently multiply multi-digit whole numbers using a standard algorithm.
6	**6.NS.B.2** **6.NS.B.3** **6.EE.A.2**	Fluently divide multi-digit numbers using a standard algorithm. Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. Write, read, and evaluate algebraic expressions.
7	**7.NS.A.1.d** **7.NS.A.2.c** **7.EE.B.4.a**	Apply properties of operations as strategies to add and subtract rational numbers. Apply properties of operations as strategies to multiply and divide rational numbers. Fluently solve one-variable equations of the form $px + q = r$ and $p(x + q) = r$
8	**8.EE.C.7**	Fluently solve linear equations and inequalities in one variable.
Algebra 1	**A1.F-IF.C.7** **A1.A-SSE.A.2**	Graph functions expressed symbolically and show key features of the graph. Use structure to identify ways to rewrite numerical and polynomial expressions.
Geometry	**G.G-SRT.B.5** **G.G-GPE.B** **G.SRT.C.8**	Use congruence and similarity criteria to prove relationships in geometric figures and solve problems utilizing a real-world context. Use coordinates to prove simple geometric theorems algebraically. Use trigonometric ratios (including inverse trigonometric ratios) and the Pythagorean Theorem to find unknown measurements in right triangles utilizing real-world context.
Algebra 2	**A2.A-SSE.A.2** **A2.F-BF.B** **A2.A-REI.B.4**	Use the structure of an expression to identify ways to rewrite it. Build new functions from existing functions. Fluently solve quadratic equations in one variable.

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Table 4: The Properties of Operations	
Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the real number system.	
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Additive inverses	$a + (-a) = (-a) + a = 0$
Associate property of multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Commutative property of multiplication	$a \cdot b = b \cdot a$
Multiplicative identity property of 1	$a \cdot 1 = 1 \cdot a = a$
Multiplicative inverses	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
Distributive property of multiplication over addition	$a \cdot (b + c) = a \cdot b + a \cdot c$
Table 5: The Properties of Equality	
Here a, b and c stand for arbitrary numbers in the real or complex number systems.	
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$
Transitive property of equality	If $a = b$, and $b = c$, then $a = c$
Addition property of equality	If $a = b$, then $a + c = b + c$
Subtraction property of equality	If $a = b$, then $a - c = b - c$
Multiplication property of equality	If $a = b$, then $a \cdot c = b \cdot c$
Division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$
Substitution property of equality	If $a = b$ then b may be substituted for a in any expression containing a.
Table 6: The Properties of Inequality	
Here a, b and c stand for arbitrary numbers in the real number systems.	
Law of Trichotomy	Exactly one of these is true: $a < b$, $a = b$, or $a > b$,
Transitive Property of Inequality	If $a > b$ and $b > c$, then $a > c$
Reversal Property	If $a > b$, then $b < a$
Additive Inverse	If $a > b$, then $-a < -b$
Addition Property of Inequality	If $a > b$, then $a + c > b + c$
Subtraction Property of Inequality	If $a > b$, then $a - c > b - c$
Positive Multiplication Property of Inequality	If $a > b$ and $c > 0$, then $a \cdot c > b \cdot c$
Negative Multiplication Property of Inequality	If $a > b$ and $c < 0$, then $a \cdot c < b \cdot c$
Positive Division Property of Inequality	If $a > b$ and $c > 0$, then $> \frac{b}{c}$
Negative Division Property of Inequality	If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$